Vicarious Liability and the Intensity Principle

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Abstract

The present paper provides an economic analysis of vicarious liability that takes information rents and monitoring costs to be borne by the principal explicitly into account. In the presence of information rents or if the principal is wealth constrained herself, vicarious liability need not generate efficient precaution incentives. Rather, precaution incentives turn out to depend on the exact quantum of damages specified by courts. I shall compare incentives under three damages regimes: strict liability, the traditional negligence rule, and proportional liability. To do so, I make use of the intensity principle that allows to rank damages regimes based on the monotonicity of differences of the principal’s expected payoff as a function of induced precaution.

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1 Introduction

Employers and firms may be held responsible for the loss caused to third parties by their employees. The law of vicarious liability may also apply to hold the corporation liable for torts committed by its directors. The aims of inducing precaution may even support vicarious liability for government bodies.\(^1\)

Vicarious liability rests on the *respondeat superior* doctrine and means the strict liability of a principal for the misconduct of her agent whose activities she controls. Vicarious liability is in contrast to secondary liability imposed on the principal under a negligence rule where the principal may be exempted if she has selected and monitored her agent with sufficient care. Secondary liability is not the subject of the present paper.

Vicarious liability increases the level of care of a wealth constrained agent provided that the principal is able to monitor the agent’s precaution. Yet, to do so, the principal may have to bear monitoring costs.

The principal may also be able to contractually alter her agent’s precaution incentives. In fact, if the principal cannot directly monitor the agent’s level of care she may offer a bonus to the agent whenever the accident has been avoided. Yet, since the agent is wealth constrained, to generate precaution incentives in this way, the principal may have to grant an information rent in excess of her agent’s outside option. Similarly, in a setting of adverse selection where an agent’s type remains hidden to the principal, she may also have to leave an information rent to her agent.

The present paper provides an economic analysis of vicarious liability that takes information rents and monitoring costs explicitly into account. It does so under weak assumptions imposed on the classical accident model. The probability of an accident is assumed to be decreasing whereas total costs as perceived by the principal are assumed increasing with the agent’s precaution level. No further assumptions will be needed to establish the main results of the paper.

In the presence of information rents or if the principal’s wealth constraint also matters, vicarious liability need not generate efficient (first best) pre-

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\(^1\)For an illuminating survey on vicarious and corporate civil liability, the reader is referred to Kraakman (1999).
caution incentives. Rather, precaution incentives will depend on the exact specification of the quantum of damages awarded by courts. The problem is of second best nature. I shall compare incentives under three damages regimes: strict liability, the traditional negligence rule, and proportional liability.

To this end, I make use of what I shall refer to as the *intensity principle*. Suppose the difference of the principal’s expected payoff under legal regime $K$ as compared to regime $J$ is monotonically increasing with precaution. Then the principal has the incentive for inducing higher precaution under regime $K$ than under $J$, that is $K$ provides more intense incentives for precaution than $J$. While monotonicity of the difference is a sufficient condition only, nonetheless, it turns out to allow for ranking the different damages regimes quite generally. Moreover, the condition is easy to check.

On intuitive grounds, the intensity principle is a straightforward extension of the externality principle: Suppose the difference between social welfare and the principal’s payoff is increasing with precaution then precaution imposes a positive externality and, according to common wisdom, the principal has insufficient precaution incentives as compared to first best. The intensity principle rests on the same intuition.

In Schweizer (2009), I have compared different damages regimes for a setting where the principal takes the precaution decision herself. Whenever the compensation principle is met in the sense of the victim being fully compensated for deviations from the due care standard and if this standard maximizes the social surplus then the injurer has the incentive to meet the standard. Incentives, however, may be distorted if courts impose inefficient standards.

Under vicarious liability, there is a second source of distortions as the principal may have to grant an information rent to induce precaution. From a social perspective, information rents constitute mere redistribution. The principal, however, perceives them as costs. This discrepancy between private and social costs may further distort incentives. In a world of second best, the damages regime in place affects both the intensity and the efficiency of incentives.

As it turns out, at common due care standards, the negligence rule and proportional liability both provide (weakly) more intense precaution incen-
tives than strict liability. If the information rent is increasing in precaution then proportional liability provides, compared with first best, insufficient precaution incentives. At fixed standards, comparing the intensity of precaution incentives under proportional liability and the negligence rule remains ambiguous. Yet, if courts are aware of the second best problematic involved, by choosing due care standards suitably, they can implement the widest range of precaution under the negligence rule and, in this sense, the negligence rule performs best. Only if the principal does not have to leave information rents to her agent, the wider range of the negligence rule is of no use for improving incentives in terms of efficiency.

Moreover, in the presence of information rents, the incentives for the principal to form the relationship need not be excessive any more as claimed by some of the previous literature whereas, in the absence of information rents, these incentives are excessive though, compared with proportional liability, less so under the negligence rule.

Not all of the present paper’s results are new as there exists a substantial literature on the law and economics of vicarious liability to build on. Yet, to the best of my knowledge, the present paper seems first to provide an analysis based on a single principle while taking monitoring costs, information rents, the principal’s wealth constraint and due care standards that need not be second best into account.

Kornhauser (1982) and Sykes (1984) examine wealth constraints of the agent as the basic condition favoring vicarious liability. Sykes also discusses the allocation of risk between the principal and the agent. Information rents as a source of distortion, however, are not considered.

Kramer and Sykes (1987) discuss the effect of imposing vicarious liability on government bodies. According to their findings, a negligence-based liability rule might be preferred as compared to the strict liability under the respondeat superior doctrine. While some of my results might be of use for the analysis of government liability as well, details are not spelled out in the present paper.

Hansmann and Kraakman (1991) deal with liability imposed on the owners of a firm. The formal setting of the present paper seems general enough to cover this kind of liability as well.

Polinsky and Shavell (1993) point out that a culpable agent faces, at
most, the loss of his job and of his limited assets. They propose criminal liability for employees as their limited assets insulate them from contractual sanctions. The present paper concentrates on remedies under civil law but explores the combination of several features as mentioned above.

Shavell (2005), deals with minimum assets requirements as solution to the judgement-proof problem. The present paper, in contrast, explores the scope of stirring precaution by agents with a given wealth constraint.

Demougin and Fluet (1999) take information rents into account. Their analysis, however, is confined to strict liability versus the negligence rule. Moreover, they consider due care standards only that are first best.

Stremitzer and Tabbach (2009), finally, consider the case of a judgement-proof injurer and establish the superiority of proportional liability among a whole set of other regimes. While they do not examine vicarious liability, the finding of the present paper uncovers that it is the missing information rent that drives their result.

The present paper is organized as follows. Section 2 introduces the accident model and the three damages regimes. Section 3 makes use of the intensity principle to compare precaution incentives in a setting of moral hazard where the agent’s actual precaution may remain hidden to the principal. Section 4 extends the intensity principle to cases of adverse selection in order then to compare precaution incentives along similar lines. Section 5 concludes. In the appendix, finally, the required monotonicity of costs as perceived by the principal will explicitly be derived for a differentiable version of the moral hazard setting.

2 Damages rules

The general setting is as follows. Let $S$ denote the social surplus that the principal and her agent can produce by forming a relationship. The agent’s reservation payoff $u \geq 0$ is assumed equal to the social surplus that he would produce outside the relationship.

The activity of the principal-agent relationship is potentially harmful to society. The probability of an accident $\varepsilon(x)$ is a decreasing function of the agent’s precaution $x \in [0, \infty)$. At precaution $x$, the agent’s effort costs amount to $c(x) \geq 0$ which are strictly increasing in $x$. To ensure that the
principal’s and the agent’s maximum problems have solutions, both $\varepsilon(x)$ and $c(x)$ are assumed to be continuous functions of precaution.\(^2\)

The social loss in case of an accident is of fixed size $L$. Effort costs and loss are expressed in monetary equivalences. The above setting is commonly referred to as the accident model.

The following three damages rules will be compared. First, under strict liability (in the narrow sense), the injurer owes $D^S = L$ to the victim whenever an accident has occurred. Second, under the negligence rule with due care standard $x^o$, the injurer owes

$$D^N(x, x^o) = \begin{cases} L & \text{if } x < x^o \\ 0 & \text{else} \end{cases}$$

to the victim in case of an accident provided that precaution $x$ has been chosen.

Notice, the negligence rule can also be interpreted as strict liability but taking causality and the but-for test into account. Under such an interpretation, $x^o$ would follow from the preponderance standard.

As a third regime, I consider proportional liability at due care standard $x^o$. Under this regime, the injurer owes damages

$$D^P(x, x^o) = \begin{cases} \frac{\varepsilon(x) - \varepsilon(x^o)}{\varepsilon(x)} \cdot L & \text{if } x < x^o \\ 0 & \text{else} \end{cases}$$

to the victim.\(^3\) For notational convenience, let $D^S(x, x^o) = L$ denote damages under strict liability even if strict liability (in the narrow sense) is neither based on a due care level nor on actual precaution.

For all of the above damages regime $J \in \{S, N, P\}$, the victim’s expected payoff under precaution $x$ amounts to $-\varepsilon(x) \cdot L + \varepsilon(x) \cdot D^J(x, x^o)$ and can never be lower than under due precaution $x^o$. All damages rules that fully compensate the victim for deviations provide efficient precaution incentives for the injurer under the following two assumptions: (i) the due care level is

\(^2\)As the intensity principle does not make use of differentiability, the case where the set of feasible precautions is a finite subset of the real numbers could easily be handled along the same line. To simplify notation, however, the case of discrete precaution choice is not spelled out explicitly.

\(^3\)In Schweizer (2009), I have shown that proportional liability may be interpreted as correct expectation damages on average over the observed event.
fixed at its efficient level and (ii) the injurer is not wealth constrained. In fact, social surplus is highest under efficient precaution. Moreover, since the victim’s payoff is never below the one under efficient precaution, the residual must be highest under efficient precaution as well and, hence, the injurer has efficient precaution incentives indeed if the two assumptions (i) and (ii) are met.

If, however, one or both of the two assumptions are violated, precaution incentives may be distorted. In particular, distortions may arise if courts specify due care at inefficient levels, if the injurer is wealth constrained or if, under vicarious liability, the principal must pay information rents to her agent in order to induce precaution. In such cases, it well matters for efficiency which damages regime is in place as will be fully explored in the next section.

3 Comparing precaution incentives under different damages regimes

In the present section, it is assumed that the agent’s precaution decision may remain hidden to the principal. Moreover, the agent’s wealth amounts to $h \geq 0$ which limits his ability to pay.

Suppose the principal wants the agent to choose precaution $x$ but cannot directly control the agent’s decision. Rather, to detect the agent’s shirking with probability $p$, the principal must bear costs amounting to $G(p)$. Yet, even if shirking is detected, the principal cannot collect more than $h$ from her agent.

By offering a suitable bonus contract, the principal can generate additional incentives for her agent. Such a bonus contract offers a non-contingent payment $t$ to the agent and, on top of it, a bonus $b$ whenever the accident is avoided. Given the agent’s wealth constraint, such a bonus contract is feasible if $t + h \geq 0$ and $t + b + h \geq 0$ both hold.

This bonus contract induces precaution $x$ at rent $r$ if the following three conditions are met:

$$t + b \cdot (1 - \varepsilon(x)) - c(x) = u + r \quad (1)$$
\[ t + b \cdot (1 - \varepsilon(x')) - c(x') \leq u + r \]  
(2)

holds for all \( x' > x \) and

\[ -p \cdot h + (1 - p) \cdot [t + b \cdot (1 - \varepsilon(x'))] - c(x') \leq u + r \]  
(3)

holds for all \( x' < x \). In fact, by choosing precaution \( x \), the agent obtains the rent \( r \) in excess of his outside option and none of the available deviations leads to a higher payoff. Under this contract, the agent has the incentive to choose precaution \( x \) indeed.

For a given detection probability \( p \), let \( R(x, p) \) denote the minimum rent \( r \geq 0 \) at which precaution \( x \) can be induced by the principal. Under perfect monitoring \((p = 1)\), the principal can induce any precaution at zero rent. In fact, she just has to offer the fixed payment \( t = u + c(x) \) at zero bonus to her agent. If, however, monitoring is less than perfect \((p < 1)\), the minimum rent may well be positive. If it is the principal will offer a pure bonus contract which requires the agent, in case of an accident, to hand over his wealth to the principal \((t + h = 0)\) who, in turn, pays damages to the victim. This result will formally be established in the appendix.

If monitoring costs are not prohibitive, to induce precaution \( x \) at lowest costs, the principal monitors the agent as to detect his shirking with probability

\[ \pi(x) \in \arg \min_p R(x, p) + G(p). \]

Her monitoring costs \( \gamma(x) = G(\pi(x)) \) constitute costs also from the social perspective. The information rent \( \rho(x) = R(x, \pi(x)) \), however, is mere redistribution. Total costs as perceived by the agent amount to \( \kappa(x) = \gamma(x) + \rho(x) \) and are assumed continuous and increasing with precaution. In the appendix, these properties of \( \kappa(x) \) will be formally established. Throughout the main part of the paper, the following assumption is imposed.

**Assumption M:** If \( x < x' \) then \( \varepsilon(x) \geq \varepsilon(x') \), \( \kappa(x) \leq \kappa(x') \) and \( 0 \leq c(x) < c(x') \). Moreover, \( \lim_{x \to \infty} c(x) = \infty \).

This assumption ensures that the first best solution as well as the set of precautions the principal may possibly be willing to induce remain bounded.

The principal may be protected by limited liability or she may be wealth...
constrained herself. In any case, an upper bound $H(x) \geq 0$ is assumed to exist which limits damages claims. I have three interpretations in mind. First, the principal’s wealth constraint never binds in which case $H(x) \equiv L$. Second, the principal is protected by limited liability in which case $H(x) \equiv H_L < L$. Third, the principal’s wealth is not sufficient to compensate for the loss caused by an accident and she is not protected by limited liability. In this case, the upper limit $H(x) \leq L$ may interfere with the principal’s payments to the agent and, hence, may depend on the precaution $x$ that was actually induced by her. It is assumed, however, that the term

$$-\varepsilon(x) \cdot [L - H(x)]$$

remains (weakly) monotonically increasing in precaution.

Wealth is assumed non-divertible. As a consequence, the victim can actually collect damages amounting to

$$\min \left[ D^J(x, x^\alpha), H(x) \right]$$

if regime $J$ is in place.

Under the above assumptions, the principal has the incentive to induce precaution $x$ from the set

$$\arg \max_{x \in [0, \infty)} \phi^J(x, x^\alpha) = S - u - c(x) - \kappa(x) - \varepsilon(x) \cdot \min \left[ D^J(x, x^\alpha), H(x) \right].$$

In fact, the principal must bear monitoring costs $\gamma(x)$ and she must compensate for effort costs $c(x)$. Moreover, on top of the agent’s outside option $u$, she may have to cover information rent $\rho(x)$ to induce precaution $x$. Without loss of generality, it is assumed that the victim directly claims damages (5) from the principal who, in turn, may collect the agent’s contribution through the appropriate bonus contract. Notice, if the costs $\kappa(x)$ as perceived by the principal vanish then the above setting is equivalent to the one where no agent is involved as the principal takes the precaution decision herself. In this sense, the present setting contains judgement-proof injurers as a special case.

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$^4$The principal’s wealth is assumed sufficient to reimburse the agent as laid down in their contract.
Due to assumption M, under any regime $J$ from \{$N, P$\} truly based on a due care standard, the principal has no incentive to induce precaution in excess of the standard. For ease of comparison, precaution choice is artificially constrained to the range $[0, x^o]$ under strict liability $S$ as well.

For later reference, notice that the principal’s payoff under the proportionality regime amounts to

$$\phi^P(x, x^o) = \max[\varepsilon(x^o) \cdot L + \sigma(x) - \rho(x), \phi^S(x, x^o)]$$

as can easily be shown where

$$\sigma(x) = S - u - c(x) - \gamma(x) - \varepsilon(x) \cdot L$$

denotes social surplus in excess of the agent’s outside option $u$. Notice, monitoring costs are included but not the information rent because this rent constitutes mere redistribution and, hence, does not affect social welfare.\(^5\) Notice further that $\phi^N(x, x^o) = \phi^S(x, x^o)$ holds in the range $x < x^o$ whereas

$$\phi^N(x^o, x^o) = \phi^P(x^o, x^o) = \varepsilon(x^o) \cdot L + \sigma(x^o) - \rho(x^o)$$

if the standard is met.

To compare precaution incentives under different damages regimes, use of the following intensity principle is made. Damages regime $K$ is said to generate higher precaution incentives than regime $J$, for short $J(x^o) \leq K(x^o)$ if, for any $x^J$ that maximizes the principal’s payoff function $\phi^J(x, x^o)$ in the range $[0, x^o]$, there exists $x^K \geq x^J$ that maximizes the principal’s payoff function $\phi^K(x, x^o)$ in the same range. Notice, if the precautions that maximize the principal’s payoff functions happen to be unique, then the order relation $x^K \geq x^J$ necessarily must hold whereas if maximizers fail to be unique then, at least, there exists an order preserving selection from the set of maximizers.

The following proposition establishes that the monotonicity of the difference of the principal’s payoff function is sufficient for $J(x^o) \leq K(x^o)$ to hold.

**Proposition 1 (intensity principle)** If the difference of the principal’s expected payoff $\phi^K(x, x^o) - \phi^J(x, x^o)$ is increasing in the range $0 \leq x \leq x^o$ then regime $K$ generates higher precaution incentives at due care standard $x^o$ than regime $J$ in the sense that $J(x^o) \leq K(x^o)$ as defined above must hold.

\(^5\)Notice, from the social perspective, the relationship between the principal and the agent operating at precaution $x$ would be desirable if $\sigma(x) > 0$ holds.
Proof. Suppose precaution \( x^J \) maximizes \( \phi^J(x, x^o) \) in the range \([0, x^o] \) and take any precaution \( x < x^J \). Then

\[
\phi^K(x, x^o) \leq \phi^K(x^J, x^o) - [\phi^J(x^J, x^o) - \phi^J(x, x^o)] \leq \phi^K(x^J, x^o)
\]

as follows from the assumed monotonicity of the difference of the two payoff functions. It follows that the principal’s payoff at precaution \( x^J \) but under regime \( K \) is at least as high as under any lower precaution and, hence, a (weakly) higher precaution must exist in the range \([x^J, x^o]\) at which \( \phi^K(x, x^o) \) attains its maximum in the range \([0, x^o]\). □

The intensity principle is a straightforward extension of the externality principle. In fact, if the difference \( \sigma(x) - \phi^J(x, x^o) \) is increasing in the range \([0, x^o]\) then precaution generates a positive externality to the rest of the world and, in accordance with the externality principle, regime \( J \) provides insufficient incentives from the social perspective. The intensity principle is a straightforward extension of this externality principle.

To make use of the intensity principle, notice that the differences

\[
\phi^N(x, x^o) - \phi^S(x, x^o) \quad \text{and} \quad \phi^P(x, x^o) - \phi^S(x, x^o)
\]

are both monotonically increasing in the range \( 0 \leq x \leq x^o \). While the first claim is obviously true, the second follows from (6) and the assumed monotonicity of the term (4).

Similarly, let me consider the payoff function \( \phi^\Delta(x) = \sigma(x) - \rho(x) \). While this function is not meant to reflect any damages regime, the monotonicity of the following difference

\[
\phi^\Delta(x) - \phi^P(x, x^o) = \min \left[ -\varepsilon(x^o) \cdot L, -\varepsilon(x) \cdot (L - H(x)) \right]
\]  

(7)

(which holds due to the assumed monotonicity of (4)) proves convenient nonetheless. In fact, if the information rent is monotonically increasing then, as follows from (7), precaution under the proportionality rule generates a positive externality. As a consequence, for any precaution \( x^P \) which maximizes the principal’s payoff function \( \phi^P(x, x^o) \), there exists \( x^{FB} \geq x^P \) which maximizes social surplus in the range \([0, x^o]\) or, for short, \( P(x^o) \leq FB(x^o) \) where \( FB \) refers to first best.

Based on the monotonicity of the above differences, the intensity principle immediately leads to the following conclusions.
Proposition 2  At any due care standard \( x^o \), the following intensity relations hold:

1. \( S(x^o) \leq N(x^o) \) and \( S(x^o) \leq P(x^o) \leq \Delta(x^o) \)

2. If, under the negligence rule \( N \), it is not optimal for the principal to meet the due care level then
   \[ S(x^o) = N(x^o) \leq P(x^o) \]

3. If, under the negligence rule \( N \), it is optimal for the principal to meet the due care level then
   \[ \operatorname{arg~max}_x \phi^P(x, x^o) = \operatorname{arg~max}_{x \leq x^o} \sigma(x) - \rho(x) \]

4. If the information rent \( \rho(x) \) is monotonically increasing then \( \Delta(x^o) \leq FB(x^o) \).

Proof. Claim (1) directly follows from the intensity principle. Since
\[ \phi^N(x, x^o) = \phi^S(x) \]
holds for any \( x < x^o \), claim (2) must obviously be true.

As for claim (3), since
\[ \phi^N(x, x^o) \leq \phi^N(x^o, x^o) = \varepsilon(x^o) \cdot L + \sigma(x^o) - \rho(x^o) = \phi^P(x^o, x^o) \]
holds for all \( x \leq x^o \), it follows from (6) that
\[ \operatorname{arg~max}_x \phi^P(x, x^o) = \operatorname{arg~max}_{x \leq x^o} \varepsilon(x^o) \cdot L + \sigma(x) - \rho(x) = \operatorname{arg~max}_{x \leq x^o} \sigma(x) - \rho(x) \]
must hold indeed.

Claim (4), finally, directly follows from the monotonicity of \( \sigma(x) - \phi^\Delta(x, x^o) = \rho(x) \).

The proposition uncovers new insights but it also generalizes known yet disparate results under the unifying frame of the intensity principle.

First, if no information rent is needed to induce precaution (that is \( \rho(x) \equiv 0 \)) then either proportional liability performs as well as first best constrained to the range \([0, x^o]\), i.e. \( P(x^o) = FB(x^o) \) or
\[ S(x^o) \leq P(x^o) \leq FB(x^o) \]
must hold. This result generalizes Kahan’s (1989) findings that proportional liability outperforms the negligence rule even if courts were known to impose inefficient standards. Notice, Kahan’s analysis did not take wealth constraints into account.
Second, still in the absence of information rents, proportional liability outperforms the negligence rule not just in terms of intensity of incentives, but also in terms of efficiency provided that the social surplus is a concave function of precaution. Such superiority of proportional liability has been established before by Stremitzer and Tabbach (2009) but for the case of a single, judgement-proof injurer only.

If, however, the information rent is strictly increasing then the superiority of proportional liability can no longer be taken for granted. In fact, due to the increasing rent, proportional liability provides precaution incentives below the first best level and, for that reason, the more intense precaution incentives possibly generated by the negligence rule may become socially desirable. This result generalizes the findings of Demougin and Fluet (1999) who have shown that, due to the information rent, strict liability leads to underprovision of care.

The above proposition rests on the corresponding differences of payoff functions being monotonically increasing in precaution. But some of the payoff functions can also be ranked in terms of value. In fact, for any precaution $x$ from the range $[0, x^o]$ the following order relations are easily seen to hold:

$$\phi^N(x, x^o) \geq \phi^S(x, x^o) \geq \sigma(x) - \rho(x)$$  \hspace{1cm} (8)

and, as follows from (6),

$$\phi^P(x, x^o) \geq \phi^S(x, x^o) \text{ and } \phi^P(x, x^o) \geq \phi^N(x, x^o)$$  \hspace{1cm} (9)

These comparisons allow to rank damages regimes from the principal’s perspective.

**Proposition 3** At any fixed due care level $x^o$, the principal prefers the negligence rule over strict liability, the latter constrained to the range $[0, x^o]$. Moreover, she prefers proportional liability over both the negligence rule and strict liability. Finally, in the absence of information rents, all four damages rules generate excessive incentives to form the relationship.

**Proof.** Suppose precaution $x^j$ maximizes the principal’s payoff function in the range $[0, x^o]$ for $j \in \{J, K\}$ and $\phi^K(x, x^o) \geq \phi^j(x, x^o)$ holds for all
precautions from the same range then, of course,

\[ \phi^K(x^K, x^o) \geq \phi^K(x^J, x^o) \geq \phi^J(x^J, x^o) \]

must also hold and the principal prefers damages regime \( K \) over regime \( J \) indeed.

Similarly, in the absence of information rents, \( \phi^S(x, x^o) \geq \sigma(x) \) holds in the range \([0, x^o]\). Therefore, under strict liability constrained to this range, the principal’s maximum payoff (weakly) exceeds the social surplus from running the activity even at the first best precaution from that range. As a consequence, parameter configurations exist where the principal’s payoff is positive whereas the social surplus is negative. As the principal’s payoff under the other damages regimes exceeds those under strict liability (constrained to the range \([0, x^o]\)), the principal’s incentives to form the relationship is even higher. This establishes the final claim of the proposition. ■

So far, the intensity of precaution incentives has been compared for a given due care standard. Yet, if courts take the second best nature of vicarious liability into account, they may adapt the due care level to the damages regime in place. The remaining part of the present section is devoted to examining the ranges of precautions that can be implemented under various damages regimes if courts define the due care standard suitably.

Recall, for liability rules \( N \) and \( P \) that are truly based on a due care level, the principal will never induce precaution in excess of that level. Moreover, due to assumption M, there exists a finite upper bound \( x_M \) for all precautions that qualify for first best or would be induced under strict liability \( S \) (in the narrow sense). The following proposition establishes that the negligence rule allows implementing the widest range of precaution levels among all of the above damages regimes.

**Proposition 4**

1. If \( x^S \) maximizes the principal’s payoff under strict liability \( S \) then \( x^S \) also maximizes the principal’s payoff under proportional liability at due care standard \( x^o = x^S \)

2. If \( x^o \) maximizes the principal’s payoff under proportional liability \( P \) at due care standard \( x^o \) then \( x^o \) also maximizes the principal’s payoff under the negligence rule \( N \) at due care standard \( x^o \)

3. If \( x^P < x^o \) maximizes the principal’s payoff under proportional liability
P at due care standard \( x^o \) then \( x^p \) maximizes the principal’s payoff under the negligence rule \( N \) either at due care standard \( x^p \) or at due care standard \( x^o \).

**Proof.** To simplify notation, let

\[
X^J(x^o) = \arg \max_{x \leq x^o} \phi^J(x, x^o)
\]

denote the set of precautions that maximize the principal’s payoff at due care level \( x^o \) if damages regime \( J \) is in place.

As for the first claim, take any \( x^S \in X^S(\infty) = X^S(x_M) \). Since \( x^S \in X^S(x^S) \) must also hold and since \( \phi^P(x, x^S) - \phi^S(x, x^S) \) is monotonically increasing in the range \([0, x^S]\) it follows that \( x^S \in X^P(x^S) \) must hold indeed. The first claim is established.

As for the second claim, suppose \( x^o \notin X^N(x^o) \). Then there exists \( x^N < x^o \) such that

\[
\phi^P(x^N, x^o) \geq \phi^N(x^N, x^o) > \phi^N(x^o, x^o) = \phi^P(x^o, x^o)
\]

and, hence, \( x^o \notin X^P(x^o) \). This establishes the second claim.

To establish the third claim, two subcases are distinguished. Recall (6) and suppose, first, that

\[
\phi^P(x^P, x^o) = \varepsilon(x^o) \cdot L + \sigma(x^P) - \rho(x^P)
\]

is valid. It follows that

\[
\varepsilon(x^o) \cdot L + \sigma(x^P) - \rho(x^P) \geq \varepsilon(x^o) \cdot L + \sigma(x) - \rho(x)
\]

and, hence,

\[
\varepsilon(x^P) \cdot L + \sigma(x^P) - \rho(x^P) \geq \varepsilon(x^P) \cdot L + \sigma(x) - \rho(x)
\]

holds for all \( x \leq x^o \). Moreover,

\[
\varepsilon(x^P) \cdot L + \sigma(x^P) - \rho(x^P) \geq \varepsilon(x^o) \cdot L + \sigma(x^P) - \rho(x^P) \geq \phi^S(x)
\]

also holds for all \( x \leq x^o \) from which it follows that \( x^P \in X^P(x^P) \) and, hence, from claim 2 that \( x^P \in X^N(x^P) \). This establishes claim 3 under the first constellation.

Suppose, second, that

\[
\phi^P(x^P, x^o) = \phi^S(x^P)
\]
is valid. It then follows that

\[ \phi^P(x^P, x^o) = \phi^S(x^P) \geq \phi^S(x) \]

and

\[ \phi^P(x^P, x^o) = \phi^S(x^P) \geq \phi^P(x^o, x^o) = \phi^N(x^o, x^o) \]

and, hence, that

\[ \phi^N(x^P, x^o) = \phi^S(x^P) \geq \phi^N(x, x^o) \]

holds for all \( x \leq x^o \). Therefore, \( x^P \in X^N(x^o) \) and claim 3 is fully established.

\[ \square \]

Let \( x^\Delta \) be the (largest) precaution that maximizes payoff function \( \phi^\Delta(x) \). No precaution in excess of \( x^\Delta \) can be implemented under proportional liability. Yet, by raising the due care standard \( x^o \) slightly above \( x^\Delta \), due to the discontinuity of the principal’s payoff function, this standard would still be kept under the negligence rule. Moreover, if the information rent is an increasing function of precaution, it would be socially desirable to raise the induced precaution beyond \( x^\Delta \). This finding provides yet another justification of the negligence rule based on efficiency considerations.

### 4 Adverse selection and the intensity principle

In the present section, the intensity principle is extended to the following setting of adverse selection. The principal expects the agent she faces to be of type \( i = 1, \ldots, n \) with probability \( f_i \) where \( f_1 + \ldots + f_n = 1 \). The agent knows his type. All types choose precaution from the non-negative real line. At precaution \( x_i \), the accident involving a social loss \( L \) of fixed size occurs with probability \( \varepsilon(x_i) \). While an agent’s type is his private information, precaution is assumed observable if adverse selection is involved.

The social surplus \( S \) that is generated by the relationship between the principal and her agent as well as the reservation payoff \( u \) do not depend on the agent’s type. The effort costs \( c_i(x_i) \), however, are type-contingent. Monotonicity assumptions are imposed as before. Moreover, a single-crossing property is also assumed to hold. Type-contingent precaution profiles are denoted by \( x = (x_1, \ldots, x_n) \in X = [0, \infty)^n \) whereas one-dimensional real
variables are denoted by $x_i$ and, where appropriate, by $y \in [0, \infty)$. The assumptions of the present section can then be summarized as follows.

**Assumptions:** Suppose $y < y'$ then

(M): $\varepsilon(y) \geq \varepsilon(y')$ and $c_i(y) < c_i(y')$. Moreover, $\lim_{y \to \infty} c_i(y) = \infty$ holds for each type.

(SCP): $0 \leq c_i(y) - c_{i+1}(y) < c_i(y') - c_{i+1}(y')$ holds for $i = 1, \ldots, n-1$.

Suppose the principal wants to induce type-contingent precaution $x \in X$. Without loss of generality (revelation principle), she may do so by offering an incentive compatible contract

$$[(x_1, t_1), \ldots, (x_n, t_n)]$$

which directly asks the agent for his type. To ensure truthful revelation, incentive compatibility constraints must be met. Due to assumption (SCP) it is enough to check for the local constraints:

$$R_i = t_i - c_i(x_i) \geq t_{i+1} - c_i(x_{i+1})$$

and

$$R_{i+1} = t_{i+1} - c_{i+1}(x_{i+1}) \geq t_i - c_{i+1}(x_i)$$

The first constraint (upwards) requires that an agent of type $i$ cannot gain from pretending to be of type $i + 1$ whereas the second constraint (downwards) requires that an agent of type $i + 1$ cannot gain from reporting type $i$. Given such an incentive compatible contract, the principal pays an expected information rent $r = f_1 \cdot (R_1 - u) + \ldots + f_n \cdot (R_n - u)$ to the agent. Let $\rho(x)$ denote the minimum expected information rent at which the principal can induce type-contingent precaution $x \in X$. The following proposition rests on the single-crossing property and is well-known from the literature.

**Proposition 5**

1. If the contract $[(x_1, t_1), \ldots, (x_n, t_n)]$ is incentive compatible, then $x_i \leq x_{i+1}$, $R_i \leq R_{i+1}$ and $t_i \leq t_{i+1}$ hold for $i = 1, \ldots, n-1$

2. If $x_i \leq x_{i+1}$ holds for $i = 1, \ldots, n-1$ then there exist payments $t_1, \ldots, t_n$ such that the contract $[(x_1, t_1), \ldots, (x_n, t_n)]$ is incentive compatible

3. The expected information rent is additively separable, that is $\rho(x) = \sum_{i=1}^{n} f_i \cdot \rho_i(x_i)$ where $\rho_n(x_n) = 0$ and $\rho_i(x_i)$ strictly increasing in $x_i$ for $i = 1, \ldots, n-1$. 17
**Proof.** For convenience, let me briefly recall the main steps of the argument. It follows from the local incentive compatibility constraints that

\[ 0 \leq c_i(x_i) - c_{i+1}(x_i) \leq R_{i+1} - R_i \leq c_i(x_{i+1}) - c_{i+1}(x_{i+1}) \]

holds for \( i = 1, \ldots, n - 1 \) and, by the single-crossing property, that the type-contingent precaution must be monotonically increasing with the type.

To minimize the expected information rent, the principal chooses an incentive compatible contract such that the downwards constraints are binding, that is \( R_i = u \) and \( c_i(x_i) - c_{i+1}(x_i) = R_{i+1} - R_i \) holds for \( i = 1, \ldots, n - 1 \). By solving recursively and by rearranging terms, it follows that

\[ f_i \cdot \rho_i(x_i) = \left[ \sum_{k=i}^{n-1} f_{k+1} \right] \cdot [c_i(x_i) - c_{i+1}(x_i)] \]

and, by making use of the single-crossing property, that \( \rho_i(x_i) \) must be increasing indeed. ■

Notice, the minimum rent \( R_i \) as constructed in the above proof depends on all precaution levels of lower types, i.e. \( R_i = R_i(x_1, \ldots, x_{i-1}) \). Therefore, if the principal were wealth constrained her ability to cover damages may also depend on all these precaution levels. Yet, to avoid difficulties that would arise from such dependence, let me assume that the principal’s wealth constraint is never binding. She may, however, by protected by limited liability. The upper bound \( H_i(x_i) \leq L \) may actually depend on the agent’s type and the precaution chosen by that type but not on the precaution potentially chosen by other types.

It follows from the above proposition that type-contingent precautions \( x \in X \) can be induced if and only if they are increasing in type. Let \( X^m = \{ x \in X : x_i \leq x_{i+1} \text{ for } i = 1, \ldots, n - 1 \} \) denote the set of such monotonic type-contingent precautions.

The same damages regimes are considered as in section 3 but the due care levels \( x^o = (x_1^o, \ldots, x_n^o) \in X \) may now also be type-contingent. Let me assume, however, that \( x^o \) is monotonic because, otherwise it would be impossible for the principal to meet the due care standards, irrespective of which type her agent happens to be.

The expected social surplus \( \sigma(x) \) in excess of \( u \) amounts to

\[ \sigma(x) = \sum_{i=1}^{n} f_i \cdot \sigma_i(x_i) = \sum_{i=1}^{n} f_i \cdot [S - u - c_i(x_i) - \varepsilon(x_i) \cdot L] \]

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and, hence, is additively separable. Due to assumption (SCP), the difference 
\[ \sigma_{i+1}(y) - \sigma_i(y) = c_i(y) - c_{i+1}(y) \]
is strictly monotonically increasing. Therefore, the first best precaution from the range \( x \leq x^o \) has to be monotonic as the following lemma establishes.\(^6\)

**Lemma 1** If \( x^{FB} \in \arg \max_{x \leq x^o} \sigma(x) \) then \( x^{FB} \in X^m \).

**Proof.** Assume the contrary such that there exists a type \( i \) for which \( x^{FB}_{i+1} < x^{FB}_i \leq x^o_i \) holds. It then follows from the single crossing property that \( \sigma_{i+1}(x^{FB}_{i+1}) - \sigma_i(x^{FB}_{i+1}) < \sigma_{i+1}(x^{FB}_i) - \sigma_i(x^{FB}_i) \) and, hence, that \( \sigma_{i+1}(x^{FB}_{i+1}) < \sigma_{i+1}(x^{FB}_i) - [\sigma_i(x^{FB}_i) - \sigma_i(x^{FB}_i)] \leq \sigma_{i+1}(x^{FB}_i) \), a contradiction to the optimality of \( x^{FB}_{i+1} \) for type \( i+1 \).

Under damages regime \( J \in \{ S, N, P \} \), the injurer owes damages \( D^J(x_i, x^o_i) \) to the victim. Due to the possibly limited liability of the principal, the victim actually recovers \( \min[D^J(x_i, x^o_i), H_i(x_i)] \) only if the agent happens to be of type \( i \) and an accident has occurred. As in the previous section, the terms \( -\varepsilon(x_i) \cdot [L - H_i(x_i)] \) are assumed (weakly) monotonically increasing in precaution. The principal’s expected payoff amounts to

\[
\phi^J(x, x^o) = \sum_{i=1}^{n} f_i \cdot \phi^J_i(x_i, x^o_i)
\]

where

\[
\phi^J_i(x_i, x^o_i) = S - u - c_i(x_i) - \rho_i(x_i) - \varepsilon(x_i) \cdot \min[D^J(x_i, x^o_i), H_i(x_i)]
\]

if damages regime \( J \) is in place and type-contingent precautions \( x \in X^m \) have been induced. Notice, this payoff is additively separable as well. Under proportional liability,

\[
\phi^P_i(x_i, x^o_i) = \max[\varepsilon(x^o_i) \cdot L + \sigma_i(x_i) - \rho_i(x_i), \phi^S(x_i, x^o_i)]
\]

holds for the same reason as in the previous section (see (6)).

Under damages regimes \( J \in \{ N, P \} \) truly based on a due care standard, the principal has no incentives to induce precaution in excess of the due care standard as the following lemma establishes.

---

\(^6\)For \( x, x' \in X = [0, \infty)^n, x \leq x' \) is defined to hold if it holds type by type, that is if \( x_i \leq x'_i \) holds for \( i = 1, ..., n \).
Lemma 2 For damages regimes $J$ truly based on a due care standard, if $x^J \in \arg \max_{x \in X^m} \phi^J(x, x^o)$ then $x^J \leq x^o$ holds type by type.

Proof. If $x^J \leq x^o$ then we are done. Otherwise, let $g$ denote the lowest type, for which $x^o_g < x^J_g$ holds. By lowering $x^J_g$ to $x^o_g$, the principal’s payoff would strictly increase, a contradiction to the assumed optimality of $x^J$. ■

Again for ease of comparison, precaution choice is artificially constrained to the range $x \leq x^o$ (type by type) under strict liability $J = S$ and if first best is at stake.

Comparing the intensity of precautions under the three damages regimes proceeds now along similar lines as in the previous section. At due care levels $x^o \in X^m$, damages regime $K$ is said to generate higher precaution incentives than regime $J$ or, for short, $J(x^o) \leq K(x^o)$ if, for any $x^J \in X^m$ that maximizes the principal’s payoff under damages regime $J$ in the range $x \leq x^o$, there exists $x^K \in X^m$ that maximizes the principal’s payoff under regime $K$ in the same range and such that $x^J \leq x^K$ holds type by type. The following proposition extends the intensity principle to the setting of adverse selection.

Proposition 6 (intensity principle under adverse selection) If the difference of the principal’s expected payoff $\phi^K_i(x_i, x^o) - \phi^J_i(x_i, x^o)$ is increasing in the range $0 \leq x_i \leq x^o_i$ then regime $K$ generates higher precaution incentives at due care standard $x^o$ than regime $J$, that is $J(x^o) \leq K(x^o)$ must hold.

Proof. Suppose $x^J \in X^m$ maximizes the principal’s payoff under damages regime $J$ whereas $x^K \in X^m$ maximizes the principal’s payoff under regime $K$ in the range $x \leq x^o$. If $x^J \leq x^K$ holds type by type then we are done.

Otherwise, let $g$ be the highest index for which $x^K_g < x^J_g$. By increasing $x^K_g$ to $x^J_g$, due to the monotonicity of the difference, the principal’s payoff under rule $K$ cannot decrease. To fully establish the proposition, the above procedure must possibly be repeated but, after finitely many repetitions, an optimizer under rule $K$ is found in the range between $x^J$ and $x^o$, type by type. ■

By making use of the above version of the intensity principle, Proposition 2 can now easily be extended to cover the case of adverse selection. In
particular, since the expected information rent in the present section has been shown to be increasing the following version of the proposition holds.

**Proposition 7** For any possibly type-contingent due care standard \( x^o \in X^m \), the following intensity relations hold:

\[
S(x^o) \leq N(x^o) \quad \text{and} \quad S(x^o) \leq P(x^o) \leq FB(x^o)
\]

Demougin and Fluet (1999) have shown that, in the presence of information rents, strict liability leads to underprovision of care and the negligence rule dominates strict liability if the due care level is defined at its first best level. The above proposition extends their findings by taking further damages regimes into account and by allowing for type-contingent due care standards other than first best. In fact, due to the second best problem involved, incentives at lower standards may lead closer to efficient precaution.

The order relations (8) and (9) and, hence, Proposition 3 based on them also remain valid in the present setting of adverse selection. Proposition 4 (1) continues to hold as well. The remaining claims of Proposition 4, however, do not generalize easily if the principal is bindingly protected by limited liability. Yet, if she is not, the negligence rule is easily seen to still allow for the widest range of precautions that can be implemented by courts.

In fact, if the principal’s wealth constraint never binds then her payoff under proportional liability simplifies to

\[
\phi^P(x, x^o) = \left[ \sum_{i=1}^{n} f_i \cdot \varepsilon(x^o_i) \right] \cdot L + \phi^S(x)
\]

as follows from (10). Suppose now that precaution \( x^P \in X \) maximizes the principal’s expected payoff under proportional liability with possibly type-contingent due care levels \( x^o \in X \). It then follows from lemma 2 above that \( x^P \leq x^o \) holds type by type and, hence, \( x^P \) must maximize the principal’s expected payoff also under strict liability artificially constrained by \( x^o \) and, a fortiori, by \( x^P \). As the negligence rule provides (weakly) stronger precaution incentives than strict liability (see Proposition 7 above), it follows that \( x^P \) has to maximize the principal’s expected payoff under the negligence rule as well provided that type-contingent precaution standards \( x^P \) are imposed by courts. In this sense, the negligence rule still offers the widest range of implementable precautions, even if adverse selection is involved.


5 Concluding remarks

This paper examines vicarious liability in a setting that takes monitoring costs, information rents and wealth constraints of the agent but also of his principal into account. Given the second best problematic involved, the exact specification of damages matters. While the negligence rule allows for the widest range of implementable precaution levels, it may no longer be optimal for courts to impose precaution standards at their efficient (first best) level.

All findings of the present paper directly follow from the intensity principle. To be sure, a damages regime may possibly provide more intense precaution incentives than another one even if the difference in the principal’s payoff functions is not monotonically increasing in the whole range of definition. But in the setting of the present paper, monotonicity if the difference follows from the monotonicity of the probability of an accident and the costs as perceived by the principal as a function of precaution quite generally. Neither differentiability nor concavity as imposed by most of the related literature is needed here.

I have checked many other results from the economic analysis of tort and contract law to find out that, whenever comparing incentives under different regimes is at stake, the monotonicity of the difference happens to hold such that these other findings could also be traced back to the intensity principle.

In Schweizer (2010), I have examined the effects of breach remedies and performance excuses in combination on investment decisions. These findings were based on the externality principle of which the intensity principle is a straightforward extension. As monotonicity is maintained under the expectation operator, integration by parts can be dispensed with which simplifies the analysis substantially and supports the intuition behind the results. The present paper adds further evidence to the claim that examining the difference of payoff functions might be the most convenient approach if the intensity of incentives under different institutional arrangements is at stake.

6 References


7 Appendix

In section 3, it has been claimed that, under less than perfect monitoring \((p < 1)\), the costs \(\kappa(x)\) as perceived by the principal are monotonically increasing in precaution. In this appendix, such monotonicity is shown to hold in the following differentiable setting indeed. By assumption, the probability of an accident is a differentiable, decreasing and convex function of precaution. Moreover, the cost \(c(x) \geq 0\) of precaution is an increasing and convex function. More precisely, the following assumption is made throughout the appendix.

**Assumption:**

1. For all \(x \in \mathbb{R}\), it holds that \(\varepsilon'(x) < 0\) and \(\varepsilon''(x) > 0\).
2. For all \(x \in \mathbb{R}\), it holds that \(c'(x) > 0\) and \(c''(x) \geq 0\).

At costs \(G(p)\), the principal is able to detect the agent’s shirking with probability \(p\). This cost function is assumed differentiable and increasing with \(p\). Suppose the feasible bonus contract with fixed payment \(t\) and bonus \(b\) induces precaution \(x\) at rent \(r \geq 0\), that is (1) – (3) are assumed to hold and let \(R(x, p)\) denotes the minimum rent \(r \geq 0\) required to induce precaution \(x\) if shirking is detected with probability \(p\).

Under less than perfect monitoring \((p < 1)\), the following lemma holds.

**Lemma 3** (1) Suppose the bonus contract \([t, b \leq 0]\) induces precaution \(x\) at rent \(r \geq 0\). Then

\[
c(x) \leq \frac{p}{1-p} \cdot (u + h + r) + \frac{c(0)}{1-p} \tag{11}
\]

must hold.

(2) Conversely, if (11) holds then the feasible bonus contract \(t = u + r + c(x)\) with zero bonus \(b = 0\) induces precaution \(x\) at rent \(r\).

**Proof.** (1) Suppose \([t, b]\) induces \(x\) at rent \(r\) where \(b \leq 0\). If \(x = 0\) then claim (11) obviously is met. Therefore, suppose \(x > 0\) and consider shirking
\[ x' = 0 \text{ at its extreme. By combining constraints (1) and (3), it follows that} \]
\[ (1 - p) \cdot c(x) + (1 - p) \cdot b \cdot (\varepsilon(x) - \varepsilon(0)) \leq p \cdot (u + h + r) + c(0) \]

from which claim (11) follows immediately.

(2) Suppose (11) is met. Then the feasible contract \( t = u + r + x \) and \( b = 0 \) is easily seen to induce precaution \( x \) at rent \( r \). ■

Monitoring is still assumed to be less than perfect but precaution \( x \) violates (11), i.e.
\[ c(x) > \frac{p}{1 - p} \cdot (u + h + r) + \frac{c(0)}{1 - p} \tag{12} \]
and the feasible contract \([t, b]\) induces \( x \) at rent \( r \geq 0 \). In this case, the following lemma holds.

**Lemma 4** The bonus must be positive, i.e. \( b > 0 \), the derivative of the payoff function must be non-positive at \( x \), i.e. \(-b \cdot \varepsilon'(x) - c'(x) \leq 0 \) and \( y < x \) where
\[ y = \arg \max_{x'} -p \cdot h + (1 - p) \cdot [t + b \cdot (1 - \varepsilon(y))] - c(y). \tag{13} \]
Moreover,
\[ -p \cdot h + (1 - p) \cdot [t + b \cdot (1 - \varepsilon(y))] - c(y) \leq u + r \tag{14} \]
must also hold.

**Proof.** Since (12) is assumed to hold, it follows from the previous lemma that both the bonus and precaution must be strictly positive, i.e. \( b > 0 \) and \( x > 0 \). It then follows from (2) that \(-b \cdot \varepsilon'(x) - c'(x) \leq 0 \) must hold indeed.

Consider \( y \) as in (13). If \( y = 0 \) then \( y < x \) clearly holds. If, however, \( y > 0 \), it follows from the corresponding first order condition that
\[ -(1 - p) \cdot b \cdot \varepsilon'(y) - c'(y) = 0 \]
and, hence, that
\[ -b \cdot \varepsilon'(x) - c'(x) \leq 0 = -(1 - p) \cdot b \cdot \varepsilon'(y) - c'(y) < -b \cdot \varepsilon'(y) - c'(y) \]
such that \( y < x \) follows from the assumed concavity of the function \(-b \cdot \varepsilon(\cdot) - c(\cdot)\). Since \( y < x \) it follows from (3) that (14) must hold indeed. The lemma is fully established. ■

Monitoring is still assumed to be less than perfect and precaution \( x \) still violates (11) and the feasible contract \([t, b]\) induces \( x \) at rent \( r \geq 0 \). In this case, the following lemma also holds.
Lemma 5 Suppose \( r > 0 \). If (14) is not binding or if (14) is binding but \( t + h > 0 \) then \( R(x, p) < r \).

Proof. Suppose, first, that (14) is not binding and consider a marginal variation of the contract such that \((1 - \varepsilon(x)) \cdot db = dr\) whereas \(dt = dx = 0\). Since \(d[-b \cdot \varepsilon'(x) - c'(x)] = -\varepsilon'(x) \cdot db\), it follows that the new contract still induces precaution \( x \) but at lower rent, provided that \( b \) is modified in the direction \( db < 0 \). Therefore, \( x \) can be induced at a lower rent indeed if (14) is not binding.

Suppose, second, that (14) is binding but \( t + h > 0 \). Since 
\[-(1 - p) \cdot b \cdot \varepsilon'(y) - 1 = 0 < -b \cdot \varepsilon'(y) - 1\]
and since 
\[b \cdot (1 - \varepsilon(x)) - c(x) \leq b \cdot (1 - \varepsilon(y)) - c(y)\]
it follows from the concavity of \(-b \cdot \varepsilon(\cdot) - c(\cdot)\) that the derivative of the principal’s payoff function at \( x \) must even be strictly negative, i.e. \(-b \cdot \varepsilon'(x) - c'(x) < 0\).

Taking this into account consider a marginal variation of the contract such that
\[dt + (1 - \varepsilon(x)) \cdot db = dr\]
and
\[(1 - p) \cdot dt + (1 - p) \cdot (1 - \varepsilon(y)) \cdot db = dr\]
both hold whereas \(dx = 0\). By eliminating \(dt\) from the above two equations, it follows that
\[(1 - p) \cdot (\varepsilon(x) - \varepsilon(y)) \cdot db = p \cdot dr\]
must hold. The new contract induces the same precaution but at a lower rent provided that \( b \) is modified in the direction \( db > 0 \). The lemma is fully established. ■

As a corollary, it follows immediately from the above lemma that the bonus contract inducing precaution at minimum rent \( R(x, p) > 0 \) requires the agent to transfer his wealth to the agent whenever an accident has occurred, i.e. \( t + h = 0 \). These findings allow to establish the following proposition.

Proposition 8 Suppose monitoring is less than perfect and \( R(x, p) > 0 \). Then the partial derivative with respect to precaution must be positive, i.e. \( R_x(x, p) > 0 \).
Proof. It follows from the last lemma that a bonus $b$ must exist such that the rent $r = R(x, p)$ is uniquely determined by the two equations

$$b \cdot (1 - \varepsilon(x)) - c(x) = u + h + r$$

and

$$(1 - p) \cdot b \cdot (1 - \varepsilon(y)) - c(y) = u + h + r$$

where $-(1 - p) \cdot b \cdot \varepsilon'(y) - 1 = 0$. It then follows from the implicit function theorem that

$$R_x(x, p) = \frac{(1 - p) \cdot (1 - \varepsilon(y)) \cdot [b \cdot \varepsilon'(x) + c'(x)]}{p \cdot (1 - \varepsilon(y)) + [\varepsilon(y) - \varepsilon(x)]} > 0$$

holds indeed. ■

With the above proposition at hand, it can now be shown that the costs $\kappa(x)$ of inducing precaution $x$ must be monotonically increasing. In fact, the principal has the incentive to detect shirking with probability

$$\pi(x) = \arg\max_p R(x, p) + G(p)$$

to be characterized by first order condition $R_p(x, \pi) + G_p(\pi) \leq 0$, and $= 0$ if $p > 0$. Since total costs as perceived by the principal amount to

$$\kappa(x) = R(x, \pi(x)) + G(\pi(x))$$

it follows from the envelope theorem that $\kappa_x(x) = R_x(x, \pi(x)) \geq 0$ holds as was to be shown.