Abstract

We consider a model with a profit maximizing rating agency, a continuum of heterogeneous sellers, and a competitive market of risk-neutral buyers. We depart from the existing literature on rating agencies by adding two ingredients: there is aggregate uncertainty about whether the state of the world is good or bad; buyers can possibly make inferences about the state of the world by observing the distribution of ratings in the economy. We consider four different possible rating strategies: full disclosure of the quality of a seller, an absolute minimum standard above which sellers get a rating, a relative minimum standard that makes sure that the same percentage of sellers gets rated in both states of the world, and complete pooling, i.e. all sellers getting a rating. We obtain the following results. First, in accordance with empirical observations, the value of an optimal absolute rating is lower in a bad state. Second, with an absolute rating, the threshold is higher than in a world with no aggregate uncertainty. Third, relative ratings can be better than absolute ones. Fourth, it can be optimal for an agency not to reveal the state of the world.

Keywords: Rating agencies, certification, aggregate uncertainty
JEL-Classification: C72, D42, D82, G20
1 Introduction

Ratings and other quality certifications by third parties play an important role in today’s economy. For instance, the volume of rated debt issues was over $8,000 billion in 2006. Financial ratings aid the decisions of investors, are used for the regulations of banks in the Basel Accords, and the aggregate distribution of ratings can be used as an indicator for crises.

Often there is both an aggregate and an idiosyncratic part of a rated firm’s quality. For example, if all banks invest in subprime mortgages, a rating agency may rate how risky a certain bank is compared to other banks (idiosyncratic part), but it may also try to assess the riskiness of subprime mortgages in general (aggregate part). Depending on whether subprime mortgages in general are considered harmless or toxic, the ratings of all banks (or all debt issues) may have to be adjusted. Other examples in financial markets include the assessment of stocks of dotcom firms at the end of the nineties. One can think of many examples outside of financial markets, just to take one consider general opinion on the health effects of saturated fats. In the past, saturated fats were considered to have little negative health effects. However, in 2004 the Center for Disease Control released a statement pointing out the health risks of saturated fats. A food quality certifier may make relative comparisons between the different products, however, it may also publish information about the (uncertain) effects of a factor (such as saturated fats) that impacts a large set of products.

Typically, the overall distribution of ratings given by a rating agency is publicly observable, so that market participants may make inferences about aggregate shocks. A further empirical observation, specific to the rating of debt, is that the market impact of ratings is typically lower during bad times (i.e. aggregate shock), as it was observed at the wake of the subprime mortgage crisis.

We ask several questions related to these observations. Do rating agencies have an incentive to truthfully and fully disclose the quality of rated firms or debt issues? What are the effects of aggregate uncertainty on the predictions of the model? When
will rating agencies give absolute ratings that depend only on the quality? And when will they give relative ratings that only depend on which quantile of the distribution a rated firm or debt issue is in, independently of the state? Are the actions of rating agencies efficient?

We want to gain a better understanding of rating agencies by considering the following setup. There is a continuum of sellers with private information about the quality of the good they are selling, a profit maximizing rating agency, and a competitive market of risk-neutral buyers who pay a product’s expected quality after observing ratings. The rating agency gets a perfect signal about the seller’s quality at no cost. There are two states of the world, in one the distribution of qualities is good, in the other bad. Rating agencies either give a firm a positive rating or no rating at all. They disclose their rating method in advance and also set a fee charged for a rating before learning the state of the world. Sellers decide whether to get a rating after getting to know their quality but without knowing the state of the world. Then the agency observes the qualities of the sellers, hence can infer the state of the world from their distribution, and gives ratings to firms. Finally, buyers observe the ratings of the firms and may or may not infer from the overall distribution of ratings the state of the world.

We will consider four possible strategies for the rating agency: full disclosure, absolute minimum standard, relative minimum standard, and complete pooling. With full disclosure, the exact qualities of the rated firms are disclosed. With an absolute minimum standard, firms above a threshold independent of the state of the world get a positive rating. With a relative minimum standard, a certain percentage of firms gets a positive rating, the absolute threshold being lower in a bad state of the world. With complete pooling, all firms get a positive rating. For full disclosure and the absolute minimum standard, buyers can infer the state of the world from the distribution of ratings. For the relative minimum standard and complete pooling, they cannot.

We obtain the following (so far mostly numerical) results. First, full disclosure is always dominated. Second, if the mass of sellers with low quality is large, an absolute
rating is profit maximizing for the rating agency. Third, if the mass of sellers with a high quality is large, a relative rating or complete pooling are optimal. Fourth, the value of a rating is lower in a bad state of the world. Fifth, with an absolute rating the threshold is higher than in a model without aggregate uncertainty. An intuition for this result is that sellers with a low quality are more inclined to believe that the state of the world is bad. Hence, their willingness to pay for a rating is lower and it is less attractive for the rating agency to rate them. Sixth, with relative ratings, the two threshold (in the good and in the bad state) may be below or above the one in a world with only one state.

Our paper relates to a large literature on rating agencies, experts, and reputation. We differ from all papers by having (i) aggregate uncertainty over the state of the world and (ii) letting buyers observe the overall distribution of ratings in the economy and hence possibly being able to make inferences about the state of the world.

Our paper is based on the framework outlined in the seminal paper by Lizzeri (1999). He uses a one period model, a continuous distribution of quality, and proves that complete pooling is a profit maximizing strategy for a monopolist. Doherty, Kartasheva, and Phillips (2009) extend the model by risk-averse consumers and show that then it can be optimal to pool sellers in rating classes and to cover the market only partially.

Two papers also consider (i), but not (ii). Bolton, Freixas, and Shapiro (2010) find that agencies understate risk in booms. They model a boom by a higher fraction of naive consumers which leads to rating inflation. Bar-Isaac and Shapiro (2010) allow for the change of economic fundamentals. They investigate the quality of ratings when accuracy is costly for the agency and reputation matters.

In a wider sense, our paper also relates to the literature on experts and reputation. Reputation gives an incentive to report truthfully. Strausz (2005) shows that reputation leads to monopolization and that honest certification may require a price above that of a monopolist. Nevertheless, reputation is often not enough to ensure accurate information transmission (Ottaviani and Sørensen, 2006; Bouvard and Levy, 2009; Mariano, 2008). Mathis, McAndrews, and Rochet (2009) show that
reputation and confidence cycles may exist, because the certifier likes to build up
to later inflate the grades and make larger profits.

The paper is structured as follows. Chapter 2 sets up the baseline model. Chapter
3-6 discuss four different rating strategies of an agency. These are compared in
Chapter 7. Chapter 8 concludes.

2 Model, Basic Setup

The paper focuses on a basic model. It includes one rating agency, a continuum of
firms and a continuum of possible investors. The model assumes that the seller pays
the rating fee $P$ upfront. In addition, the agency commits to a rating/disclosure
strategy. The quality of a firm $t$ is from the interval $[\ell, l]$. This quality is known to
the firm but not to the buyers. Furthermore, buyers are risk neutral and willing to
pay a price equal the expected quality of the good. Finally, there are two different
states $z$ of the world. The world can be in good or bad economical condition,
$z \in \{g, b\}$. The difference between the two states is that the distribution of quality
is different and that the expected quality in the good state is higher than in the bad
state. In a “good” state the distribution of $t$ is $G(t)$ and in a “bad” one it is $B(t)$. The probability for the good state of the world is $\bar{q}$. This probability and the two
different distributions of quality are known to all players.

The timing of moves is as follows:

- The agency sets the rating fee $P$ and its rating strategy $s$, $s(t, z) = r$, $s : \mathbb{R} \times \{g, b\} \rightarrow \mathbb{R} \cup \{\emptyset\}$.

- Nature draws quality $t$ of each firm and the state of the economy.

- The firms observe their own qualities and decide whether to go to the agency to
ask for a rating or not. This decision depends on the own type $t$, the strategy
of the agency $s$ and the price $P$.

- the agency observes the state, the quality of the firms asking for ratings and
gives ratings according to its strategy
Observing the distribution of ratings $H, H : \mathbb{R} \rightarrow [0, 1]$, the buyers decide how much to bid in a second price auction for a product. The consumer’s bid is equal to her belief $\mu$ about the expected quality given all information, which is $\mu(t \mid s, P, r, H)$. Since it is a second price auction, the consumers bid their own valuation.

The expected quality in the good state is larger than the one in the bad state: $E_g(t) > E_b(t)$. Further, we assume that the monotone likelihood ratio property holds for the densities $g$ and $b$, i.e. $\frac{\partial g(t)}{\partial t} \geq 0$ which is equivalent to $\frac{\partial (g/b)}{\partial t} \geq 0$, i.e. the individual updated probability to be in the good state is increasing in the own quality.

Later we will also use $\frac{\partial E_g[t \leq x]}{\partial x} \geq 0, \frac{\partial E_b[t \leq x]}{\partial x} \geq 0$, which follows from the two distribution functions. This means the expected quality below the threshold is increasing in $x$ in both states of the world. \footnote{Proof is in the appendix.} To solve the setup for equilibria we use perfect Bayesian Equilibrium. First we solve for the equilibria in the subgames, given the fee $P$ and the rating strategy. Then given each strategy we solve for the profit maximizing price $P$. Afterwards we can compare the profits of the agency to find the equilibria of the game. We restrict the strategy space to some possible strategies, which we discuss and compare in the different sections of the paper. Thus we do not give a complete list of all possible equilibria in this setup. Furthermore, we restrict the strategy of the firms to pure strategies and look at symmetric equilibria.

3 Full disclosure

The first possible strategy we are looking at is that the agency gives precise ratings. This strategy we use as a benchmark for the later ones. Formally this strategy can be written

$$s(t, z) = \begin{cases} 
 t & \text{if } t \geq x, \\
 \emptyset & \text{else}.
\end{cases}$$
This means that the rating matches with the real quality. A strategy of full disclosure can be a different function \( s \), which has to be injective, because it is only important that the consumers can infer the true quality from the rating.

The expected payoff of a firm of type \( x \) of being pooled with firms below a threshold \( \underline{x} \) is

\[
E_x[E[t \mid t \leq \underline{x}]^+] = E_g[t \mid t \leq \underline{x}]^+ q(x) + E_b[t \mid t \leq \underline{x}]^+ (1 - q(x))
\]

where \( [y]^+ \) is a shorthand for \( \max\{y, 0\} \) and \( E_g[t \mid t \leq \underline{x}]^+ \) is the maximum of the expected quality of firms below \( \underline{x} \) in the good state and 0. Since the firm prefers not to sell the good at all, when the expected quality and consequently also price are negative. In this case a firm without rating do not sell the good and get the outside option 0. \( E_g[t \mid t \leq \underline{x}]^+ \) is calculated

\[
E_g[t \mid t \leq \underline{x}]^+ = \max\left\{ \int_{\underline{x}}^{\underline{t}} t \ast g(t)/G(\underline{x}) dt, 0 \right\}
\]

Similar \( E_b[t \mid t \leq \underline{x}]^+ \) is either the expected quality of firms below \( \underline{x} \) in the bad state or 0:

\[
E_b[t \mid t \leq \underline{x}]^+ = \max\left\{ \int_{\underline{x}}^{\underline{t}} t \ast b(t)/B(\underline{x}) dt, 0 \right\}
\]

The probability for the good state of the world is \( \overline{q} \) and is known to everybody. However, each firm is updating this probability after it knows its own quality. Thus the individual probability of the states which we need to calculate expectations over the different states depend on the individual quality of the firm. \( q(t) \) is the updated probability of the world to be in the good state given the own quality \( t \)

\[
q(t) = \frac{g(t)\overline{q}}{g(t)\overline{q} + b(t)(1 - \overline{q})}
\]

Given the threshold \( \underline{x} \) for ratings, the valuation of type \( x \geq \underline{x} \) for a rating is

\[
V(x, \underline{x}) = x - E_x[E[t \mid t \leq \underline{x}]^+]
\]

\[
= x - E_b[t \mid t \leq \underline{x}]^+ (1 - q(x)) - E_g[t \mid t \leq \underline{x}]^+ q(x)
\]
The agency can set
\[ P = V(\underline{x}, \bar{x}) \]

Given the price \( P \), the seller of type \( \underline{x} \) is indifferent between paying price \( P \) and getting a rating of \( \underline{x} \) or not going to the agency and being pooled with the lower types. To ensure that \( t > \underline{x} \) go to the agency as well, given this price, it has to hold \( \frac{\partial V}{\partial x} \geq 0 \), which is \( 1 > q'(\underline{x})(E_g[t \mid t \leq \underline{x}]^+ - E_b[t \mid t \leq \underline{x}]^+) \). Then there is an equilibrium in which the consumers’ beliefs \( \mu \) about the quality are

\[
\mu(t \mid s, P, H, r) = \begin{cases} 
    t & \text{if } r = t \\
    E_g[t \mid t \leq \underline{x}]^+ & \text{if } r = \emptyset \text{ and } H(t) > 1 - B(\underline{x}) \\
    E_b[t \mid t \leq \underline{x}]^+ & \text{else.}
\end{cases}
\]

and the firm goes to the agency if the quality level \( t \in [\underline{x}, \bar{x}] \) and otherwise not.

In equilibrium, the profit of the agency is
\[
\Pi = P(\bar{q}(1 - G(\underline{x})) + (1 - \bar{q})(1 - B(\underline{x})))
\]
and the agency chooses \( \underline{x} \) and \( P \) to maximize it.

In the following a numerical example is given in which we solve for the optimal strategy to maximize the profit. We assume two density functions of quality \( g(t) \) and \( b(t) \).

\[
g(t) = \frac{2(t - \bar{t})}{(\bar{t} - \underline{t})^2}
\]

\[
b(t) = \frac{2(\bar{t} - t)}{(\bar{t} - \underline{t})^2}
\]

These densities \( g(t) \) and \( b(t) \) are straight lines; an increasing one in the good state and a decreasing one in the bad state. Furthermore, we can adapt the support of \( t \) without changing this property. In this example \( t \) is assumed to be \( t \in [-1, 3] \).

Figure 1 shows the profit as a function of \( \underline{x} \) using the full disclosure strategy. The agency chooses the \( \underline{x} \) which maximizes the profit. Given this threshold a different mass of sellers is coming to the agency in the two different states of the world, which is shown in figure 2.
Figure 1: Profit for full disclosure as a function of $x$. Parameter value: $\overline{q} = 0.95$, $\underline{t} = -1$, $\overline{t} = 3$. Densities: $g(t) = 2(t - \underline{t})/(\overline{t} - \underline{t})^2$, $b(t) = 2(\overline{t} - t)/(\overline{t} - \underline{t})^2$

Figure 2: Optimal threshold and mass of rated firms in the two states. Parameter value: $\overline{q} = 0.95$, $\underline{t} = -1$, $\overline{t} = 3$. Densities: $g(t) = 2(t - \underline{t})/(\overline{t} - \underline{t})^2$, $b(t) = 2(\overline{t} - t)/(\overline{t} - \underline{t})^2$
An agency faces the tradeoff of setting a lower threshold, which leads to more rated firms but a lower price $P$, or setting the threshold higher, which allows to charge a higher price but reduces the number of rated firms.

4 Absolute Minimum Standard

In this section the rating agency gives a certificate which just states the firm as “being certified”. It does not give a precise rating about the quality. Formally

$$s(t, z) = \begin{cases} 
1 & \text{if } t \geq x, \\
\emptyset & \text{else}.
\end{cases}$$

The expected payoff of firm $t$ of being pooled with firms below a threshold $x$ is again

$$E_x[E[t \mid t \leq x]^+] := E_b[t \mid t \leq x]^+ (1 - q(x)) + E_g[t \mid t \leq x]^+ q(x)$$

The expected return for a firm $t$ when it is pooled with firms above the threshold $x$ is

$$E_x[E[t \mid t \geq x]^+] := E_b[t \mid t \geq x]^+ (1 - q(x)) + E_g[t \mid t \geq x]^+ q(x)$$

where $E_g[t \mid t \geq x]^+$ is the maximum of the expected quality of firms above $x$ and 0. This is calculated

$$E_g[t \mid t \geq x]^+ = \max \left\{ \int_{\underline{x}}^{x} t * g(t) / (1 - G(x)) dt, 0 \right\}$$

Similar $E_b[t \mid t \geq x]^+$ is the maximum of the expected quality of firms above $x$ and 0 in the bad state and is calculated

$$E_b[t \mid t \geq x]^+ = \max \left\{ \int_{\underline{x}}^{x} t * b(t) / (1 - B(x)) dt, 0 \right\}$$

The probability $q$ for the good state of the world is updated by each individual firm after knowing its own quality in the same way as before, leading to the individual
probability for the good state $q(t)$

$$q(t) = \frac{g(t)\overline{q}}{g(t)\overline{q} + b(t)(1 - \overline{q})}$$

If the firm is certified, it does not receive anymore its own quality from the consumers, but it is pooled with the other certified firms. A firm with quality $x$ which is pooled with the quality levels above $x$ receives in expectation

$$E_x[E[t \mid t \geq x]] = E_b[t \mid t \geq x]^+ (1 - q(x)) + E_g[t \mid t \geq x]^+ q(x)$$

from the consumers.

The valuation of a firm of type $x$ for being certified (given the threshold above which a firm is certified is $\overline{x}$) is

$$V(x, \overline{x}) = E_x[E[t \mid t \geq \overline{x}]^+] - E_x[E[t \mid t < \overline{x}]^+]$$

$$= E_b[t \mid t \geq \overline{x}]^+ (1 - q(x)) + E_g[t \mid t \geq \overline{x}]^+ q(x)$$

$$- (E_b[t \mid t < \overline{x}]^+ (1 - q(x)) + E_g[t \mid t < \overline{x}]^+ q(x))$$

$$= (E_g[t \mid t \geq \overline{x}]^+ - E_g[t \mid t < \overline{x}]^+)q(x)$$

$$+ (E_b[t \mid t \geq \overline{x}]^+ - E_b[t \mid t < \overline{x}]^+)(1 - q(x))$$

We define $D_g(\overline{x})$ as the additional revenue a firm gets in the good state when it is certified in comparison to not being certified, given the threshold for certification is $\overline{x}$

$$D_g(\overline{x}) = E_g[t \mid t \geq \overline{x}]^+ - E_b[t \mid t \leq \overline{x}]^+$$

and equivalent for the bad state

$$D_b(x) = E_b[t \mid t \geq x]^+ - E_g[t \mid t \leq x]^+$$

If we want $x$ to be indifferent between being certified or not, it has to hold

$$P = V(x, \overline{x}) = D_g(\overline{x})q(x) + D_b(x)(1 - q(x))$$
In the equation above \( x \) is indifferent between being pooled with the qualities below \( x \) or paying the price and being pooled with the qualities above \( x \). Thus, it is assumed, that the quality levels above \( x \) go to the agency. For this to be true, it has to hold that \( V(x, x) \geq P \) for all \( x \geq \bar{x} \).

If \( D_g(x) \geq D_b(x) \), then the agency should set \( P = V(x, x) \). Otherwise, if \( D_g(x) < D_b(x) \), a firm with quality \( t > x \) would not go to the agency given this price, because it has a higher updated probability for the high state, which now gives a lower return for having a certificate. \( \frac{\partial V(x, x)}{\partial x} < 0 \).

Consequently the equations of the expected revenue after receiving a certification do not hold anymore. In this case we have to make the highest quality level \( \bar{t} \) indifferent between being rated or not, because this one puts the lowest probability on the bad state. This means that the agency should set \( P = V(\bar{t}, x) \). In this case \( \bar{t} \) is indifferent between going to the agency or not, and the type \( x \) is the threshold above which firms get the certification. \(^{2}\)

There is an equilibrium in which the consumers’ beliefs \( \mu \) about the quality are

\[
\mu(t | s, P, H, r = 1) = \begin{cases} 
E_g[t | t \geq \bar{x}]^+ & \text{if } H \text{ means good state} \\
E_b[t | t \geq \bar{x}]^+ & \text{else.}
\end{cases}
\]

\[
\mu(t | s, P, H, r = 0) = \begin{cases} 
E_g[t | t \leq \bar{x}]^+ & \text{if } H \text{ means good state} \\
E_b[t | t \leq \bar{x}]^+ & \text{else.}
\end{cases}
\]

\( H \) means good state if

- \( H(1) \neq 1 - B(x) \) for \( D_g(x) \geq D_b(x) \)
- \( H(1) = 1 - G(x) \) for \( D_g(x) < D_b(x) \)

and a firm goes to the agency if its quality \( t \in [\bar{x}, \bar{t}] \), otherwise not.

\(^{2}\)Solving the equations in the two cases to get the type \( x \) who either is indifferent between going to the agency or not or who is the threshold, we have the same problems as before. Given \( \bar{x} \) we can calculate \( P \). However, given the price \( P \) it is harder to solve for \( \bar{x} \). For uniqueness, we need monotonicity of \( E_{\bar{x}2}[E[t | t \geq \bar{x}]^+] - E_{\bar{x}2}[E[t | t \leq \bar{x}]^+] \) or in the second case of \( E_{\bar{t}1}[E[t | t \geq \bar{x}]^+] - E_{\bar{t}1}[E[t | t \leq \bar{x}]^+] \).
Then the profit of the agency is

$$\Pi = P(\bar{q}(1 - G(x)) + (1 - \bar{q})(1 - B(x)))$$

In figure 3 the numerical example from chapter 3 is used again. It shows the profit for the two cases \(D_g(x) \geq D_b(x)\) and \(D_g(x) < D_b(x)\) as a function of \(x\). The feasible profit is marked in green.

Red: \(D_g \geq D_b\), Blue: \(D_g < D_b\), Green: \(\min\{\text{Red, Blue}\}\)

Figure 3: Profits for the different cases as a function of \(x\). Parameter value: \(\bar{q} = 0.05\), \(t = -1\), \(\bar{t} = 3\). Densities: \(g(t) = 2(t - \bar{t})/((\bar{t} - t)^2\), \(b(t) = 2(\bar{t} - t)/((\bar{t} - t)^2\)

Figure 4 shows the mass of sellers going to the agency given the optimal \(x\). In the good state more sellers are rated and thus the buyers can infer the state of the world. An interesting point is that the optimal \(x\) is above 0 even though the sellers between 0 and \(x\) should be sold in the first best, what is not happening in this example. Furthermore, in contrast to this result, with only one state of the world the threshold would be exactly 0. 3

\[^3\text{see Lizzeri (1999)}\]
5 Relative Minimum Standard

In this section the rating agency gives again a certificate which just states that the firm is certified. In the previous section there was a threshold $x_\text{above}$ above which the firms were certified and below not. By observing the quantity of certified firms, the consumers could infer the state of the world. In this section now, the agency certifies the same quantity of firms in each state. In this way the consumers cannot infer the state of the world. Consequently they are willing to pay the same price in each state. Since in both states the same quantity of firms is certified, the threshold is higher in the good state than in the bad one. We define $\pi$ the threshold above which firms are certified in the good state and equivalent $\underline{x}$ for the bad state. We know that $\pi \geq \underline{x}$ and $1 - G(\pi) = 1 - B(\underline{x})$. It follows that $\underline{x} = B^{-1}(G(\pi))$.

Formally, the strategy of the agency is

$$s(t, z) = \begin{cases} 1 & \text{if } t \geq \pi, \\ 1 & \text{if } \underline{x} < t < \pi \text{ and } z = b, \\ \emptyset & \text{else.} \end{cases}$$

Since the firms do not know the state of the world, their decisions whether to go to
the agency or not cannot depend on it. We want all \( t \in [x, T] \) to ask for a rating. However, \( t \in [x, T] \) are always certified, while \( t \in [x, \bar{x}] \) only in the bad state. Firms with a quality \( t \in [\underline{t}, \bar{x}] \) do not go to the agency, since they are never certified.

Consumers are willing to pay

\[
[q_E g[t | t \geq \bar{x}] + (1 - q) E_b[t | t \geq \underline{x}]]^+
\]

for a firm having a rating. Without a rating they pay

\[
[q_E g[t | t < \bar{x}] + (1 - q) E_b[t | t < \underline{x}]]^+.
\]

This means \( t \in [\bar{x}, T] \) pay at most for a rating the difference between the consumers’ willingness to pay having a rating and the one without rating

\[
[q_E g[t | t \geq \bar{x}] + (1 - q) E_b[t | t \geq \underline{x}]]^+ - [q_E g[t | t < \bar{x}] + (1 - q) E_b[t | t < \underline{x}]]^+
\]

However, we also want \( t \in [\underline{x}, \bar{x}] \) to go to the agency, even if they are certified only in the bad state. Their willingness to pay to go to the agency is

\[
V(x, \underline{x}, \bar{x}) = (1 - q(x)) ([q_E g[t | t \geq \bar{x}] + (1 - q) E_b[t | t \geq \underline{x}]]^+ - [q_E g[t | t \leq \bar{x}] + (1 - q) E_f[t | t \leq \underline{x}]]^+)
\]

This is the additional return having a rating (see above) times the probability of the bad state, which means here the probability to get the rating. The type \( \underline{x} \) is indifferent between going to the agency and being only sometimes certified and not going at all, if \( P = V(x, \underline{x}, \bar{x}) \)

Since \( \frac{\partial(1 - q(t))}{\partial t} \leq 0 \) firms with \( \underline{x} < t < \bar{x} \) would not go to the agency given this price, because their updated probability for the bad state is lower. This means the probability to get the certificate is too low. That is why we have to make the type with the lowest \( V(x, \underline{x}, \bar{x}) \) indifferent between going or not, which is equivalent to
the type with the lowest \(1 - q(t)\), and this is \(\overline{x}\). It is indifferent if

\[ P = V(\overline{x}, x, \overline{x}) \]

There is an equilibrium in which the consumers’ beliefs \(\mu\) about the quality are

\[
\mu(t \mid s, P, H, r = 1) = \overline{q}E_g[t \mid t \geq \overline{x}] + (1 - \overline{q})E_b[t \mid t \geq x]
\]

\[
\mu(t \mid s, P, H, r = 0) = \overline{q}E_g[t \mid t \leq \overline{x}] + (1 - \overline{q})E_b[t \mid t \leq x]
\]

and all \(t \in [x, \overline{t}]\) always go to the agency and the rest not. The profit of the agency is

\[
\Pi = P(\overline{q}(1 - G(\overline{x})) + (1 - \overline{q})(1 - B(\overline{x})))
\]

Maximizing this profit the agency has to set \(x, \overline{x}\) and \(P\). Using the same numerical example as above figure 5 shows the profit depending on the threshold in the bad state \(x\). Figure 6 illustrates the threshold in the good state \(\overline{x}\) resulting from the optimal \(x\) such that the same number of sellers is rated in both states. The minimum quality to be rated in the good state of the world is higher than in the bad one. In this example the agency certifies even negative qualities in the bad state in order to let the same number of sellers pass and for the buyers not to be able to infer the true state of the world.

### 6 Complete Pooling

In this strategy, the agency certifies everybody who is asking for a rating. This means it lets everybody pass. Formally, the strategy is

\[
s(t, z) = 1 \text{ for all } t
\]

Since the consumers cannot infer the state of the world, they are willing to pay the expected quality

\[
E(t) = \overline{q}E_g(t) + (1 - \overline{q})E_b(t)
\]
Figure 5: Profits as a function of $x$. Parameter value: $\overline{q} = 0.05$, $t = -1$, $\overline{t} = 3$. Densities: $g(t) = 2(t - \overline{t})/(\overline{t} - \overline{t})^2$, $b(t) = 2(\overline{t} - t)/(\overline{t} - \overline{t})^2$

Figure 6: Optimal thresholds $x$ and $\overline{x}$. Parameter value: $\overline{q} = 0.05$, $t = -1$, $\overline{t} = 3$. Densities: $g(t) = 2(t - \overline{t})/(\overline{t} - \overline{t})^2$, $b(t) = 2(\overline{t} - t)/(\overline{t} - \overline{t})^2$
and the sellers’ valuation for a rating is

\[ V(x) = E(t) - [t]^+ \]

Consequently they pay at most the price \( P \) for the rating

\[ P = [V(x)]^+ \]

There is an equilibrium where everybody is going to the agency and everybody gets the certificate and the consumers’ beliefs are

\[ \mu(t \mid s, P, H, r = 1) = E(t) \]

\[ \mu(t \mid s, P, H, r = \emptyset) = [t]^+ \]

and since always all firms go to the agency and pay the price \( P \), the profit of the agency is

\[ \Pi = P \]

In the subgame there exists also another equilibrium. In this second equilibrium the consumers’ beliefs are

\[ \mu(t \mid s, P, H, r = 1) = E(t) \]

\[ \mu(t \mid s, P, H, r = \emptyset) = E(t) \]

Now \( V(x) = 0 \) for all \( t \) and for any \( P > 0 \) no one is going to the agency and the consumers pay \( E(t) \) for a firm without rating. This leads to \( \Pi = 0 \). Of course this is not an equilibrium in the whole game.

7 Comparison of Different Rating Strategies

When comparing the different rating strategies, we find the following.

With an absolute rating, the value of getting a rating is lower in a bad state of the world, as shown in the following proposition.
**Proposition 1.** The value of an absolute rating is im optimum larger in the good state of the world.

*Proof. ... [to be completed]*

We can also make the following statement about the threshold over which firms get a rating with an absolute minimum standard.

**Proposition 2.** The threshold for absolute minimum standard is greater than 0 for \( \bar{q} \) strictly between 0 and 1 (given monotone likelihood ratio).

*Proof. ... [to be completed]*

Note that Prop. 2 stands in contrast to Lizzeri (1999) where the threshold is always 0. The following intuition can be given for this result. If there is uncertainty about the state of the world, firms with a higher type \( t \) attach a larger probability to being in a good state of the world. Since a rating is more valuable in a good state of the world (Prop. 1), increasing the threshold has an additional positive effect on profits compared to Lizzeri (1999). This increases the optimal threshold from 0 to a positive value.

The following result shows that the general result that full disclosure is not optimal also holds in this case.

**Proposition 3.** Full disclosure is never optimal, is dominated by absolute minimum standard

*Proof. Setting the optimal price using absolute threshold we distinguish two cases. Either \( \bar{x} \) is indifferent or \( \bar{t} \).

Case 1: \( \bar{x} \) is indifferent

\( \bar{x} \) is the optimal threshold using full disclosure. Now we can show that absolute threshold makes higher profit even when it uses the same threshold.

Profit full disclosure:

\[
\Pi_f = P_f(\bar{q}(1 - G(\bar{x})) + (1 - \bar{q})(1 - B(\bar{x})))
\]

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Profit absolute threshold:

\[ \Pi_a = P_a(\bar{q}(1 - G(x)) + (1 - \bar{q})(1 - B(x))) \]

It is easy to show that \( P_a > P_f \)

\[ E_b[t \mid t \geq x]^+ (1 - q(x)) + E_g[t \mid t \geq x]^+ q(x) > x \]

\[ E_b[t \mid t \geq x]^+ (1 - q(x)) + E_g[t \mid t \geq x]^+ q(x) \]
\[ - E_b[t \mid t < x]^+ (1 - q(x)) - E_g[t \mid t < x]^+ q(x) > \]
\[ x - E_b[t \mid t \leq x]^+ (1 - q(x)) - E_g[t \mid t \leq x]^+ q(x) \]

\[ \Rightarrow \Pi_a > \Pi_f \]

Case 2: \( \tilde{t} \) is indifferent

to be completed

One can further show that if all firms are worth buying, complete pooling, i.e. giving all firms a rating, is an optimal strategy for the rating agency.

**Proposition 4.** If \( t > 0 \) complete pooling is the optimal strategy.

**Proof.** ... [to be completed]
Red: full, Blue: absolute, Green: relative, Black: complete

Figure 7: Thresholds for different rating strategies as a function of $q$. Parameter values: $\ell = -1$, $\bar{t} = 3$. Densities: $g(t) = 2(t - \ell)/(\bar{t} - \ell)^2$, $b(t) = 2(\bar{t} - t)/(\bar{t} - \ell)^2$

Red: full, Blue: absolute, Green: relative, Black: complete

Figure 8: Profits for different rating strategies as a function of $q$. Parameter values: $\ell = -1$, $\bar{t} = 3$. Densities: $g(t) = 2(t - \ell)/(\bar{t} - \ell)^2$, $b(t) = 2(\bar{t} - t)/(\bar{t} - \ell)^2$
“Disclosure” is never optimal (compare to Prop.3). In figure 9 the range of $t$ is changed to $t \in [-3, 3]$. Now “absolute minimum standard” is always the optimal strategy.

![Figure 9: Profits for different rating strategies as a function of $\bar{q}$](image)

Figure 9: Profits for different rating strategies as a function of $\bar{q}$. Parameter values: $\underline{t} = -3$, $\overline{t} = 3$. Densities: $g(t) = 2(t - \underline{t})/((\overline{t} - \underline{t})^2)$, $b(t) = 2(\overline{t} - t)/(\overline{t} - \underline{t})^2$

8 Conclusion

We have looked at different rating strategies of a rating agency if there are different states of the world and if the aggregate distribution of ratings is observed by market participants. We have found the following. First, the value of a rating is larger in a good state of the world. Second, the threshold of absolute ratings is positive rather than 0, which stands in contrast to previous findings in the literature. Third, relative ratings can be better than absolute ones, i.e. changing the threshold depending on the state of the world, even though the threshold should be independent of the state of the world in first best. Fourth, it can be optimal for an agency not to reveal the state of the world.

9 Further Results

**Proposition 5.** The expected quality for firms below $\overline{x}$ in one state of the world $(\int_{L}^{\overline{x}} \frac{g(t)}{g(x)} dt)$ is increasing in $\overline{x}$.
Proof. $\frac{\partial}{\partial \xi} \int_{\xi}^{\eta} g(t)dt = G(\xi)g(\xi) - G(\eta)g(\eta) \int_{\xi}^{\eta} t g(t)dt$ since $g(t) > 0 \forall t \in [\xi, \eta]$

The proof is the same for the bad state of the world.

References


