Liquidating Illiquid Collateral*

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Abstract

Defaults of financial institutions can cause large, disorderly asset liquidations. I model the dynamics of such liquidations as a continuous-time trading game, in which, following a default, balance-sheet constrained lenders unwind illiquid collateral positions. I show that (i) the equilibrium price of the collateral asset overshoots during liquidation when the lenders who liquidate the collateral asset are sufficiently balance-sheet constrained; (ii) when collateral is spread across multiple lenders this alleviates balance sheet constraints, but can cause inefficient 'racing to the market', potentially reducing expected liquidation proceeds; (iii) lenders should take into account a borrower's creditor structure and their own balance sheet constraints when setting margins to manage counterparty risk; (iv) the model allows to determine the block price and expected profit at which a 'deep pocket' buyer can purchase the entire collateral position.

Keywords: Illiquidity, Collateralized Lending, Strategic Trading, Prime Brokers, Hedge Funds, Fire Sales, Counterparty Risk Management, Block Trades

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Defaults of financial institutions can cause large and disorderly asset liquidations. Following a default, lenders to a defaulted institution usually need to unwind collateral assets on a large scale to recover their losses. These liquidations can cause significant shocks to the financial system—leading to low recovery values for lenders, fire sales, spillovers to other market participants and, ultimately, the real economy. While this was already an important concern during the demise of the hedge fund LTCM, the credit crisis of 2007-08 has brought these concerns back to the center of attention. Bear Stearns was rescued by the Federal Reserve since, according to Chairman Bernanke, "a bankruptcy filing would have forced Bear's secured creditors and counterparties to liquidate the underlying collateral, and given the illiquidity of markets, those creditors and counterparties might have sustained losses."[1] Lehman Brothers, on the other hand, went into liquidation, causing financial market turmoil in the aftermath. But what determines the dynamics of collateral liquidations that follow the default of a financial institution? What price dynamics and recovery should one expect, and how should lenders protect themselves ex ante from losses they may incur?

In this paper, I develop a model of large-scale collateral liquidations following the default of a levered non-bank financial institution. I develop a framework in which the collateral asset is illiquid, and the lenders that unwind collateral—e.g. the prime brokers in the case of a hedge fund default—face real-world balance sheet constraints. Upon default, lenders unwind the collateral asset strategically, taking into account their own price impact. This means that when multiple lenders sell at the same time, liquidation takes the form of a dynamic trading game. Solving for the equilibrium of this liquidation game, I generate four main results: (i) the equilibrium price of the collateral asset overshoots during the liquidation when the liquidating lenders are sufficiently balance-sheet constrained; (ii) when collateral is spread across multiple lenders this alleviates balance sheet constraints, but at the cost of creating inefficient 'racing to the market'—the creditor structure of financial institutions thus involves a fundamental tradeoff between risk sharing and 'rushing to the exits' after a default; (iii) rather than relying on purely statistical models, lenders should take into account creditor structure, strategic interaction, and their own balance constraints when setting margins to

[1] Speech given at the Federal Reserve Bank of Atlanta, May 13, 2008; available at http://www.federalreserve.gov/newsevents/speech/bernanke20080513.htm.

manage counterparty risk; and (iv) the model pins down the block price and expected profit at which a 'deep pocket' buyer can purchase the entire collateral position from the lender(s) or the ailing financial institution.

The model takes the form of a continuous-time trading game, in which risk-neutral lenders choose trading strategies to maximize the expected payoff from liquidating the collateral asset. Because of limited balance sheet size, the lenders are subject to a risk management constraint, which takes the form of a limit on the standard deviation of the liquidation payoff. The collateral asset is risky (its fundamental follows a Brownian motion) and illiquid, in the sense that trading affects the price during the liquidation process, both permanently and temporarily, through price pressure effects.

Because of illiquidity frictions, the efficient liquidation strategy absent any constraints would be to unwind the collateral position slowly over time. However, there are two forces that prevent the liquidating lenders from doing so. First, the ability to reduce short-term price impact costs by trading slowly is limited, since the lenders need to liquidate the position quickly enough not to violate their risk management constraints. Second, a 'racing to the market' effect arises. When the demand curve for the collateral asset is downward-sloping, competing sellers of collateral have an incentive to sell before the other sellers drive down the price. This means that, while a monopolistic seller always uses its entire risk-bearing capacity to reduce price impact costs, competition may in equilibrium lead multiple lenders not to use their balance sheets fully during liquidation.

After solving this continuous-time trading game, I first show that the price overshoots during the liquidation when the lenders are sufficiently constrained by their own risk management. Price overshooting is thus 'balance sheet driven;' it results from the lenders' need to quickly offload risk from their books, and is thereby distinct from price overshooting that results from predatory trading. Importantly, price overshooting emerges as part of the optimal liquidation strategy of a constrained seller—the Lagrange multiplier on the variance constraint acts like a holding cost on the collateral position that remains on the lender's balance sheet. Overshooting leads to fire sales with low recovery values for the liquidating lenders, and may cause spillovers to other market participants.

Next, I examine the impact of creditor structure on the liquidation. When a collateral position is spread among multiple lenders, this can reduce balance-sheet-driven price overshooting, since the risky position each lender needs to unwind upon default is smaller relative to the lender's balance sheet. This allows each lender to unwind the collateral position in a more orderly fashion. However, racing to the market among competing sellers creates a tradeoff, such that more risk-bearing capacity will not always result in higher expected liquidation values.

I illustrate this tradeoff in a stylized financial system with two lenders and two levered investors (for concreteness, I will refer to them as hedge funds), in which financing can either be *monopolistic* (each of the two hedge funds has one lender) or *distributed* (each of the two hedge funds borrows from both lenders). Thus, while the allocation of collateral across the two lenders differs between the two regimes, the aggregate collateral position and the risk-bearing capacity of the lending sector is held constant. The model shows that monopolistic finance leads to higher expected liquidation values than distributed finance when the collateral position is sufficiently small relative to the lenders' variance constraint, and when the ratio of the size of the permanent to temporary price impact parameters is large. The intuition for this result is that, under these conditions, the incentive for competing traders to race to the market is particularly large, such that the competitive pressure between the two liquidating lenders prevents them from using their joint risk bearing capacity in equilibrium. In that case, liquidation by a monopolist, who always uses its entire risk-bearing capacity, leads to higher expected recovery.

The model has important implications for counterparty credit risk management. Since the expected collateral liquidation proceeds after default depend crucially on the lenders' balance sheet constraints and the strategic interaction during liquidation, the lenders should take this into account when setting margins. For example, when a hedge fund has a large number of creditors, the margin charged to the hedge fund should reflect that liquidations following a default will be 'crowded trades,' with multiple sellers rushing to the exit at the same time. The collateral asset's effective liquidity following a default will thus be significantly lower than during normal times. Likewise, a lender who has similar exposures to its the parties he is extending credit to (think, for example, of a prime broker that also runs his own trading book) should take into account that, due to losses on its own positions, its risk-bearing capacity is likely to be low just at the time its lending counterparty defaults. This is an important consideration since nowadays many prime brokerages also have trading operations on their own.

Finally, the model provides a new framework to analyze block trades, in which a 'deep pocket' or 'vulture' buyer purchases the entire collateral position from the lender(s) or the ailing hedge fund or financial institution. Recent examples of these types of trades include Citadel's acquisitions of portfolios from Sowood Capital Management and Amaranth. The 'deep pocket' buyer—by definition less balance-sheet constrained than the liquidating lender(s)—can purchase the collateral position *en bloc* at the constrained lenders' outside option of selling into the downward-sloping demand curve, and then unwind it orderly, using its capacity to bear risk. Block trades occur when the difference in liquidation payoffs exceeds the vulture buyer's cost of executing the trade, which may reflect the cost of holding spare balance sheet capacity or the human capital needed to swiftly value the position. This view of block trades also provides a framework to analyze asset acquisition or guarantees issued by the government (or another buyer of last resort) during financial crises, e.g. the credit crisis of 2008.

The paper builds on the recent literature on strategic trading in the presence of liquidity frictions. The closest related paper is Carlin, Lobo, and Viswanathan (2007), whose continuous-time trading game I adapt in my model. Their paper analyzes how episodic liquidity crises result from breakdowns of cooperation among traders that interact repeatedly. However, since they do not allow for risk management constraints, the tradeoff between balance sheet constraints and strategic racing analyzed in this paper cannot emerge in their framework. Brunnermeier and Pedersen (2005) develop a strategic trading game with price impact to show that, when a large trader needs to liquidate, other traders may sell at the same time to withdraw liquidity, which can lead to price overshooting and systemic risk. While in their paper price overshooting is driven by additional units that are sold by predatory traders and then bought back at a later stage, price overshooting in my model is a consequence of the traders' risk management constraint and does not rely on the strategic selling of an opportunistic trader. More generally, the paper is related to the literature on optimal trading and liquidation strategies when traders have price impact, in particular Almgren and Chriss (2001), Bertsimas and Lo (1998), Huberman and Stanzl (2005), and Engle and Ferstenberg (2007). However, these papers do not consider strategic interaction among multiple sellers, which is central in this paper. The paper is also related to the literature on the optimal creditor structure in corporate finance settings. Bolton and Scharfstein (1996) discuss the tradeoff that emerges in the choice between one or more creditors in an optimal contracting framework. While they focus on how the number or creditors affects bargaining during the liquidation of a real investment project, this paper focuses on the impact of the number of creditors on the liquidation of illiquid financial assets (albeit not in an optimal contracting framework). Finally, the paper is related to the literature on margin setting and the market microstructure literature on block trades.[2]

The remainder of the paper is organized as follows. Section 1 introduces the model. Section 2 analyzes collateral liquidations as a continuous-time game among balance-sheet constrained sellers. Section 3 studies price overshooting during liquidations, and section 4 discusses the tradeoff between spreading risk and inefficient strategic behavior inherent in a financial institution's creditor structure. Section 5 discusses the model's implications for margin setting and counterparty risk management, and section 6 analyzes block trades. Section 7 concludes.

1 Model Setup

The model has two types of players, lenders and margin investors. One may think of the lender as prime broker or broker dealer, and the margin investor as a levered non-bank financial institution or a hedge fund. Note, however, that the model extends to more general settings—it applies to any lender that extends credit on a collateralized basis and may have

[2] The recent literature on margins includes Chowdhry and Nanda (1998), Geanakoplos (2003), and Brunnermeier and Pedersen (2008). The theoretical literature on block trades is summarized in chapter 8 of O'Hara (1995).

to liquidate collateral in the case of default.[3] The model is set in continuous time, and time runs on the interval $[0, \infty)$. The analysis in this paper centers around what happens conditional on the default of a financial institution or hedge fund, and I thus normalize the time of default to $t = 0$. There is no discounting.

Collateralized borrowing. The financial institutions in this model are leveraged investors and borrow from their lenders in order to invest in a risky asset. This risky asset may also be interpreted more broadly as a risky trading strategy, or a portfolio of assets. Borrowing is done on a collateralized basis: financial institutions purchase a position in the risky asset and use this position to obtain a collateralized loan from their lenders. Usually the amount of the loan is less than the market value of the position, such that the financial institution has to invest some of its own capital to finance the position. This difference is referred to as the *margin* or *haircut*. When the financial institution defaults, the lenders seize the collateral position and liquidate it to cover their losses. These liquidations are the focus of this paper.

I assume that, upon default of the financial institution, lenders always liquidate the entire collateral position. I thus do not allow the lenders to permanently hold on to the collateral or sell only part of it. There are a number of reasons why this assumption is reasonable. First, in many cases the lenders, e.g. broker dealers, have no direct use for the collateral asset in their own portfolio. For example, a hedge fund may have held the collateral asset as part of a larger-scale trading strategy. By itself, however, the asset may be of little use to its prime broker. Second, even if the collateral asset by itself was an attractive investment from the financial institution's perspective, the lender may have relatively little expertise in hedging this asset and may thus be reluctant to hold it. Third, since the lender's main business is margin lending, it may simply want to liquidate the collateral asset in order to use the proceeds for its core margin lending business.

Two features of the model make the collateral liquidation interesting. First, the collateral asset is illiquid, meaning that the liquidating lenders move the price of the asset when they sell it. The lenders act strategically and take these price effects into account when choosing

[3]In fact, if one reinterprets the collateral asset to be commercial property, machines, or inventory, the analysis may also be applied to non-financial market settings.

their liquidation strategies. Second, lenders have limited risk-bearing capacity—thus even though they are risk-neutral, there is a limit to how much risk they are willing to take onto their balance sheets during the liquidation process. This constraint limits their ability to unwind the collateral position slowly over time.

Illiquidity. To capture illiquidity of the collateral asset, I assume that its price is affected by the lenders' selling, both temporarily (through temporary price pressure effects) and permanently (by walking down a downward-sloping demand curve). Let $X(t)$ be the aggregate amount of collateral still held by the liquidating lenders at time t, and $Y(t)$ the lenders' aggregate trading rate at time t, which implies that $dX(t) = Y(t)dt$.[4] This means that when the lenders liquidate the collateral asset, $Y(t)$ is negative. I assume that the price of the collateral asset at time t consists of the sum of three components,

$$P(t) = F(t) + \gamma[X(t) - X(0)] + \lambda Y(t). \qquad \gamma, \lambda \geq 0 \tag{1}$$

The first term, $F(t)$, is the fundamental value of the asset at time t. I assume that $F(t)$ follows a Brownian motion without drift and with instantaneous variance σ_F, i.e. $dF(t) = \sigma_F dW(t), F(0) = F_0$. The zero drift assumption is made for simplicity of notation; a drift term could be added without materially changing the analysis in this paper. Changes in the fundamental term may, for example, result from publicly observable news about the value of a final dividend that is paid at maturity T. In my analysis below I will take the limiting case and let the maturity date T tend to infinity. However, this is purely to make the expressions derived as simple as possible; the analysis does not change for finite T.

The two following terms reflect the illiquidity of the collateral asset. Since $X(t)$ is the aggregate amount of the collateral asset still held by the lenders at time t, $X(t) - X(0)$ is the amount of collateral that has been liquidated by the lenders up to time t. Thus the second term in the pricing function implies that, when the lenders sell the collateral position, the price of the collateral asset drops permanently. The parameter γ measures the slope of this downward sloping demand curve—selling a block of Δ shares permanently reduces the price of the asset by $\gamma\Delta$. This permanent price effect may be caused by the residual demand of a

[4]Or, equivalently, $X(t) = X(0) + \int_0^T Y(t)dt$.

continuum of long-term investors, which absorb the selling from the large risk-neutral traders and must be compensated for the risk they hold, either since they are risk averse, or because they have only limited capital. Papers that generate a downward sloping demand curve for risky assets in this manner include Brunnermeier and Pedersen (2005) or Xiong (2001). Alternatively, the permanent price effect may be the outcome of informational differences, such as the permanent price impact of trading in the Kyle (1985) model. The third term reflects temporary price pressure effects. When the asset is sold at an aggregate rate of $Y(t)$, its price temporarily decreases by $\lambda Y(t)$. This effect is purely temporary and captures that traders may not have access to the full demand curve at any point in time. Importantly, the presence of this temporary price pressure effect implies that the faster the lenders need to unwind positions, the lower the price the asset fetches.[5][6]

Both temporary and permanent price impact of large trades have been well documented empirically. For example, Shleifer (1986), Chan and Lakonishok (1995), and Wurgler and Zhuravskaya (2002) provide evidence for downward sloping demand curves for stocks. Kraus and Stoll (1972), Holthausen, Leftwich, and Mayers (1987), Holthausen, Leftwich, and Mayers (1990), and Madhavan and Cheng (1997) document permanent and temporary price effects for block trades on the New York Stock Exchange. The type of illiquidity effects assumed above are thus economically significant even for relatively liquid securities, such as U.S. equities. They are likely to be even more pronounced for more exotic collateral assets often posted by hedge funds and other levered financial institutions, such as mortgage-backed securities, CDOs or other structured products. There is also ample evidence that financial institutions take these illiquidity frictions into account when trading—they often rely on trading systems that estimate permanent and temporary price impact parameters and then calculate trading schedules based on these estimates. One example is Citigroup's Best Execution Consulting Services (BECS) software described in Almgren, Thum, Hauptmann, and Li (2005).

[5] A pricing function of the same linear structure is used in Carlin, Lobo, and Viswanathan (2007). In order to facilitate comparison across models, I retain their convention of labelling the coefficient determining the size of the short-term price effect λ, and the coefficient determining the size of the permanent price effect γ. Note however that λ cannot be readily interpreted as the 'Kyle lambda' in Kyle (1985).

[6] While the focus on linear price impact is mostly for tractability, it can be justified theoretically, at least for the permanent price effect. Huberman and Stanzl (2004) show that permanent price impact has to be linear to rule out arbitrage. The temporary price impact, however, could take a more general form.

Balance sheet constraint. The second assumption that makes the liquidation interesting is that the lenders face balance sheet constraints similar to those they face in practice. This means that there is a limit to how much risk they can put onto their balance sheets while liquidating the collateral asset. More specifically, I assume that upon default of the financial institution, the lenders cannot take on more than $\overline{V}$ units of variance during the liquidation phase. In effect, this constraint limits the amount of time a liquidating lender can keep the risky collateral on its balance sheet, since the longer the collateral asset is held, the more variance the lender incurs due to changes in the risky fundamental F. Thus, while illiquidity gives the lenders an incentive to liquidate the collateral position slowly over time, the variance constraint limits their ability to do so. In particular, when $\overline{V}$ is low, a lender needs to liquidate the collateral quickly to get it off its balance sheet.

This balance sheet constraint can be interpreted in a number of ways. First, prime brokers, banks and broker-dealers only have a limited amount of capital that allows them to absorb losses. To avoid default or costs of financial distress, they thus have an incentive to limit their risk taking. For example, assume that a lender wants limit the probability of a loss that exceeds a critical amount to a certain percentage, say one percent. This roughly translates into a constraint on the standard deviation (or equivalently the variance) of its positions. A variance limit of the type assumed here may then emerge as a self-imposed risk limit as part of the privately optimal risk management of the lender. Second, regulated financial institutions need to hold a certain amount of regulatory capital against their risky assets. Thus even in the absence of privately optimal risk management constraints, there may be a limit to how much risk a financial institution can take on. Irrespective of the particular interpretation, what is important for the model is that the lenders face realistic constraints that limit the amount of risk they are willing to take during liquidation.[7] As section 2 shows, both illiquidity and the balance sheet constraint are crucial in determining the lenders' liquidation strategies following a default.

Discussion of assumptions. While the model's assumptions are meant to capture as

[7]The variance constraint can also be reinterpreted in terms of risk aversion: The Lagrange multiplier of the variance constraint enters the maximization problem the same way as the cost of bearing risk would in a mean-variance optimization (see appendix for details).

closely as possible the institutional setup and financial market frictions that exist in practice, some simplifications have to be made to keep the model tractable within a continuous-time framework that can capture the dynamics of collateral liquidations. In this section I briefly discuss these simplifying assumptions and possible alternative specifications of the model. The section can be skipped without loss of continuity.

First, price impact is assumed to be linear. While one can theoretically justify linearity of the permanent component of price impact (see Huberman and Stanzl (2004)), the temporary component of price impact could take more general forms. However, as long as the temporary price impact is increasing in $Y(t)$, it will give the sellers of collateral an incentive to liquidate more slowly, which is what matters for the tradeoff discussed in the model. In that sense, assuming that the temporary component is linear allows for a closed-form solution, but does not fundamentally drive the results of the paper.

Second, there are alternative ways to specify the risk management constraint. One alternative would be to set it up as a true value-at-risk constraint, in which case rather than just on the variance, the constraint would depend on the sum of the expected liquidation loss and its standard deviation. Including the expected loss in the risk management constraint, however, makes the problem intractable. But since what matters most for risk management are tail losses which are mainly driven by the variance, leaving the mean loss out of the risk management constraint should only have a second order effect in terms of the implications that emerge from the model.

Another extension would be to allow for the risk management constraint to be dynamic. One may imagine that, if the collateral asset initially rises in value, the liquidating lenders have a larger 'capital cushion' that allows them to hold the collateral position for a longer period of time. In this setup up there would thus be some paths for which the risk management constraint becomes less binding over time, while for other realizations of the collateral asset's value the risk management constraint will become more binding over time. This would complicate the analysis significantly since it would require the addition of another state variable. It would, however, still be the case that—like the static constraint used in the model—this dynamic constraint would in expectation limit the lenders' ability to sell slowly over time. The fundamental tradeoff between risk sharing and strategic racing described in this paper would thus still be present even with this more complicated constraint. Finally, the risk-management constraint in the model could be formulated in terms of downside variance, rather than pure variance (as is sometimes the case in practice). However, due to the symmetry of Brownian motion, in the setup used in this paper these two constraints would coincide, and thus the problem would not change.

2 Collateral Liquidation

I model liquidation as a continuous-time (or differential) game, in which the n lenders to a defaulted financial institution strategically liquidate their collateral holdings, taking into account their own price impact, the trading of the other liquidating lenders, and their variance constraints. Upon default of the financial institution, the lenders each seize $\overline{X}_i$ units of collateral. They then choose liquidation strategies that maximize the expected payoff from collateral liquidation taking into account the illiquidity of the collateral asset, subject to not exceeding their balance sheet constraint.[8] The collateral positions and the variance constraints are common knowledge, and are symmetric across lenders.

The liquidating lenders' objective function can be constructed as follows. To liquidate the collateral position lender i' chooses a schedule of trading rates $\{Y_i(t)\}$. The proceeds from liquidating $Y_i(t)dt$ shares over the interval $[t, t+dt)$ at price $P(t)$ are equal to $-Y_i(t)P(t)dt$, where the minus sign appears, since for liquidations the trading rate is negative. The payoff from the liquidation strategy $\{Y_i(t)\}$ can thus be written by integrating the instantaneous proceeds from 0 to infinity. To take account of the variance constraint, we need to calculate the payoff variance generated by a particular liquidation strategy $\{Y_i(t)\}$. Rewriting the payoff and applying Itô isometry, the payoff variance that results from following a particular trading strategy $\{Y_i(t)\}$ over the interval $t \in [0, \infty)$ is given by $V = \int_0^\infty \sigma_F^2 [X_i(t)]^2 dt$. The details of this calculation are in the appendix.

[8] In practice, if a lender can sell the collateral for more than the financial institution owed, the remainder is given back to the financial institution. I disregard this complication. One justification for this assumption is that at the time or default the collateral value is already sufficiently 'under water', such that the liquidating lender(s) will effectively act like a residual claimant when selling the collateral position.

Using these results, lender i's maximization problem is to maximize the expected payoff,

$$\max_{Y_i(t)\in\mathcal{Y}} E\int_0^\infty -Y_i(t)P(t)dt, \tag{2}$$

subject to starting with an initial collateral position $\overline{X}_i$ and eventually selling the entire collateral position,

$$\begin{aligned} X_i(0) &= \overline{X}_i & (3)\\ \lim_{T\to\infty} X_i(T) &= \overline{X}_i + \int_0^\infty Y_i(s)ds = 0, & (4) \end{aligned}$$

and subject to the variance constraint (with associated Lagrange multiplier ϕ)

$$\int_0^\infty \sigma_F^2[X_i(t)]^2 dt \leq \overline{V}. \tag{5}$$

I impose standard restrictions on the strategy space $\mathcal{Y}$ to make the problem well defined. First, I restrict my attention to continuous strategies. Second, for a strategy to be admissible, the lender's expected profit has to be integrable, i.e. $E\int_0^\infty -Y_i(t)P(t)dt < \infty$. This is guaranteed, for example, when $\{Y_i(t)\}$ lies in $\mathcal{L}^2$ (i.e. $\int_0^\infty [Y_i(t)]^2 dt < \infty$), which is a standard assumption.

In the above maximization problem, the price is affected not only by lender i's trading rate, but also by the trading of all other lenders. The strategic interaction created through the sellers' influence on the price of the collateral asset means that the liquidation takes the form of a differential game. I solve this game for an equilibrium in time-dependent trading strategies, as outlined in the following definition.[9]

Definition 1 ***Equilibrium.*** *An equilibrium in time-dependent trading strategies is given by a set of admissible trading strategies* $\{Y_i(t)\}$*, such that each lender i maximizes expected profit* (2)*, subject to the pricing equation* (1)*, the individual trading constraints* (3) *and* (4)*, and*

[9]Time-dependent strategies imply that the equilibrium is 'open-loop', or weakly time consistent. There is no closed-form solution for the closed-loop equilibrium, but Carlin, Lobo, and Viswanathan (2007) show numerically that open-loop and closed-loop equilibria are qualitatively very similar in trading games of this type. For more on different equilibrium concepts in differential games see Dockner, Jorgensen, Long, and Sorger (2000) and Basar and Olsder (1995).

the variance constraint (5)*, taking the strategies of the other lenders* $\{Y_{-i}(t)\}$ *as given.*

Proposition 1 ***Equilibrium trading strategies for n liquidating lenders.*** *The unique equilibrium time-dependent trading strategies* $\{Y_i(t)\}$ *in the case when* n *lenders liquidate* $\overline{X}_i$ *units of collateral each, subject to an individual variance constraint* $\overline{V}$*, are given by*

$$Y_i(t) = -ae^{-at}\overline{X}_i, \tag{6}$$

where

$$a = \frac{1}{2}\frac{(n-1)\gamma + \Gamma}{(n+1)\lambda} \tag{7}$$

$$\Gamma = \sqrt{\gamma^2(n-1)^2 + 8(n+1)\phi\sigma_F^2\lambda}. \tag{8}$$

The variance incurred by following this strategy is given by $V = \frac{\sigma_F^2 \overline{X}_i^2}{2a}$.

When $\overline{X}_i > \sqrt{\frac{n-1}{n+1}\frac{2\gamma\overline{V}}{\lambda\sigma_F^2}}$ *the variance constraint is binding and the strategy simplifies to*

$$a = \frac{\sigma_F^2 \overline{X}_i^2}{2\overline{V}}. \tag{9}$$

When $\overline{X}_i < \sqrt{\frac{n-1}{n+1}\frac{2\gamma\overline{V}}{\lambda\sigma_F^2}}$ *the variance constraint is not binding and*

$$a = \frac{(n-1)\gamma}{(n+1)\lambda}. \tag{10}$$

The amount of collateral still held by each lender t *periods after default is given by*

$$X_i(t) = \overline{X}_i e^{-at}. \tag{11}$$

Proof. See appendix. ■

Proposition 1 shows that equilibrium trading strategies take an exponential form,[10] as

[10]When the variance constraint does not bind, the equilibrium strategies are equivalent to those in Carlin, Lobo, and Viswanathan (2007). When the variance constraint is binding, on the other hand, the trading strategies have an additional term that is driven by the Lagrange multiplier ϕ.

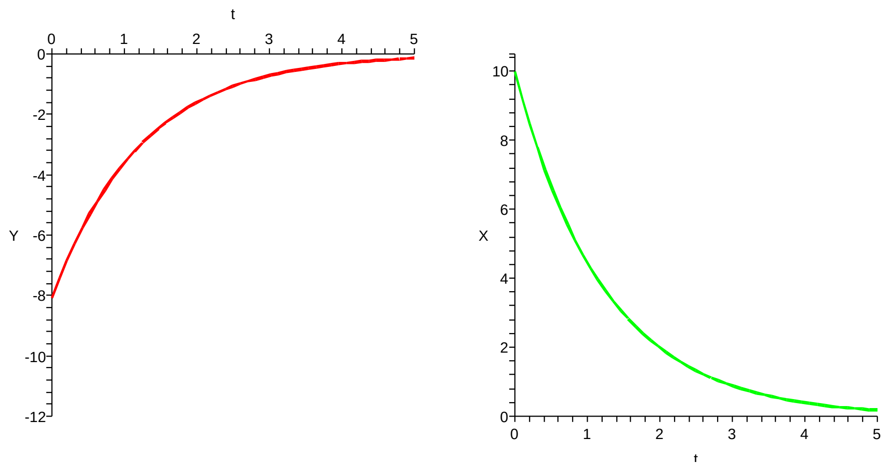

Figure 1: Trading rate $Y_i(t)$ (left) and remaining collateral position $X_i(t)$ (right). The trading rate and the remaining collateral position of each individual lender decrease exponentially over time. The parameters in this example are $n = 3, \gamma = 1, \lambda = 1, \phi = 0.5, \overline{X}_i = 10$.

illustrated in Figure 1. As shown by equation (6), the equilibrium value of a completely summarizes the strategic interaction between the liquidating lenders. The initial rate of trading by lender i at time 0 is given by $-a\overline{X}_i$. After that, the trading rate decays exponentially at rate a. This means that when the equilibrium value of parameter a is large, selling is heavier in early periods and then dies out at a faster rate. Trading is more front-loaded and, because of the additional temporary price impact costs generated by the heavy early trading, liquidation is more 'disorderly'. When a is small, on the other hand, trading is more equally spread out over time, and the liquidation is more 'orderly'.

The underlying parameters enter the equilibrium trading rate in intuitive ways. The trading intensity a is decreasing in the short-term price pressure parameter λ, as short-term price impact gives the liquidating lenders an incentive to spread out their trades. In contrast, a is increasing in the permanent price impact parameter γ when $n > 1$. This is the case since long-term price impact leads to racing to the market among the sellers of collateral. Finally, a is also increasing in the Lagrange multiplier ϕ, i.e. the more balance-sheet constrained the lenders are, the earlier trading occurs.

Once we substitute in the equilibrium value of the Lagrange multiplier, we see that there are two cases. When the variance constraint is not binding, the equilibrium trading strategy is fully characterized by the liquidity parameters γ and λ and the number of liquidating lenders, n. Neither the variance constraint nor the fundamental variance of the collateral asset matter for the determination of a (and thus the equilibrium trading strategies) in that case. Importantly, when γ is positive and $n > 1$, racing to the market arises and liquidation is more disorderly. This is because the incentive to race to the market is stronger the steeper the demand curve and when more traders liquidate at the same time. On the other hand, when the lenders are constrained in equilibrium, the trading intensity is determined by the fundamental variance, the variance constraint, and the size of the position, but is independent of the number of liquidating lenders and the illiquidity parameters. In this case, the liquidation is disorderly when $\overline{V}$ is small.

Having solved equilibrium trading strategies, the expected liquidation value of the collateral position can be calculated by substituting back into the objective function.

Proposition 2 *Lender i's equilibrium expected unwind value* Π *of collateral position* $\overline{X}_i$*, when* n *lenders unwind an aggregate amount of* $\overline{X} = n\overline{X}_i$ *units of the collateral asset, is given by*

$$\Pi(\overline{X}_i, \overline{V}) = F_0\overline{X}_i - \frac{\gamma}{2}\overline{X}_i\overline{X} - \frac{\lambda}{2}a\overline{X}_i\overline{X}, \tag{12}$$

where a *is the trading intensity parameter defined above.*

This implies that the aggregate expected unwind value is given by

$$n\Pi(\overline{X}_i, \overline{V}) = F_0\overline{X} - \frac{\gamma}{2}\overline{X}^2 - \frac{\lambda}{2}a\overline{X}^2. \tag{13}$$

Proof. See appendix. ■

Proposition 2 shows that the expected unwind value consists of three terms. The first term, $F_0\overline{X}_i$, is the fundamental value of the collateral position at the beginning of the liquidation process. This term can be interpreted as the marked-to-market value of the collateral position at the time of default. The second term, $\frac{\gamma}{2}\overline{X}_i\overline{X}$, is the loss due to permanent price effects during the liquidation process, i.e. it reflects the cost to the lender from walking down the demand curve during liquidation. Intuitively, this term does not depend on the variance constraint or the strategic interaction between liquidating lenders. The third term, $\frac{\lambda}{2}a\overline{X}_i\overline{X}$, is the loss the lender incurs due to temporary price pressure. This term is increasing in a, and thus depends on the strategic interaction during liquidation—the more front-loaded the equilibrium trading strategies, the larger the losses due to temporary price effects. Both of these terms are functions of the respective liquidity parameters (γ and λ) and the product of the aggregate position liquidated by all lenders and the position liquidated by each individual lender.

An important implication of Proposition 2 is that the aggregate illiquidity loss from the lenders' limited ability to spread their trades over time (either due to binding variance constraints or competitive pressure) is fully characterized by the equilibrium value of the trading intensity parameter a. This fact will be used extensively in the analysis of different financing arrangements in section 4.

3 Price Overshooting

The above analysis shows that the collateral liquidation can be disorderly in two ways. First, lenders may be balance-sheet constrained, forcing them to unwind the collateral asset quickly. Second, lenders may race to the market due to competition in the liquidation process. In this section, I show that the price of the collateral asset can overshoot during the liquidation, and that overshooting is driven by the balance sheet constraint, not by racing to the market.

To analyze price overshooting we need to consider the price path during liquidation. Reinserting the optimal trading strategies (6) and (11) into the price function (1), we get the following expression.

Corollary 1 *The equilibrium price during the liquidation of an aggregate position of size* $\overline{X}$ *is given by*

$$P(t) = F(t) - \gamma(1 - e^{-at})\overline{X} - \lambda a e^{-at}\overline{X}. \tag{14}$$

Proof. See appendix. ■

Corollary 1 shows that while the permanent price effect dominates in the long run, initially, when the lenders' selling is strongest, there can be significant short-term price pressure. When

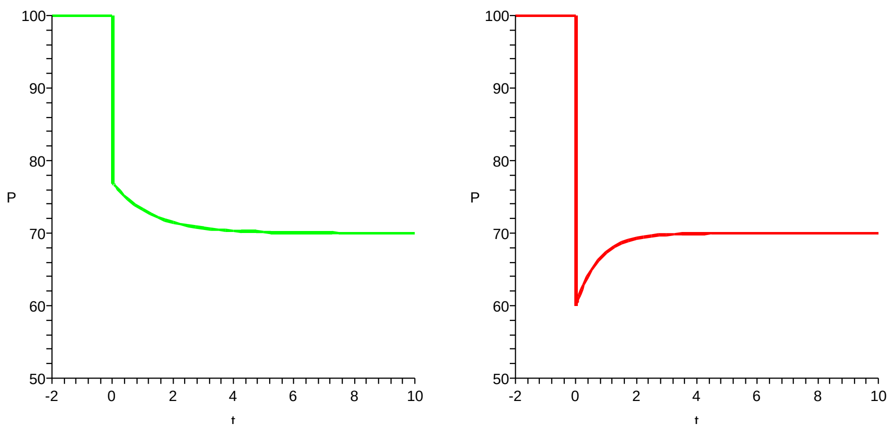

Figure 2: **Price overshooting.** The figure shows that depending on the relative size of ϕ, γ and λ, overshooting can occur. Liquidation starts at time zero, at which point the price drops discontinuously. In this example the initial price is 100, the final price is 70. The parameters are $n = 2, \gamma = 1, \lambda = 1, \overline{X} = 30$. On the left panel $\phi = 0.5$, in which case the price does not overshoot. On the right panel $\phi = 2$, which leads to overshooting.

this initial price pressure is large enough, the price overshoots its new expected long-run level.

Proposition 3 *The expected price path overshoots (it drops below its expected post-liquidation level) if and only if*

$$\phi > \frac{\gamma^2}{\sigma_F^2 \lambda}, \tag{15}$$

i.e. when ϕ, the Lagrange multiplier on the variance constraint, is sufficiently large.

Proof. See appendix. ■

As proposition 3 shows, overshooting occurs when the equilibrium value of the Lagrange multiplier on the balance sheet constraint is sufficiently large. When the balance sheet constraint is not binding (and $\phi = 0$), or when it is just slightly binding, price overshooting does not occur. This means that price overshooting is 'balance-sheet driven'—it is generated by the lenders' need to offload the collateral asset quickly due to their own risk management constraints. The lenders' financial position is thus a crucial determinant of the price path and expected recovery during collateral liquidations after default of a leveraged investor. Moreover, when lenders themselves have weak balance sheets at the time that a hedge fund defaults, the resulting price overshooting can lead to spillovers—it may force other traders to also unwind, a process that can ultimately lead to systemic crises.

Price overshooting is illustrated in Figure 2. The left panel shows the price path in the case in which price does not overshoot. In that case, the price drops discontinuously at time $t = 0$ due to short-term price pressure, but not sufficiently to overshoot its long-run expected post liquidation value. This means that after the initial drop the price smoothly decreases to this new expected long-run level. In the right panel, on the other hand, the collateral position is sufficiently large for the price to overshoot in expectation. In this case the lenders' need to offload the asset leads to a downward jump in the expected price at time $t = 0$ that is large enough to cause the price to overshoot before eventually moving back up to its expected long-run level.

When the price of the collateral asset overshoots, early trades are executed at a price below the expected long-run price after liquidation. However, the liquidating lenders are still maximizing their expected payoff, which means that overshooting emerges in this model as an equilibrium outcome in markets with illiquidity frictions and balance-sheet constrained traders. The reason is that the Lagrange multiplier on the balance sheet constraint acts like a holding cost on the collateral position still remaining on the lenders' books—holding a unit of collateral on the balance sheet longer is costly, since it uses up risk-bearing capacity that could otherwise be used later in the liquidation process. To understand the intuition why overshooting occurs as part of the optimal strategy, consider shifting one unit that is sold early during the liquidation to a later point when the price is higher. By selling one less unit early on, the price would overshoot less initially. However, holding one more unit on the balance sheet for longer makes the variance constraint tighter and forces faster selling and lower prices in the future. It is this future price decline that makes the deviation unprofitable.

4 One or Two Lenders?

Since overshooting is caused by the lenders' balance sheet constraints, one possible way to alleviate potential price overshooting is to spread a collateral position across multiple lenders. In that case, the collateral position each lender needs to liquidate in the case of default is smaller relative to its balance sheet. In this section I show that while spreading a collateral position over a number of lenders always reduces price overshooting, it may also decrease the expected liquidation value of the collateral position. This means that the creditor structure of a financial institution involves an important tradeoff between risk-bearing capacity and strategic racing.

To analyze this tradeoff, I consider a stylized financial system with two prime brokers (lenders) who lend to two hedge funds (financial institutions). Each hedge fund owns a collateralized position of size X in the risky asset. I compare two different settings. In the first, the hedge funds have only one prime broker who holds and, in the case of default of the hedge fund, liquidates the entire collateral position. This is illustrated in the left panel of Figure 3. In the alternative setting, the hedge funds spread their collateralized lending between the two prime brokers, such that, in the event of default, each prime broker receives a smaller share of the outstanding collateral. This is illustrated in the right panel of Figure 3. I assume that each of the two hedge funds defaults individually with probability p, and that with probability q both funds will default. With probability $1 - 2p - q$ there is no default. Since the hedge funds are invested in the same asset, the individual default event is best interpreted as an exogenous wealth shock, potentially stemming from other investments the hedge fund has made. As before, the two prime brokers can each take an amount $\overline{V}$ of risk onto their balance sheet during the liquidation process.

Importantly, the comparison is set up such that the aggregate position of the hedge funds (both of them hold a position of size X) and the aggregate risk bearing capacity of the prime broking sector (each prime broker has a variance constraint $\overline{V}$) are held fixed across the two settings. Only the allocation of collateral between the two prime brokers changes, which means that the results are not driven by an implicit change in aggregate risk or aggregate risk bearing capacity.

First consider the setting in which each prime broker lends to only one hedge fund. I will refer to this setup as 'monopolistic finance'.[11] In the monopolistic case, the default of a hedge fund means that the prime broker to that hedge fund receives the entire collateral position

[11] Not that the term 'monopolistic' is refers only to the number of prime brokers used by a hedge fund, but not to monopoly power or monopoly rents extracted by the prime broker, which are not considered here.

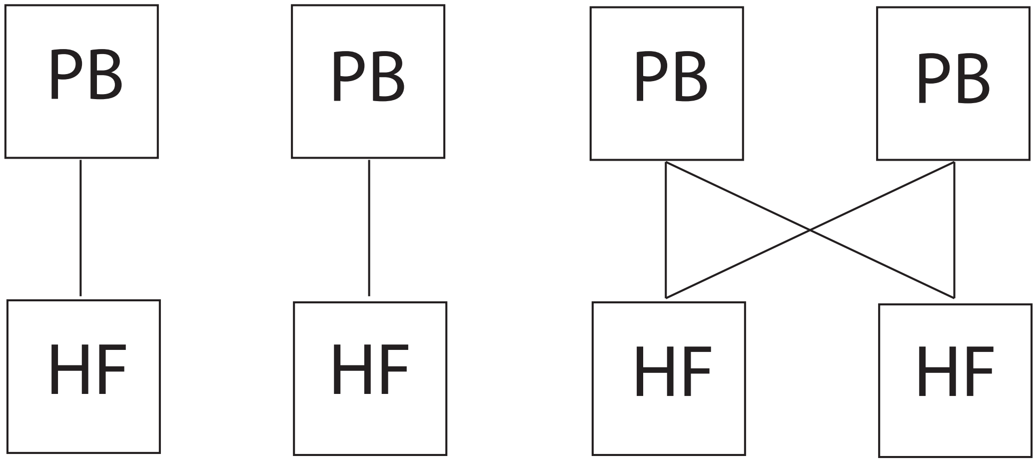

Figure 3: **A stylized financial system with two hedge funds and two prime brokers.** In the left panel, each prime broker lends to just one hedge fund (monopolistic finance). In the case of default of one hedge fund, the affected prime broker unwinds the entire position of the defaulted fund, X, as a monopolist. In the right panel, each prime broker is a counterparty to both hedge funds (distributed finance). In the case of default of one hedge fund both prime brokers unwind $\frac{X}{2}$ units of collateral as duopolists.

X and liquidates it subject to its balance sheet constraint $\overline{V}$. Note that when only one hedge fund defaults, the prime broker can liquidate the collateral as a monopolist. When both hedge funds default, on the other hand, the two prime brokers unwind X units of collateral each, which means that they act as duopolists when liquidating.

Now consider the case in which the hedge funds' positions are spread equally across both prime brokers, which I call 'distributed finance'. When just one fund defaults, each prime broker receives $\frac{X}{2}$ units of collateral. When both hedge fund defaults, the prime brokers seize a total of X units of collateral, $\frac{X}{2}$ from each defaulted fund. Note that in the distributed setup, the prime brokers *always* sell as duopolists.

Price overshooting under monopolistic and distributed financing. I first show that, as conjectured above, distributing the collateral position among multiple prime brokers reduces price overshooting. This follows from a direct application of Proposition 3.

Corollary 2 *In the monopolistic setup the price overshoots if and only if*

$$X > \sqrt{2\frac{\gamma}{\lambda}\frac{\overline{V}}{\sigma_F^2}}. \tag{16}$$

Under distributed finance (i) when one hedge fund defaults the price overshoots if and only if

$$X > 2\sqrt{2\frac{\gamma}{\lambda}\frac{\overline{V}}{\sigma_F^2}}, \tag{17}$$

and (ii) when both funds default, the overshooting condition is the same as in the monopolistic setup.

When the price overshoots, the amount of price overshooting is always strictly larger under monopolistic finance than under distributed finance.

Proof. See appendix. ■

Corollary 2 shows that in the case that only one of the two hedge funds defaults, overshooting occurs for a larger set of parameter values under the monopolistic setup than under the distributed setup. The reason is that overshooting can only occur when the liquidating prime brokers are constrained, i.e. when $\phi > 0$. Since the constraint is always more binding in the monopolistic setting than in the distributed setting, the expected price overshoots for a larger set of parameter values under monopolistic finance. Moreover, when overshooting occurs under both setups, the extent of overshooting is always strictly larger under monopolistic finance.

Comparing expected liquidation values across financing arrangements. While distributing the position across multiple prime brokers alleviates balance sheet constraints and reduces overshooting when it occurs, this does not imply that it will always raise the expected liquidation value of the collateral position. This is the case, since the racing to the market that occurs when multiple prime brokers liquidate at the same time may mean that the additional risk-bearing capacity generated by adding another prime broker may not be used in equilibrium. In fact, racing the market can cause the competing prime brokers to liquidate more disorderly than a monopolist, despite their large joint balance sheet capacity. In this subsection I analytically characterize this tradeoff and show for which parameter values on or two prime brokers lead to higher expected liquidation proceeds.

First, consider the monopolistic setup. When either of the two hedge funds defaults individually in the monopolistic setup, the expected liquidation payoff to the liquidating prime broker is given by $\Pi^M(X, \overline{V})$. This is the payoff to a monopolist with variance constraint $\overline{V}$, who liquidates an aggregate position of size X. When both hedge funds default simultaneously, the two prime brokers need to liquidate a position of size X each, such that the aggregate amount liquidated is $2X$. Moreover, in this case the two prime brokers act as duopolists selling into the same market, and take this strategic interaction into account when choosing their trading strategies. The joint expected liquidation payoff is then given by $2\Pi^D(X, \overline{V})$, twice the payoff of a duopolist who liquidates a position of size X, given its variance constraint $\overline{V}$.

Now consider the distributed setup. When just one fund defaults, each prime broker receives $\frac{X}{2}$ units of collateral. The joint liquidation payoff to the prime brokers is then given by $2\Pi^D(\frac{X}{2}, \overline{V})$, the expected unwind value for two duopolists who liquidate a position of size $\frac{X}{2}$ each, given their individual variance constraints $\overline{V}$. When both hedge funds default, on the other hand, the expected aggregate liquidation payoff is given by $2\Pi^D(X, \overline{V})$, twice the liquidation payoff to a duopolist liquidating an aggregate position of X ($\frac{X}{2}$ from each defaulted fund), given its variance constraint $\overline{V}$. To facilitate comparison, the payoffs in the two settings are summarized in the following table.

	Monopolistic Finance	Distributed Finance
HF1 defaults	$\Pi^M(X, \overline{V})$	$2\Pi^D(\frac{X}{2}, \overline{V})$
HF2 defaults	$\Pi^M(X, \overline{V})$	$2\Pi^D(\frac{X}{2}, \overline{V})$
Both HF default	$2\Pi^D(X, \overline{V})$	$2\Pi^D(X, \overline{V})$

The first thing to notice is that in the case in which both hedge funds default, the expected liquidation payoff does not depend on the financing arrangement; in both cases—financing with a single prime broker or across multiple prime brokers—the joint liquidation payoff is given by $2\Pi^D(X, \overline{V})$. In order to determine which setting generates a higher expected liquidation value, it is thus sufficient to compare the two regimes in the case in which just one hedge fund defaults. In that case, as the table shows, monopolistic financing leads to an aggregate expected payoff of $\Pi^M(X, \overline{V})$ while distributed financing leads to an expected liquidation payoff of $2\Pi^D(\frac{X}{2}, \overline{V})$. This implies that spreading the collateral position across the two prime brokers leads to a higher expected liquidation value if and only if

$$2\Pi^D(\frac{X}{2}, \overline{V}) > \Pi^M(X, \overline{V}). \tag{18}$$

Equation (18) captures the fundamental tradeoff in choosing the financing arrangement. Spreading the collateral position across multiple prime brokers allows each prime broker to liquidate more orderly, since it reduces the size of the collateral position to be liquidated by each prime broker relative to their variance constraint. However, at the same time it forces the prime brokers to liquidate as duopolists, which results in strategic inefficiencies from racing to the market whenever $\gamma > 0$. This leads to the first conclusion: when there is no permanent price impact ($\gamma = 0$), spreading a collateral position across multiple prime brokers always raises the expected liquidation payoff to the prime brokers, as it allows to spread risk across prime brokers without causing strategic inefficiencies during the liquidation process. In other words, when competition among sellers does not affect trading strategies, no real tradeoff emerges.

Once we allow for permanent price effects, i.e. $\gamma > 0$, a non-trivial tradeoff emerges. In addition to the diversification benefit of spreading the position across two prime brokers, we now need to consider the change in strategic behavior that results from the prime brokers' incentive to race to the market. Recall from equation (13) that the liquidation inefficiency is completely determined by the equilibrium value of the selling intensity parameter a. This means that, in order to compare the change in the expected unwind value across the two regimes, we only need to determine whether the equilibrium value of a when a monopolist sells a position of size X is smaller or larger than the equilibrium level of a when two duopolists sell $\frac{X}{2}$ of the collateral asset each, i.e. $a^M(X, \overline{V}) \lesseqgtr a^D(\frac{X}{2}, \overline{V})$. This leads to the following proposition.

Proposition 4 *In a setting with two prime brokers and two hedge funds, in which each prime broker can liquidate assets subject to its variance constraint $\overline{V}$, the expected liquidation value of the collateral position is larger under distributed finance than under monopolistic finance* *if and only if*

$$X > \sqrt{\frac{2}{3}\frac{\gamma}{\lambda}\frac{\overline{V}}{\sigma_F^2}}. \tag{19}$$

Proof. See appendix. ■

Proposition 4 shows that a financial system in which the two hedge funds spread their positions across the two prime brokers leads to a higher expected post-default liquidation value for the collateral position when the position size X is sufficiently large or, equivalently, when the risk-bearing capacity of the prime brokers is sufficiently small. Under these conditions, the benefit from reducing the amount of collateral each prime broker needs to unwind relative to the size of its balance sheet outweighs the inefficiencies that result from racing to the market.

Equation (19) shows that the critical position size above which distributed finance leads to higher expected liquidation values depends on the liquidity parameters of the collateral asset. Distributed finance leads to higher expected liquidation values when the short-term illiquidity parameter λ is large relative to the permanent price effect γ. This is because when λ is large relative to γ, the duopolists have relatively little incentive to race each other to the market. In contrast, when the permanent price effect γ is large relative to λ, there is a strong incentive for the duopolists to race to the market, such that splitting the position among multiple prime brokers is less likely to raise the liquidation value. Proposition 4 also shows that splitting the position is more likely to be raise the expected liquidation value when prime brokers are relatively variance constrained, i.e. when the ratio of their variance limit to the fundamental volatility of the asset, i.e. $\frac{\overline{V}}{\sigma_F^2}$, is low, since in that case the benefits from diversification are particularly large.

The key insight is that distributing positions across the two prime brokers raises the expected liquidation value, unless the competitive forces that lead to racing to the market are sufficiently large to prevent the prime brokers from using their risk-bearing capacity effectively. Adding risk-bearing capacity thus only raises the expected liquidation value if it gets used in equilibrium. This is illustrated in Figure 4. The left panel shows the case

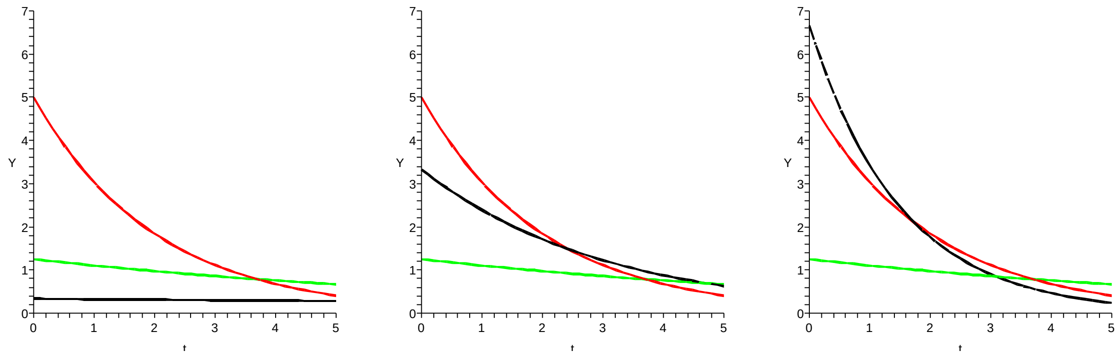

Figure 4: The figure shows the equilibrium trading rates (constrained and unconstrained) for three different cases. Whether using two lenders raises liquidation values depends on the location of the black line, which is the unconstrained duopoly solution. In the left panel it lies below the trading rate a monopolist would choose (top) and the constrained solution for duopolists (middle). Hence duopolists use all their risk-bearing capacity in equilibrium. The middle panel shows the situation in which the duopolists will not use all of their risk-bearing capacity, but spreading the position across two prime brokers is still beneficial. The right panel shows the situation in which the duopolists would trade faster than a monopolist, i.e. the racing effect dominates. The black line tilts upward as $\frac{\gamma}{\lambda}$ increases.

in which both the monopolist and the duopolists are constrained in equilibrium, i.e. both use all of their risk-bearing capacity during liquidation. The top line is the trading rate a monopolist would choose. The middle line is the trading rate duopolists could theoretically choose if they used all their risk-bearing capacity. The bottom line is the trading rate that duopolists would choose if they were unconstrained. Since the unconstrained solution would imply a trading rate that violates the variance constraint, the duopolists are constrained in equilibrium and their aggregate trading rate is given by the middle line. In this case distributed finance clearly raises the expected liquidation value. The middle panel shows the situation in which the duopolists will not use all of their risk-bearing capacity in equilibrium—the unconstrained duopoly trading strategy now lies above the constrained duopoly solution. However, the aggregate duopoly trading rate still lies below the trading rate a monopolist would choose, such that spreading the position across two prime brokers still raises expected liquidation proceeds. The right panel shows the situation in which competitive pressure forces duopolists to trade faster than a monopolist would. This is the situation in which a monopolistic setting leads to a higher expected unwind value.

Proposition 4 also implies that when balance sheets are strong, monopolistic finance is more likely to lead to higher collateral liquidation values. This is the case since, when $\overline{V}$ is large, competing prime brokers will not be able to use their risk-bearing capacity effectively when liquidating. When balance sheets are weak, on the other hand, the benefits from added risk-bearing capacity are likely to outweigh the losses from racing to the market. This suggests that, if balance sheet size varies across the financial cycle, as documented by Adrian and Shin (2008), a creditor structure with multiple prime brokers is likely to be more desirable during a financial bust, whereas during financial booms, the monopolistic setup is preferable.

This result is illustrated in Figure 5. The figure depicts the expected liquidation value of a given aggregate position $\overline{X}$ as a function of the variance constraint $\overline{V}$. The left panel shows the expected liquidation value for a monopolist, and the right panel the expected (aggregate) liquidation value for two duopolists. Since a monopolist always uses all of its risk-bearing capacity in the liquidation process, the expected payoff for a monopolist is monotonically increasing in $\overline{V}$. Duopolists, on the other hand, are only using their entire risk-bearing capacity for low values of $\overline{V}$. For sufficiently high levels of $\overline{V}$, the variance constraint does not bind in equilibrium and the expected profit curve is flat. In that region relaxing the variance constraint of the duopolists does not increase the expected value of the liquidation payoff. The figure shows that when $\overline{V}$ is small relative to the size of the position, distributed finance leads to a higher expected liquidation value. When the risk-bearing capacity is large, on the other hand, monopolistic finance leads to a higher expected aggregate liquidation payoff.

Default correlation. Recall that up to now we have focused on the case in which only one hedge fund defaults. This was sufficient to determine a ranking of expected liquidation payoffs between the two regimes, since when both hedge funds default, expected liquidation payoffs do not differ between the two financing arrangements. Which financing regime leads to the higher expected liquidation value is thus independent from the joint default state. However, the extent to which the expected liquidation payoff differs across the two regimes depends on the likelihood of the joint default state and, consequently, on the default correlation between the two hedge funds.

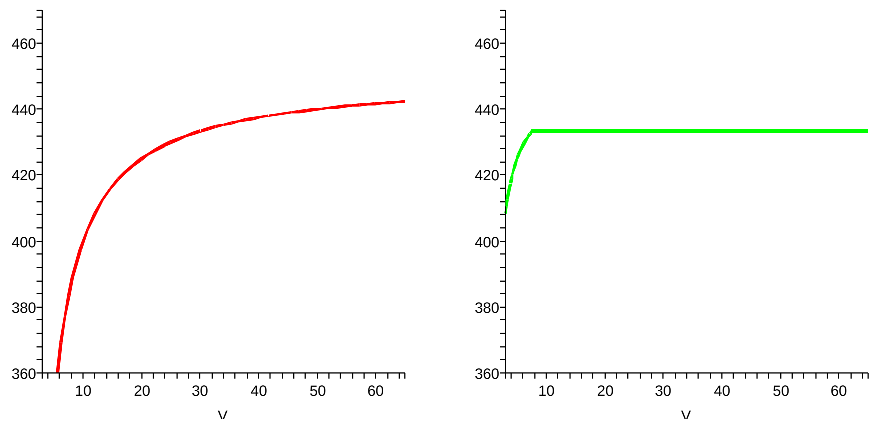

Figure 5: **Expected aggregate liquidation payoff Π as a function of the variance constraint in the monopolistic case (left panel) and the duopoly case (right panel).** The monopolist always uses its entire risk-bearing capacity. This means his payoff increases monotonically in $\overline{V}$. The duopolists are constrained for low values of $\overline{V}$, but unconstrained above a critical value. In the unconstrained region the curve is flat. For large values of $\overline{V}$ the monopolist's expected profit is higher, whereas for low values of $\overline{V}$ the expected duopoly liquidation value is larger. The parameter values are $F = 50, \overline{X} = 10, \gamma = 1, \lambda = 0.2, \sigma_F^2 = 1$.

Corollary 3 *The expected difference of liquidation proceeds under distributed and monopolistic finance is decreasing in ρ, the default correlation of the two funds. When $\rho = 1$ individual defaults never occur and monopolistic and distributed finance lead to the same outcome. When $\rho < 1$ the two frameworks will generally differ and the difference, irrespective of its sign, grows as ρ gets smaller.*

Proof. See appendix. ■

5 Margin Setting

The above analysis has important implications for counterparty risk management. When the expected liquidation value of a collateral position depends on the illiquidity of the collateral asset, the lenders' balance sheet conditions, and their strategic interaction during the liquidation phase, lenders should take this into account *ex ante* when setting margins to manage their counterparty credit exposure. In this section I illustrate the model's implications for margin setting through a simple example.

Assume that the lenders set their margins ex-ante such that the margin covers the expected loss in the case of default. Of course, in practice margins may not be set to just cover expected losses; often they are set to cover a risk measure, such as value-at-risk, which is a combination of expected loss and the distribution of losses. Nevertheless, even the stylized case, in which margins just depend on expected losses, suffices to illustrate that illiquidity, balance sheet constraints and strategic interaction must be taken into account. Assume also (to save on notation) that the only expected loss in the case of default comes from illiquidity losses incurred while unwinding the asset, i.e. that the expected value of the fundamental conditional on default is the unconditional expectation. This assumption is unrealistic, but could easily be relaxed by adding another term to the margin expression. Given these assumptions, Proposition 5 shows how the model can be used to calculate margins for the two financing arrangements introduced in section 4.

Proposition 5 *The per share margin charged by lender i to cover its expected loss given default in the monopolistic setup is given by*

$$M^M = (p+2q)\frac{\gamma}{2}X + [pa^M(X,\overline{V}) + 2qa^D(X,\overline{V})]\frac{\lambda}{2}X. \tag{20}$$

The per share margin charged by lender i to cover its expected loss given default in the duopolistic setup is given by

$$M^D = (p+2q)\frac{\gamma}{2}X + [pa^D(\frac{X}{2},\overline{V}) + 2qa^D(X,\overline{V})]\frac{\lambda}{2}X. \tag{21}$$

Proof. See appendix. ■

In both cases, the margin expression has two parts. The first term captures the expected loss from walking down the demand curve during a potential liquidation. The second term represents the expected loss due to short-term illiquidity during the liquidation process.

Importantly, this second term depends on the equilibrium trading parameters a^M and a^D, which capture strategic interaction and the effect of balance sheet constraints during liquidation. This implies that when managing counterparty risk, the factors that determine the equilibrium value of the trading intensity a should be explicitly taken into account by lenders. To give a concrete example, when a hedge fund has a large number of creditors, the prime broker's margin should reflect that liquidations after a potential default will be 'crowded trades'. The collateral asset's effective liquidity following a default will thus be significantly lower than during normal times. Likewise, a lender who has similar exposures to the financial institution it is extending credit to, should take into account the covariance of its own balance sheet constraint with the default of the financial institutions it is lending to. If the constraint is correlated with the default state, the lender's ability to bear the risk during a liquidation may be impaired (low $\overline{V}$) just when the financial institution defaults. This is an important consideration since, to continue the above example, many large broker dealers also have trading operations on their own, whose returns may be significantly correlated to returns in the hedge fund industry.

6 Block Trades

The model also provides a framework to analyze large block transactions that occur when a financial institution defaults. To see this, assume that rather than liquidating the collateral asset into a downward-sloping demand curve, the lender(s) (or the troubled financial institution) can alternatively sell the collateral position to a buyer with a strong balance sheet (large $\overline{V}$). Recently Citadel has acted as such a 'vulture' or 'deep pocket' buyer a number of times. In September 2006 Citadel, together with JPMorgan, purchased the entire energy portfolio of Amaranth Advisors, a hedge fund that had collapsed after a loss of more than \$4.6 billion in natural gas positions. In July 2007 Citadel took over the entire portfolio of Sowood Capital Management, a Boston-based hedge fund that, after heavy losses in the credit markets, was struggling to meet margin calls from creditors. Another interpretation of the 'deep pocket' buyer are asset purchases made by the government as a buyer of last resort. In this case the government takes on the entire position upon default and unwinds it slowly, using its large public balance sheet.

To incorporate a 'deep pocket' buyer into the model, consider the default of an individual financial institution, say a hedge fund with a collateral position of size X. Assume that the vulture buyer does not face a balance sheet constraint. This assumption is not crucial;

the analysis works similarly as long as the vulture buyer's balance sheet is stronger than that of the lender or distressed financial institution. However, while the vulture buyer is not balance-sheet constrained, it has to incur a cost c to bid for the collateral position. This cost may reflect, among other things, the effort and human resources needed to learn about the collateral asset and to value the position. For example, commenting on Citadel's investment in Sowood, the Financial Times notes that "Citadel is among the best positioned hedge funds for such large, complex trades. Unlike some funds that manage many billions of dollars with small staffs, Citadel employs more than 1,000 people. That gives it firepower to evaluate complex assets quickly and manage them after pouncing."[12] This quote suggests that vulture buyers need to incur costs, both to keep spare capital, but also in terms of human resources and spare 'valuation capacity'. Alternatively, c may reflect the fact that the 'deep pocket' buyer has less expertise in the collateral asset than the original owner.

Since the vulture buyer is not balance-sheet constrained, it can liquidate the collateral asset infinitely slowly, such that no temporary price impact costs are incurred.[13] The vulture buyer's valuation of the entire collateral position is thus given by $F(0)X - \frac{\gamma}{2}X^2 - c$. The block trade results in gains from trade whenever the loss from temporary price pressure to the constrained lender(s) is larger than the lump-sum investment to be made by the vulture investor. To pin down the transaction price, I assume that the vulture buyer has all bargaining power, i.e. it can fully extract the gains from trade from the block trade.[14]

Proposition 6 *In the presence of a deep-pocket buyer, a block trade results in positive gains from trade when the vulture buyer's fixed cost c is smaller than the price pressure cost incurred by the lender(s), i.e.*

$$c < \frac{\lambda}{2} a X^2, \tag{22}$$

[12] "Citadel's Appetite", The Lex Column, Financial Times, August 1, 2007.

[13] I assume that the vulture buyer liquidates the asset, rather than holding it until until maturity. However, the analysis can easily be adapted to also allow for this case; the only difference would be that the vulture buyer would not have to incur the loss due to permanent price effects of trading.

[14] While giving all the bargaining power to the vulture buyer may be slightly extreme, it is reasonable to assume that it will extract most of the surplus, since in equilibrium it needs to be compensated for holding spare capital that allows him to buy when an opportunity arises.

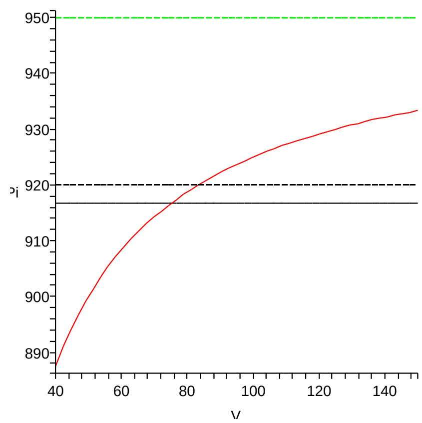

Figure 6: **Gains from block trades.** The figure shows that when the lender's variance limit $\overline{V}$ is sufficiently low, there are positive gains from trade when the whole collateral position is sold to a vulture buyer with a strong balance sheet. The top line is the expected liquidation value for an unconstrained seller. The dotted line below is the vulture buyer's valuation, which takes into account the fixed cost c. The upward sloping line is the expected liquidation value for the constrained trader as a function of the variance constraint $\overline{V}$. The solid horizontal line is the upper bound of the constrained traders' valuation under predatory trading. The parameters are $F = 100, \gamma = 1, \lambda = 1, \sigma_F^2 = 1, c = 30$.

where a is the equilibrium trading intensity of the constrained trader(s). The price for the block is determined by the lender(s)' outside option $F(0)X - \frac{\gamma}{2}X^2 - \frac{\lambda}{2}aX^2$ and the vulture buyer's expected profit from the block trade is given by

$$\Pi^V = \frac{\lambda}{2}aX^2 - c. \tag{23}$$

Proof. See appendix. ■

Note that the deep pocket buyer creates value in two ways. First, its stronger balance allows to unwind the collateral position more slowly. Second, if assets are purchased from a number of parties that would have otherwise liquidated at the same time, the deep pocket buyer concentrates the position before unwinding it, and thus reduces 'racing to the market' during the sell-off. Importantly, this second, strategic effect is absent when the government just injects equity into institutions rather than buying the troubled positions. While, in terms of the model, injecting equity may raise $\overline{V}$, 'racing to the market' among sellers may still lead to a disorderly liquidation.

Figure 6 illustrates block trades for the case of a monopolistic lender. The upward-sloping line is the valuation of the constrained monopolistic seller as a function of the balance sheet constraint $\overline{V}$. The line is upward-sloping, since relaxing the variance constraint allows the lender to liquidate in a more orderly fashion. The top horizontal line in the figure is the expected liquidation value of an unconstrained seller who does not incur temporary price impact costs. Note that when $\overline{V}$ is very large, the constrained monopolist's liquidation payoff converges to that of the unconstrained seller. The middle line is the valuation of the vulture buyer, which is given by the unconstrained valuation less the fixed cost c. The figure shows that when the lender is sufficiently constrained, the block transaction leads to gains from trade. When all bargaining power lies with the vulture buyer, it fully appropriates these gains from trade. Substituting into equation (23), the vulture buyer's expected profit in the case of a monopolistic lender is given by $\frac{\lambda\sigma_F^2 X^4}{2\overline{V}} - c$.

Finally, note that the vulture buyer can potentially make an even larger profit from the block transaction if it can threaten to act as a 'predatory trader'. Predatory trading is defined by Brunnermeier and Pedersen (2005) as trading that 'induces and/or exploits the need of other investors to reduce their positions'. In this case, it means that the vulture buyer threatens to exploit the lender's need to liquidate if it does not agree to the block trade. The vulture buyer would do this by initially also selling the collateral asset, driving down the price even further, and repurchasing the asset later on. Predatory trading reduces the amount that the variance constrained seller can raise when liquidating the asset into the market even below the liquidity losses caused by the variance constraint. By lowering the constrained trader's outside option, the vulture buyer increases its profits from the block trade, and can potentially increase the range of parameter values for which block trades occur.

The solid horizontal line in Figure 6 is the upper bound to how much the constrained seller expects to raise by liquidating the collateral position under predatory trading by the vulture buyer. This upper bound is calculated by assuming the, when predation occurs, the variance constraint of the seller is not binding in equilibrium. It is an upper bound since the expected liquidation payoff when the variance constraint binds would be even lower. Note that in the example depicted in the figure, the threat of predatory trading makes the block trade profitable irrespective of the variance limit $\overline{V}$.

7 Conclusion

This paper provides a new model to analyze collateral liquidations following the default of a financial institution. Liquidation strategies are driven both by strategic considerations and by lenders' balance sheet constraints. When the liquidating lenders' risk-bearing capacity is small, the price can overshoot during liquidation, possibly causing spillovers to other market participants. Overshooting is balance sheet driven and is mitigated when the collateral position is spread across multiple lenders, but only at the cost of potential strategic racing, which can lower expected liquidation values. This means that there is a fundamental trade-off between risk sharing and liquidation inefficiencies when choosing the number of lenders a financial institution or hedge fund borrows from. I show explicitly how this tradeoff depends on the size of the collateral position, the lenders' balance sheet constraints, and the illiquidity of the collateral asset. The model also implies that, rather than relying on purely statistical models, lenders should take into account their own balance sheet constraints and strategic interaction during collateral sell-offs when managing counterparty exposure through margin setting. Finally, the model provides a framework to analyze block trades in which 'deep pocket' buyer acquires the entire collateral position from a constrained lender or ailing financial institution or hedge fund.

There are a number of directions in which the model could be extended in future research. First, the model assumes that the exposures of each lender are common knowledge. Relaxing this assumption would allow us to analyze how uncertainty about exposures affects liquidation dynamics. Second, the model takes the size of the financial institutions' positions as given. Ideally, this position should be modeled as the outcome of an ex ante stage in which lenders compete on margins to generate lending business.

8 Appendix

Proof of Proposition 1: In this section I derive the optimal liquidation strategies for the case of n lenders. The monopolistic solution follows straightforwardly from setting $n = 1$. Lender i chooses a liquidation strategy to maximize the expected liquidation payoff subject to the variance of the liquidation payoff remaining below $\overline{V}$. The control variable is $Y_i(t)$, the trading rate of lender i at time t. The state variable is $X_i(t)$, lender i's remaining collateral position at time t. $\overline{X}_i$ denotes the overall amount of the collateral asset to be liquidated by lender i. Recall that $\overline{X}_i$ is symmetric across lenders.

The liquidation payoff that results from a trading strategy $\{Y_i(t)\}$ is given by $\int_0^\infty -Y_i(t)P(t)dt$. To take into account the variance constraint we need to calculate the variance of this expression. Since $X_i(t)$ is continuously differentiable, we can apply the integration by parts formula and rewrite the liquidation payoff as

$$-\int_0^\infty Y_i(t)P(t)dt = X_i(0)P(0) - X_i(\infty)P(\infty) + \int_0^\infty \sigma_F X_i(t)dP(t),$$

which, using the boundary condition $X_i(\infty) = 0$, simplifies to

$$-\int_0^\infty Y_i(t)P(t)dt = X_i(0)P(0) + \int_0^\infty \sigma_F X_i(t)dP(t).$$

Using this result, we can calculate the variance of the liquidation payoff as

$$\begin{aligned} V\left[-\int_0^\infty Y_i(t)P(t)dt\right] &= V\left[X_i(0)P(0) + \int_0^\infty \sigma_F X_i(t)dP(t)\right] \\ &= V\left[\int_0^\infty \sigma_F X_i(t)dP(t)\right]. \end{aligned}$$

Since the only random component of the price is the Brownian motion of the fundamental, we can rewrite the variance as

$$V\left[\int_0^\infty -Y_i(t)P(t)dt\right] = V\left[\int_0^\infty \sigma_F X_i(t)dW(t)\right] = \int_0^\infty \sigma_F^2 \left[X_i(t)\right]^2 dt,$$

which is the expression given in the text. The last step follows from the isometry property of Brownian motion.

Using this result we can write lender i's maximization problem as

$$\max_{Y_i(t)} E \int_0^\infty -Y_i(t)P(t)dt$$

subject to

$$\begin{aligned} dX_i(t) &= Y_i(t)dt \text{ (evolution of state variable)} \\ P(t) &= F(t) + \gamma \sum_{i=1}^n [X_i(t) - \overline{X}_i] + \lambda \sum_{i=1}^n Y_i(t) \text{ (pricing equation)} \\ dF(t) &= \sigma_F dW(t) \text{ (evolution of fundamental)} \\ \int_0^\infty \sigma_F^2 [X(t)]^2 dt &\leq \overline{V} \text{(variance constraint)} \end{aligned}$$

and subject to the trading constraints

$$\begin{aligned} X_i(0) &= \overline{X}_i \\ \overline{X}_i + \int_0^\infty Y_i(t)dt &= 0. \end{aligned}$$

The variance constraint makes this maximization problem an isoperimetric problem (see, for example, section 7 in Kamien and Schwartz (1991)). We can write the Lagrangian as

$$\begin{aligned} \mathcal{L} &= -\int_0^\infty Y_i(t) \left[F(0) + \gamma \sum_{i=1}^n [X_i(t) - \overline{X}_i] + \lambda \sum_{i=1}^n Y_i(t) \right] dt - \phi \left[\int_0^\infty \sigma_F^2 [X_i(t)]^2 dt - \overline{V} \right] \\ &= -\int_0^\infty \left[Y_i(t) \left[F(0) + \gamma \sum_{i=1}^n [X_i(t) - \overline{X}_i] + \lambda \sum_{i=1}^n Y_i(t) \right] + \phi \sigma_F^2 [X_i(t)]^2 \right] dt + \phi \overline{V}, \end{aligned}$$

where ϕ is the Lagrange multiplier associated with the variance constraint. This allows us to solve the problem using Hamiltonian methods. The Hamiltonian is given by

$$\mathcal{H} = -Y_i(t) \left[F(0) + \gamma \sum_{i=1}^n [X_i(t) - \overline{X}_i] + \lambda \sum_{i=1}^n Y_i(t) \right] - \phi \sigma_F^2 [X_i(t)]^2 + q(t) Y_i(t),$$

where $q(t)$ is the costate variable. Taking first order conditions, $\frac{\partial \mathcal{H}}{\partial Y_i(t)} = 0$ and $\frac{\partial \mathcal{H}}{\partial X_i(t)} = -\dot{q}(t)$, we get:

$$\begin{aligned} F(0) + \gamma \sum_{i=1}^{n} [X_i(t) - \overline{X}_i] + \lambda \sum_{i=1}^{n} Y_i(t) + \lambda Y_i(t) - q(t) &= 0 \\ -\gamma Y_i(t) - 2\phi \sigma_F^2 X_i(t) &= -\dot{q}(t). \end{aligned}$$

Taking the time derivative of the first equation and substituting in the second we get, imposing the symmetry assumption,

$$(n-1)\gamma Y_i(t) + \lambda(n+1)\dot{Y}_i(t) - 2\phi\sigma_F^2 X_i(t) = 0.$$

Solving this ordinary differential equation yields the open-loop Nash equilibrium strategies:

$$X_i(t) = Ae^{-\frac{1}{2}\frac{(n-1)\gamma - \Gamma}{(n+1)\lambda}t} + Be^{-\frac{1}{2}\frac{(n-1)\gamma + \Gamma}{(n+1)\lambda}t},$$

where

$$\Gamma = \sqrt{\gamma^2(n-1)^2 + 8(n+1)\phi\sigma_F^2\lambda}.$$

We can determine the constants A and B from the boundary conditions. $\lim_{T\to\infty} X(T) = 0$ implies that $A = 0$. Imposing in addition $X(0) = \overline{X}_i$ we know that $B = \overline{X}_i$. This yields the result stated in the proposition. Uniqueness follows from the concavity of the objective function. The variance of the equilibrium trading strategy is given by

$$V = \int_0^\infty \sigma_F^2 [X_i(t)]^2 dt = \int_0^\infty \sigma_F^2 [\overline{X}_i e^{-at}]^2 dt = \sigma_F^2 \overline{X}_i^2 \int_0^\infty e^{-2at} dt = \frac{\sigma_F^2 \overline{X}_i^2}{2a}.$$

Now we distinguish two cases. In the first case, the variance constraint does not bind in equilibrium, i.e. $V < \overline{V}$. In that case, $\phi = 0$ and (7) simplifies to $a = \frac{(n-1)\gamma}{(n+1)\lambda}$. The variance constraint is non-binding in equilibrium whenever

$$\overline{X}_i < \sqrt{\frac{2\overline{V}}{\sigma_F^2} \frac{n-1}{n+1} \frac{\gamma}{\lambda}}.$$

In the second case, when $\overline{X}_i > \sqrt{\frac{2\overline{V}}{\sigma_F^2}\frac{n-1}{n+1}\frac{\gamma}{\lambda}}$, the variance constraint is binding in equilibrium $\phi > 0$. This condition is more likely to be met for smaller n. This is the case since when sellers compete, equilibrium selling is faster and the payoff variance lower. In fact, a monopolist will always be at the constraint since his unconstrained solution would be to liquidate by trading at a constant rate until infinity, which would imply infinite variance.

We can then solve for the lagrange multiplier associated with the variance constraint,

$$\phi = \lambda(n+1)\frac{\left[\sigma_F \overline{X}_i^2\right]^2}{8\overline{V}^2} - \gamma(n-1)\frac{\sigma_F \overline{X}_i^2}{4\overline{V}}.$$

The interpretation of the Lagrange multiplier ϕ is that of a shadow price. Relaxing the variance constraint by one unit results in an increase in the payoff to the seller by ϕ units.

Proof of Proposition 2: The expression is obtained by substituting the equilibrium trading rate (6) and the equilibrium collateral position (11) into the objective function. The resulting expression simplifies to

$$\Pi(\overline{X}_i, \overline{V}) = F(0)\overline{X}_i - \frac{\gamma}{2}\overline{X}_i\overline{X} - \frac{\lambda}{2}a\overline{X}_i\overline{X}$$

where a depends on whether the variance constraint is binding, i.e. whether $\phi > 0$. We can substitute in for a for the different cases. When the variance constraint is binding in equilibrium we have

$$\Pi^C(\overline{X}_i, \overline{V}) = F(0)\overline{X}_i - \frac{\gamma}{2}\frac{2n}{n+1}\overline{X}_i\overline{X} - \frac{\lambda}{4}\frac{\sigma_F^2 X_0^2}{\overline{V}}\overline{X}_i\overline{X}.$$

When the variance constraint is not binding,

$$\Pi^U(\overline{X}_i, \overline{V}) = F(0)\overline{X}_i - \frac{n}{n+1}\gamma\overline{X}_i\overline{X}.$$

Proof of Corollary 1: The expression is obtained by substituting (6) and (11) into the price function (1) and simplifying the resulting expression.

Proof of Proposition 3: When overshooting occurs, the price will always overshoot instantly at time 0. To see this, assume that the price does not overshoot at time 0, but at some later time. Using equations (11) and (14), the initial price drop is given by $\lambda a\overline{X}$, and the eventual expected price drop, after short-term price pressure has subsided, is given by $\gamma\overline{X}$. When the price does not overshoot at time 0, we must have that $\lambda a < \gamma$. However, when $\lambda a < \gamma$, the time derivative of the expected price path, $\frac{dP}{dt} = [\lambda a - \gamma]ae^{-at}$, is negative for all t, which means that if the price does not dip below the new expected long-term price at time 0 and then move back up to the new long-run level. Hence, to show overshooting, we only need to compare the instantaneous price drop at time 0 to the final price after liquidation has taken place, which means that the price overshoots if and only if $\lambda a > \gamma$. Using equations (7) and (8), the expression simplifies to

$$\phi > \frac{\gamma^2}{\sigma_F^2 \lambda}, \tag{24}$$

the expression in the proposition. From equation (24) it is clear that overshooting will only occur when the variance constraint is binding.

Proof of Corollary 2: From Proposition 3 we know that the price only overshoots when $\lambda a - \gamma > 0$, and that overshooting can only occur when the variance constraint binds to a sufficienct extent. Substituting in the equilibrium values of a for that case , i.e. $a^M = \frac{\sigma_F^2 X^2}{2\overline{V}}$ for a monopolist, and $a^D = \frac{\sigma_F^2 X^2}{8\overline{V}}$ for duopolists, yields the expressions in the corollary.

The overshooting amount is given by $z = \lambda a\overline{X} - \gamma\overline{X}$. When overshooting occurs under both the monopolistic and the distributed setup, it has to be the case that $a^M > a^D$, which means that the extent of overshooting is larger in the monopolistic setup.

Proof of Proposition 4: From the discussion in the text it follows that distributed finance leads to a higher expected liquidation payoff when $a^D(\frac{X}{2}, \overline{V}) < a^M(X, \overline{V})$. Morever, we have seen that using multiple creditors can only reduce liquidation proceeds when racing occurs, i.e. when lenders do not use all their risk-bearing capacity in equilibrium. Substituting in the equilibrium values of a in the case that the variance constraint is not binding yields

$$\frac{\gamma}{3\lambda} < \frac{\sigma^2 X^2}{2\overline{V}},$$

which simplifies to the expression given in the proposition.

Proof of Corollary 3: Recall that each hedge fund is assumed to default individually with probability p and that a joint default occurs with probability q. We will now write these probabilities as a function of the default correlation between the two funds. Let π be the probability that one fund, i.e. either hedge fund 1 or hedge fund 2, defaults. Then we can express p and q as

$$\begin{aligned} p &= (1-\rho)\pi(1-\pi) \\ q &= \pi^2 + \rho(1-\pi)\pi. \end{aligned}$$

We can then write the expected payoff difference between distributed and monopolistic finance as

$$\begin{aligned} \Delta &= 2p[2V^D(\frac{X}{2},\overline{V}) - V^M(X,\overline{V})] \\ &= 2\left[(1-\rho)\pi(1-\pi)\right]\left[2V^D(\frac{X}{2},\overline{V}) - V^M(X,\overline{V})\right]. \end{aligned}$$

This expression shows that the expected difference between the two regimes decreases in the default correlation ρ. When $\rho = 1$ individual defaults never occur and monopolistic and distributed finance lead to the same outcome. When $\rho < 1$ the two frameworks will generally differ and the difference, irrespective of its sign, increases as ρ gets smaller.

Proof of Proposition 5: As assumed in the text, margins are set to cover the ex-ante expected illiquidity loss that results from liquidation. This can be calculated by using the results from Proposition 2. Consider the monopolistic finance case. With probability p the lender has to liquidate a collateral position of size X as a monopolist. In that case $\overline{X}_i = X$ and $\overline{X} = X$. With probability q, when both hedge funds default, it has to liquidate a collateral position of size X as a duopolist, i.e. $\overline{X}_i = X$ and $\overline{X} = 2X$. Substituting this into the expression for the unwind value yields the margin. The proof is analogous in the distributed case and is omitted for brevity.

Proof of Proposition 6: Assume that one hedge fund of financial institution has defaulted, i.e. the aggregate size of the liquidation is X. The vulture buyer's valuation of the block is given by $F(0)X - \frac{\gamma}{2}X^2 - c$, while the valuation for the liquidating lender(s) is given by $F(0)X - \frac{\gamma}{2}X^2 - \frac{\lambda}{2}aX^2$. The difference of these two expressions (the gain from trade) is positive when $c < \frac{\lambda}{2}aX^2$. If the vulture buyer has all bargaining power, the price of the block transaction is given by the outside option of the lender(s), i.e. liquidating into the market, $F(0)X - \frac{\gamma}{2}X^2 - \frac{\lambda}{2}aX^2$.

The payoff to the lender of liquidating in the presence of predatory trading is calculated by solving a two-player version of the liquidation game, in which the lender liquidates X units of collateral while the predator chooses a trading strategy that exploits the lender's need to liquidate. The predator does this by first selling along with the lender and then buying back later. The proof is analogous to the proof of Proposition 1, but allows for asymmetric trading targets, i.e. the lender has a trading target of selling X units, while the predator has a trading target of 0, i.e. it first sells and then buys back an equivalent amount. The details are omitted for brevity, but are available on request.

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