Search Brokers

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August 24, 2010

Abstract

We consider a market with dynamic random matching and bargaining with two-sided private information. Traders know their valuation for the good before entering the market and steady state distributions in the market are endogenously determined in equilibrium. The market is organized by a profit maximizing broker. We compare the case where the broker can only charge participation fees to buyers and sellers and can influence neither the matching technology nor the bargaining protocol with two other cases. In the first alternative case, the broker can choose the bargaining protocol, but not the matching. In the second case, he can choose both (fully centralized mechanism). We find that the broker gets the same level of profits in optimum in all three cases. Further, the broker makes sure that the same mass of buyers and sellers enters the market in each period and that buyers and sellers trade immediately after entering. We further find that the ratio of (participation) fees in the fully decentralized setup is equal to the ratio of bargaining weights of the buyer and seller and independent of the elasticities of demand. Further, the price structure (i.e. ratio of fees) matters even if bargaining (or price setting) between buyers and sellers is not restricted by the broker.

1 Introduction

Intermediaries providing search platforms for buyers and sellers (or two other groups) potentially willing to trade (or interact) play an important role in many markets. Examples include online trade or auction websites, dating agencies, and credit card issuers. Such platforms have recently received increased interest by both academics and practitioners. For example, there is a controversy on the regulation of the fee structure of credit cards. Other markets show a higher level of centralization by the intermediary, e.g. financial markets, where bid and ask prices are determined centrally.

We consider such search platforms putting attention on two aspects which we consider particularly important. First, buyers and sellers have private information about their valuation for the traded good before they decide to join the platform. Hence the details of the trade or

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bargaining mechanism will have an impact on entry decisions. Second, we consider dynamics in such markets. Hence buyers and sellers have an option value of future trade and distributions in the market are endogenous, because inefficient traders tend to cumulate.

We will also consider different levels of centralization: the intermediary influences both the matching process and the bargaining protocol; only the bargaining protocol; or neither.

We model a search platform in an infinite horizon steady state. In every period, mass one of buyers and sellers enters the market. Buyers and sellers have private information about their valuation for the good. Sellers have one unit of the indivisible good, buyer have unit demand. Matching and bargaining may or may not be centralized, i.e. determined by the broker running the platform. In the completely decentralized platform, we consider random bilateral matching (with a linear matching technology), where a matched buyer and seller play a random proposer game. In this setup, the broker sets participation fees for the buyers and sellers. The ratio of the fees for the buyer and seller is independent of the elasticity of demand and only depends on the bargaining weights. We show that the broker can achieve the same optimal profits with all levels of centralization.

This paper relates to three strands of literature: dynamic random matching with private information (see e.g. Wolinsky (1988); Satterthwaite and Shneyerov (2007, 2008)), intermediaries (see e.g. Gehrig (1993); Yavas (1992); Spulber (1996)), and two-sided markets (see e.g. Caillaud and Jullien (2003); Armstrong (2004); Rochet and Tirole (2006)). It is closest to the dynamic random matching literature. We depart from this strand of literature by assuming that the search platform is owned by a profit maximizing broker. It turns out that if the search costs incurred by traders (which equal the participation fees charged by the broker) are endogenously determined by a profit maximizing broker rather than exogenously given, the equilibrium becomes simpler and can give helpful characterizations of e.g. the fee structure. We depart from the intermediation literature by considering buyers and sellers who bargain bilaterally, by letting the intermediary design a mechanism that takes into account the possibility of delay, of changing the steady state distributions in the market, and traders' option values of future trade. Our main difference to the two sided markets literature is that we analyze the entry decision of traders who know their private types *before* choosing whether to enter the market, and again, by allowing the platform owner to influence distributions and option values.[1] Our main difference in predictions is that the (participation) fee structure depends on

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[1] This is the difference to the part of the two-sided markets literature closest to our paper, where buyers and sellers have private information about their valuations of the good and payments between them are possible and

relative bargaining weights rather than elasticities of demand, that trade occurs in equilibrium in every match, and that the fee structure is non-neutral even if unrestricted transfers are possible between buyers and sellers (i.e. not only the sum of the buyer's and the seller's fee matters, but also the individual composition).

2 Setup

The agents in our model are potential buyers and sellers of a homogeneous, indivisible good. Each buyer has a unit demand for the good, while each seller has unit supply. All traders are risk neutral. Potential buyers are heterogeneous in their valuations (or types) v of the good. Potential sellers are also heterogeneous in their costs (or types) c of providing the good. For simplicity, we assume $v, c \in [0, 1]$. Time is discrete and infinite horizon. The details of the model are as follows:

- Entry: Mass 1 of potential buyers and sellers are born in each period. The types of new potential buyers are drawn i.i.d. from the c.d.f. $F_B(v)$ and the types of new sellers are drawn i.i.d. from the c.d.f. $F_S(c)$. Each trader's type will not change once it is drawn. Entry (or participation, or being active) is voluntary. Potential traders decide whether to enter the market once they are born. Those who do not enter will get zero payoff. Those who enter will drop out of the market for exogenous reasons with probability $1 - \delta$ in each period.

- Matching: Active buyers and sellers are randomly and continuously matched pairwise with the instantaneous rate of matching given by a matching function $M(B, S) = \min\{B, S\}$, where B and S are the masses of active buyers and active sellers currently in the market.

- In the decentralized market we consider, the following bargaining takes place: Once a pair of buyer and seller is matched, they bargain without observing the type of their partner. The bargaining protocol is *random-proposal*: with probability $\alpha_S \in [0, 1]$, the seller makes a take-it-or-leave-it offer to the buyer, then the buyer chooses either to accept or reject. And with probability $\alpha_B = 1 - \alpha_S$, the buyer proposes and the seller responds. We also assume the market is anonymous, so that the bargainers do not know their partners' market history, e.g. how long they have been in the market, what they proposed previously, and what offers they rejected previously.

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unrestricted.

- If a type v buyer and a type c seller trade at a price p, then they leave the market with payoff $v-p$, and $p-c$ respectively. If bargaining between the matched pair breaks down, both traders can either stay in the market waiting for another match as if they were never matched, or simply exit and never come back.

3 Centralized Market with Exogenous Exit Rates

Before we consider a decentralized market where buyers and sellers randomly meet, it is useful to consider a centralized market as a benchmark. In the centralized market, the broker designs a mechanism where all entering buyers and sellers report their valuations and costs, respectively, and the broker can arbitrarily match buyers and sellers.[2] A buyer's or seller's probability of trade and transfers are determined by his report. The highest profits that can be achieved in such a centralized market will serve as an upper bound for the profits in the decentralized market, where the broker cannot influence how buyers and sellers are matched.

Denote the steady state distributions of buyers and sellers in the market as Φ_B and Φ_S and the steady state masses as B and S.

The broker designs a mechanism where a buyer's expected probability of trade is $q_B(v)$, his per period transfer to the broker is $t_B(v)$. Denote the "ultimate discounted probability of trade" as $Q_B(v) = q_B(v)/(1-\delta+\delta q_B(v))$ and the expected net present value of his transfers as $T_B(v)$, where

$$t_B(v) = (1-\delta+\delta q_B(v))T_B(v).$$

Denote analogously for the seller $q_S(c)$, $t_S(c)$, $Q_S(c)$, and

$$t_S(c) = (1-\delta+\delta q_S(c))T_S(c).$$

The steady state condition for the buyers is

$$\begin{aligned} f_B(v) &= [q_B + (1-q_B)(1-\delta)]B\phi_B(v)] \\ &= [1-\delta+\delta q_B]B\phi_B(v) \\ &= \frac{q_B(v)}{Q_B(v)}B\phi_B(v) \end{aligned}$$

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[2] Since the good is homogeneous, this can be expressed in the following equivalent way: the goods of sellers who sell enter a pool, buyers get their goods from this pool. The only constraint is that the same number of sellers sell as buyers buy.

This condition and the analogous condition for the seller can be rewritten as

$$\begin{aligned} f_B(v)Q_B(v) &= Bq_B(v)\phi_B(v) \\ f_S(c)Q_S(c) &= Sq_S(c)\phi_S(c). \end{aligned}$$

Because trade occurs in matched pairs[3],

$$B\int q_B(v)d\Phi_B(v) = S\int q_S(c)d\Phi_S(c)$$

B and S are such that ϕ_B and ϕ_S add up to one, i.e. $B = \int d\Phi_B$ and $S = \int d\Phi_S$.

By a standard revenue equivalence and envelope theorem argument

$$t_B(v) = (1-\delta+\delta q_B(v))\left[vQ_B(v) - \int_0^v Q_B(x)dx\right] \tag{1}$$

$$t_S(c) = (1-\delta+\delta q_S(c))\left[\int_c^1 Q_S(x)dx - cQ_S(c)\right] \tag{2}$$

The expected revenue of the broker is

$$\begin{aligned} \bar{T} &= B\int t_B(v)d\Phi_B(v) - S\int t_S(c)d\Phi_S(c) \\ &= \int t_B(v)\frac{Q_B(v)}{q_B(v)}dF_B - \int t_S(c)\frac{Q_S(c)}{q_S(c)}dF_S \end{aligned} \tag{3}$$

Substituting (1) and (2) into (3), we get

$$\begin{aligned} \bar{T} =& \int \frac{1-\delta+\delta q_B}{q_B}Q_B(v)\left[vQ_B(v) - \int_0^v Q_B(x)dx\right]dF_B(v) \\ & - \int_0^1 \frac{1-\delta+\delta q_S}{q_S}Q_S(c)\left[\int_c^1 Q_S(x)dx - cQ_S(c)\right]dF_S(c) \\ =& \int_0^1 \left[vQ_B(v) - \int_0^v Q_B(x)dx\right]dF_B(v) \\ & - \int_0^1 \left[\int_c^1 Q_S(x)dx - cQ_S(c)\right]dF_S(c) \\ =& \int_0^1 J_B(v)Q_B(v)dF_B(v) \\ & - \int_0^1 J_S(c)Q_S(c)dF_S(c) \end{aligned}$$

where J_B and J_S are the virtual valuation and virtual cost functions

$$\begin{aligned} J_B(v) &= v - \frac{1-F_B(v)}{f_B(v)} \\ J_S(c) &= c + \frac{F_S(c)}{f_S(c)} \end{aligned}$$

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[3] Alternatively: because in a centralized mechanism the constraint is only that the same mass of buyers and sellers trade.

and the last equality follows from a Myerson and Satterthwaite (1983) type of argument. The broker maximizes his profit $\bar{T}$ with respect to the constraint[4] that the same number of buyers and sellers trade:

$$\int Q_B(v)dF_B(v) = \int Q_B(c)dF_S(c)$$

Note that both the objective function $\bar{T}$ and the constraint can be expressed in terms of Q_B and Q_S alone. And there is a one-to-one mapping between Q_B and q_B. The same applies to Q_S and q_S. Hence the broker can maximize profits by choosing Q_B and Q_S.

Denote the Lagrange multiplier of the constraint as λ. Pointwise maximization with respect to $Q_B(v)$ and $Q_S(c)$ gives the following first order conditions. The first order condition with respect to $Q_B(v)$ is

$$J_B(v) - \lambda = 0$$

and with respect to $Q_S(c)$ is

$$\lambda - J_S(c) = 0.$$

This clearly gives a bang-bang solution, where the broker sets $Q_B(v) = 1$ for $J_B(v) \geq \lambda$ and $Q(v) = 0$ else. Similarly, $Q_S(c) = 1$ for $J_S(c) \leq \lambda$ and $Q_S(c) = 0$. Denote the marginal buyer and seller with $\underline{v}$ and $\overline{c}$, where buyers with $v \geq \underline{v}$ and sellers with $c \leq \overline{c}$ trade. The optimal allocation rule is given by

$$\begin{aligned} J_B(\underline{v}) &= J_S(\overline{c}) \\ 1 - F_B(\underline{v}) &= F_S(\overline{c}) \end{aligned}$$

for the optimal marginal types. This allocation rule can be implemented by letting buyers pay $\underline{v}$ and paying $\overline{c}$ to sellers. Since traders either never enter or trade immediately after entering, the steady state distributions Φ_B and Φ_S are the same as the static distributions F_B and F_S, save for truncation at $\underline{v}$ and $\overline{c}$.

The implied profit maximizing spread $\theta^* = \underline{v} - \overline{c}$ is the same as in Spulber (1996) (p. 572),

$$\theta^* = \frac{1 - F_B(\underline{v})}{f_B(\underline{v})} + \frac{F_S(\overline{c})}{f_S(\overline{c})} = \frac{\underline{v}}{\eta_B(\underline{v})} + \frac{\overline{c}}{\eta_S(\overline{c})}, \tag{4}$$

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[4] Note that this constraint is different from Myerson and Satterthwaite's with respect to two aspects. First, in Myerson and Satterthwaite the probability of trade for a buyer and the seller matched to him has to be the same, $Q_B(v, c) = Q_S(v, c)$ for each realization of v and c. For us, they only have to be equal in expectations. Second, and connected to the previous point, since we are dealing with a centralized mechanism, we do not need to care about which buyer is matched to which seller. Sold goods are simply put into a common pool, buyers get their goods from this pool, without having to care which seller sold it. Hence a buyers probability of trade will only depend on his own valuation v. The same applies for the seller. [AN: we might want to talk about this earlier...]

where the $\eta_B(v) = f_B(v)/[v(1 - F_B(v))]$ and $\eta_S(c) = -f_S(c)/[cF_S(c)]$ are the elasticities of demand. This shows that the mechanism of setting a bid and an ask price considered in Spulber (1996) is the optimal (stationary) mechanism in a dynamic random matching model.[5]

Obviously, the profit generated by this centralized mechanism is an upper bound for the decentralized mechanism with random matching. This is because decentralization adds a further constraint to the maximization problem: trade can only occur between a buyer and a seller who are randomly matched.

An obvious way to implement the same allocation rule and the same profits with a decentralized matching and centralized bargaining (or exchange) protocol is the following. Buyers and seller are matched randomly. A matched buyer has to pay the price $\underline{v}$ and a seller gets the price $\overline{c}$. The broker keeps the spread $\underline{v} - \overline{c}$. A trader cannot get a better deal in the future, hence he has no interest to delay. Buyers below $\underline{v}$ and seller above $\overline{c}$ do not enter, since they would have zero utility from participation. Hence buyers and sellers trade straight away and we get the same allocation rule and the same payments as in the centralized mechanism.

While a decentralized mechanism (in the sense that matching is decentralized) with an ask and a bid price given by the broker is simple to derive, it is of interest to see whether one can implement the mechanism with even more decentralization. In particular, we will consider a bargaining mechanism where a matched buyer and seller play a random proposer game. Quite interestingly, it will turn out that we can still implement the broker optimal allocation rule.

4 Decentralized Market with Bargaining

Assume that upon being matched, the buyer gets to make a take-it-or-leave-it price offer with probability α_B. With probability $\alpha_S = 1 - \alpha_B$ the seller gets to make the offer. The broker charges a per period participation fee K_B and K_S to the buyer and seller, respectively.

Buyers with $v \geq \underline{v}$ and sellers with $c \leq \overline{c}$ enter for some $\underline{v}$ and $\overline{c}$. The marginal buyer $\underline{v}$ gets a zero net expected utility from participating, hence his option value of future trade is also zero. Further, a seller would never set a price below $\underline{v}$, hence the buyer's utility if the seller makes the offer is zero as well. Therefore, we only need to consider the buyer's utility in case

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[5] While this appears intuitive, it this is not completely obvious a priori. The broker is willing to accept efficiency losses by excluding buyers and sellers who would trade in a Walrasian equilibrium. It is not obvious at first sight that he would not also incur some efficiency losses by delaying trade for some of the traders and thus extract rents through price discrimination.

he makes the offer, which gives us

$$\alpha_B \frac{M(B,S)}{B}(\underline{p}_B - \underline{v}) = K_B,$$

where $M/B = \min\{B,S\}/B$ is the buyer's probability of being matched and $\underline{p}_B$ the optimal price set by the marginal buyer. Similarly, for the marginal seller $\overline{c}$

$$\alpha_S \frac{M(B,S)}{S}(\overline{c} - \overline{p}_S) = K_S,$$

While deriving the market equilibrium is complicated in general in such setups, since there may be multiple equilibria and a full trade equilibrium may not exist for all values of K_B and K_S (see e.g. Satterthwaite and Shneyerov (2007)), it turns out that the analysis is strongly simplified by only looking at the profit maximizing fees.

We know from the analysis in the previous section that if a broker can implement the allocation rule of the centralized mechanism, then he cannot do better. In the following we will show that choosing K_B and K_S indeed enables the broker to do this. We will hence focus our attention on the optimal allocation rule, which has the properties that there is full trade (anyone who gets matched, trades with probability 1, here: $\underline{p}_B = \overline{c}$ and $\overline{p}_S = \underline{v}$) and that the market is balanced ($1 - F_B(\underline{v}) = F_S(\overline{c})$ or, in this case also, $M = B = S$).

For full trade balanced market equilibria, the conditions for the marginal types reduce to

$$\alpha_B \theta = K_B,$$
$$\alpha_S \theta = K_S,$$

where $\theta = \underline{v} - \overline{c}$ is the spread. Dividing the first with the second equation gives us

$$\frac{\alpha_B}{\alpha_S} = \frac{K_B}{K_S}. \tag{5}$$

Adding the two equations gives us

$$\theta^* = K_B + K_S,$$

where θ^* is the profit maximizing spread chosen by the broker. Following the previous analysis this can also be written in terms of the elasticities

$$K_B + K_S = \frac{\underline{v}}{\eta_B(\underline{v})} + \frac{\overline{c}}{\eta_S(\overline{c})}, \tag{6}$$

where $\underline{v}$ and $\overline{c}$ are given by the optimal allocation rule. These results mean that the sum of the fees is equal to the sum of the semi-elasticities of demands for marginal traders. It further means that the ratio of the fees (the price structure) is independent of the elasticities and is equal to the ratio of the bargaining weights. Therefore, the side of the market with the stronger bargaining power will be charged a higher fee. Take e.g. the special case where the sellers set the price (i.e. $\alpha_S = 1$). In this case participation is free for buyers, $K_B^* = 0$, and sellers bear the full burden of the fee, $K_S^* = \theta^*$.

It is interesting to compare these results with those in the two-sided markets literature. E.g. in Rochet and Tirole (2006) the sum of the optimal fees is given by

$$K_B + K_S = \frac{K_B}{\eta_B} = \frac{K_S}{\eta_S}.$$

The difference is mainly due to a quadratic rather than a linear matching technology in their model. A far more interesting comparison is that of the ratio of fees. This is given in Rochet and Tirole (2003) Prop. 1:

$$\frac{K_B}{K_S} = \frac{\eta_B}{\eta_S},$$

i.e. by the ratio of the elasticities.[6] In our setup, the (participation rather than transaction) fee structure is independent of the elasticities of demand, but only depends on the relative bargaining weights. Another result from the two-sided markets literature that is interesting to compare with, is

$$\frac{K_i - (-K_{-i})}{K_i} = \frac{1}{\eta_i}, \qquad i = B, S, \quad -i = S, B,$$

which is similar to the standard Lerner formula.

It is also interesting to note that there is no neutrality of fees (i.e. not only the sum of fees, but also the composition of K_B and K_S matters), even though there are no restrictions on price setting/bargaining imposed by the broker. This is in contrast to the findings described in Rochet and Tirole (2006) (p. 665).

Before moving on to the existence and the (almost complete) uniqueness proof for the full trade balanced market equilibrium, it is worth mentioning that the model can be easily extended to include both participation fees k_i and transaction fees τ_i and to further include exogenously given search costs x_i per period for $i = B, S$. The conditions for the marginal types would be

$$\alpha_B(\theta^* - \tau_B) = k_B + x_B$$

$$\alpha_S(\theta^* - \tau_S) = k_S + x_S.$$

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[6] The interpretation of this is more subtle that it might appear at first sight: the equality holds in equilibrium at the optimal fees and optimal elasticities. It should not be interpreted as charging a higher fee to the *more* elastic side of the market.

This would give the broker four control variables k_B, k_S, τ_B, and τ_S, even though only two are (typically) needed to achieve the optimum.

5 Existence and Uniqueness

We will first prove the existence of a full trade balanced market equilibrium. Then we will proceed to the proof of uniqueness of this equilibrium in a static model. Uniqueness in the dynamic setup is work in progress.

5.1 Existence

The profit maximizing spread of the broker is given by

$$J_B(\underline{v}) = J_S(\overline{c}). \tag{7}$$

Conditions for the existence of a full trade equilibrium adapted to our setup

$$(1-\delta)J_B(\underline{v}) + \delta\underline{v} \geq \overline{c} \tag{8}$$

$$(1-\delta)J_S(\overline{c}) + \delta\overline{c} \leq \underline{v} \tag{9}$$

Using (7) and $J_S(c) \geq c$ we can find a lower bound for the LHS of (8):

$$(1-\delta)J_B(\underline{v}) + \delta\underline{v} = (1-\delta)J_S(\overline{c}) + \delta\underline{v} \geq (1-\delta)\overline{c} + \delta\underline{v}$$

which is greater or equal $\overline{c}$, since $\underline{v} \geq \overline{c}$ for a profit maximizing broker. Hence, condition (8) is always fulfilled for the profit maximizing spread. By an analogous reasoning, condition (9) is also always satisfied. Since the ratio of the fees K_B/K_S is chosen such that a balanced market is achieved if there is full trade, we have shown existence of a full trade balanced market equilibrium that maximizes the brokers profit.

The following intuition can be found for this in a static model ($\delta = 0$). A seller faces the following trade-off when considering raising the price: a higher price increases profits in case of trade, but it also decreases the probability of trade. A broker considering raising the price faces the same trade-off, but can additionally lower his costs as he raises the price for buyers. This is because less entry on the buyer side means that he can decrease the number of sellers entering while keeping the trade volume constant, which lowers his cost. Hence, if the broker is not willing to deviate from full trade in the centralized mechanism, neither are the sellers. The same reasoning applies to the buyers.

5.2 Uniqueness in a Static Setup

First, we will show that no other equilibrium with expansion on at least one side of the market exists (i.e. there is either more entry by buyers, more entry by sellers, or both). Then we will show that there is no other equilibrium with contraction (i.e. less entry by both buyers and sellers).

In the following we will take a non-full-trade-equilibrium where buyers $v \in [\underline{v}, 1]$ and sellers with $c \in [0, \overline{c}]$ enter. As a comparison, we will denote the marginal types in the full-trade-equilibrium that maximizes the broker's profits as v^* and c^*. We will show that any other equilibrium leads to a contradiction.

Recall that for full trade, the marginal types are given by

$$\alpha_B(v^* - c^*) = K_B$$
$$\alpha_S(v^* - c^*) = K_S$$

Similarly for the alternative non-full-trade-equilibrium

$$\alpha_B(\underline{v} - p_B)\frac{M}{B}\overline{F}_S(p_B) = K_B$$
$$\alpha_S(p_S - \overline{c})\frac{M}{S}(1 - \overline{F}_B(p_B)) = K_S$$

where $1 - \overline{F}_B(v) = (1 - F_B(v))/(1 - F_B(\underline{v}))$ and $\overline{F}_S(c) = F_S(c)/F_S(\overline{c})$ are the truncated distributions.

We will first consider a market where the mass of entrants is larger than in full trade both for buyers and sellers, i.e. $\underline{v} < v^*$ and $\overline{c} > c^*$ has to hold. In the full trade equilibirum the utility of a type v^* buyer is

$$\max_{p_B \in [0, c^*]} \alpha_B(v^* - p_B)\left\{1\right\}\left\{F_S^*(p_B)\right\} = K_B \tag{10}$$

where $F_S^*(c) = F_S(c)/F_S(c^*)$ is the truncated distribution of sellers.

Compare this with the utility of the v^* buyer in the non-full-trade-equilibrium:

$$\max_{p_B \in [0, \overline{c}]} \alpha_B(v^* - p_B)\left\{\frac{M}{B}\right\}\left\{\overline{F}_S(p_B)\right\} \tag{11}$$

Clearly, the two expressions in curly braces are both weakly less than the corresponding expressions in the previous equation, hence the maximized function in (11) is weakly less than the maximized function in (10) for all p_B, hence (11) is weakly less than K_B. Therefore, the marginal seller in the non-full-trade-equilibrium, $\underline{v} < v^*$, will have a utility strictly less than K_B, which is a contradiction. Intuitively, v^* is less likely to be matched in the non-full-trade-equilibrium and in case of being matched he is less likely to sell at a given price. Hence entering is less attractive to him. The same reasoning applies to the marginal seller $\overline{c}$.

Next, consider the case where quantity is expanded for one side and contracted for the other side of the market. There are two subcases, (i) $\underline{v} > v^*$ and $\overline{c} > c^*$ and (ii) $\underline{v} < v^*$ and $\overline{c} < c^*$. We will only consider case (i), the same reasoning applies for case (ii). For case (i), the sellers are the long side of the market, i.e. $S > B$, which is equivalent to $F_S(\overline{c}) > 1 - F_B(\underline{v})$.

Consider the profit of seller c^* in the full trade equilibrium if he makes the offer:

$$\max_{p_S}(p_S - c^*)(1 - F_B^*(p_S)) = \max_{p_S}(p_S - c^*)\frac{1 - F_B(v^*)}{F_S(c^*)}\frac{1 - F_B(p_S)}{1 - F_B(v^*)} = \max_{p_S}(p_S - c^*)\frac{1}{F_S(c^*)}(1 - F_B(p_S)) \tag{12}$$

where the first equality follows from $1 - F_B(v^*) = F_S(c^*)$ in the full trade equilibrium and the definition of the truncated distribution F_B^*.

The profit of the seller c^* in the non-full-trade-equilibrium if he makes the offer is

$$\max_{p_S}(p_S - c^*)\frac{M}{S}(1 - \overline{F}_B(p_S)) = \max_{p_S}(p_S - c^*)\frac{1 - F_B(\underline{v})}{F_S(\overline{c})}\frac{1 - F_B(p_S)}{1 - F_B(\underline{v})} = \max_{p_S}(p_S - c^*)\frac{1}{F_S(\overline{c})}(1 - F_B(p_S)) \tag{13}$$

where the first equality follows from the definitions of M/S when sellers are on the long side of the market and the truncated distribution $\overline{F}_B$. Comparing the maximized functions on the RHS of (12) and (13) reveals that profits are lower in the non-full-trade-equilibrium, since $\overline{c} > c^*$.

The profit of the marginal seller in the non-full-trade-equilibrium $\overline{c}$

$$\max_{p_S}(p_S - \overline{c})\frac{1}{F_S(\overline{c})}(1 - F_B(p_S))$$

are even lower than in (13). Neither the c^* seller in the full trade equilibrium nor the $\overline{c}$ seller in the non-full-trade-equilibrium make any profits in case the buyer makes the offer. Hence, the profit of the $\overline{c}$ seller in the non-full-trade-equilibrium are below K_S, which is a contradiction.

Now we look at contraction on both sides, i.e. $\overline{c} < c^*$ and $\underline{v} > v^*$. We will show that the marginal utility of increasing the price is positive for the marginal buyer in any equilibrium with two sided contraction. Since the same argument holds also for sellers, we know that buyers price at $\overline{c}$ and sellers at $\underline{v}$. Then we will show that an equilibrium with this pricing cannot exist for $\theta < \theta^*$.

The marginal buyer's utility when setting price p_B is

$$\pi_B(p_B) = (\underline{v} - p_B)\frac{F_S(p_B)}{F_S(\overline{c})}\frac{M}{B}$$

Looking at the marginal utility of pricing, leaving aside constants, we get

$$\begin{aligned}\pi'_B(p_B) &\propto -F_S(\overline{c}) + (\underline{v} - \overline{c})f_S(\overline{c})\\ &= (\underline{v} - J_S(\overline{c}))f_S(\overline{c})\\ &> (v^* - J_S(\overline{c}))f_S(\overline{c})\\ &\geq (v^* - J_S(c^*))f_S(\overline{c})\\ &\geq 0\end{aligned}$$

where the first two inequalities follow from $\underline{v} > v^*$ and $\overline{c} < c^*$ and the third from the fact that there is full trade at v^*, c^*. A positive π'_B means that the buyer will set a price equal to the cost of the marginal seller $\overline{c}$. By monotonicity, all other buyers (who have $v > \underline{v}$), will also price at $\overline{c}$. By an analogous argument, all sellers will price at $\underline{v}$. Since the probability of trading conditional on being matched is 1, the utility of the marginal types is

$$\alpha_B(\underline{v} - \overline{c})\frac{M}{B} = K_B \tag{14}$$

$$\alpha_S(\underline{v} - \overline{c})\frac{M}{S} = K_S \tag{15}$$

Dividing the two equations and using the fact that the broker makes sure that $\alpha_B/\alpha_S = K_B/K_S$ we get $B/S = 1$. Substituting this back into (14) and (15) gives us a contradiction to the full trade marginal type conditions

$$\alpha_B(v^* - c^*) = K_B$$

$$\alpha_S(v^* - c^*) = K_S$$

since $\underline{v} - \overline{c} > v^* - c^*$. Hence, a contraction equilibrium cannot exist.

Putting these parts together, we get the result that the full trade equilibrium is unique in the static model.

5.3 Uniqueness of the Full Trade Equilibrium in a Dynamic Setup

The argument that is to be worked out in detail is that if uniqueness holds for the static model than it also has to hold in the dynamic model.

We already have the proofs for the cases of expansion on both sides and contraction on both sides. The case where there is contraction on one and expansion on the other side of the market has to be worked out. [see separate file]

6 Conclusions

We have derived the broker optimal mechanism in a setup of dynamic random matching with private information. We have described optimal mechanisms for different levels of centralization: (i) matching and the exchange mechanism are centralized, (ii) only the trade mechanism is centrally prescribed, (iii) both matching and the trade mechanism are decentralized. Interestingly, the same profits can be achieved in all setups. Further, even though existence and uniqueness of a full trade balanced market equilibrium does not have to hold in general in a completely decentralized setup, it does hold at the optimal level of fees (the proof of uniqueness for the dynamic setup has to be finished). Considering a full trade equilibrium gives a tractable model of brokerage, with clear implications on total fees and fee structure.

The main differences to the two-sided markets literature is that we have a multi-period setup where the decision to enter is made after observing one's own valuation for the good to be traded and a linear (rather than a quadratic) matching technology. We get different predictions from the two-sided markets literature: fee structure depends on relative bargaining weights rather than elasticities of demand. Further, neutrality of the fee structure also breaks down if traders are not restricted in their price setting/bargaining behavior.

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