Oil Shocks and External Adjustment*

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Abstract

In a two-country DSGE model, a permanent rise in the oil price reduces the relative wealth of an oil-importing country, causing its non-oil trade balance to improve, and its terms of trade to worsen. The magnitude of adjustment in the non-oil trade balance and the terms of trade hinges on structural parameters that affect the divergence in wealth effects between oil importers and exporters, such as the oil price elasticity of demand. For the United States, an oil-importing country that also has a significant oil endowment, our model implies that the improvement in the non-oil trade balance and exchange rate depreciation are fairly small; hence, a permanent oil price hike generates a protracted overall trade deficit that only narrows gradually as households and firms substitute away from oil.

Keywords: oil-price shocks, trade, DSGE models

JEL Classification: F32, F41

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1 Introduction

The quadrupling of the dollar price of crude oil over the past several years has been accompanied by a significant widening of the U.S. trade deficit on petroleum products. As seen in Figure 1, the petroleum deficit – at 3 percent of GDP in early 2008 – exceeded half of the U.S. trade deficit on goods and services of 6 percent of GDP, and from an accounting perspective, could be regarded as responsible for nearly all of the 2 percentage point widening in the overall U.S. trade deficit between 2002 and 2008.

However, merely applying an accounting relationship between the trade balance and its components would fail to recognize that the oil and the non-oil trade balance react to movements in oil prices also through general equilibrium channels. We examine the effects of oil price shocks on the trade balance through a two-country dynamic stochastic general equilibrium model (DSGE). This approach highlights the economic channels through which trade adjustment occurs, including the role of changes in wealth and the non-oil terms of trade in affecting the non-oil trade balance. Our work offers a theoretical interpretation to recent work by Kilian, Rebucci, and Spatafora (2007) who examined empirically how the trade balance of oil-importers and oil-exporters responds to oil demand and supply shocks. Kilian, Rebucci, and Spatafora (2007) found that oil shocks that raise the oil price generate a deficit on the oil balance of oil-importing countries, but that an improvement in the non-oil balance partly offset the effects of the oil price rise on the overall balance. Our benchmark results imply this kind of interaction between the oil and the non-oil trade balance.

The model in this paper builds on the open economy DSGE model of Backus and Crucini (1998). In particular, each country produces a distinct tradable output good that is used as an input into the production of consumption and investment goods both at home and abroad. Oil serves as an input into the production of the domestic tradable good, and also enters directly into the household consumption bundle. One country is an oil-importer, while the other is an oil-exporter.

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1 The oil demand and supply shocks are identified from a recursively-ordered vector autoregression following the approach of Kilian (2007).

2 An extensive literature (mainly empirical) has attempted to identify the effects of energy supply shocks on real output and prices, and how monetary policy may influence the response; examples include Hamilton (1983), Hamilton (2003), Kilian (2006), Cavallo and Wu (2006), Bernanke, Gertler, and Watson (1997) and Hamilton and Herrera (2004).
The model differs from Backus and Crucini (1998) along two important dimensions. Given our interest in wealth effects, we depart from their complete markets framework by assuming that financial markets are incomplete across national borders. In addition, we introduce convex costs of adjusting the share of oil used in production and consumption. Adjustment costs provide a tractable way of capturing the putty-clay nature of oil demand (see Atkeson and Kehoe (1999) and Wei (2003)), which allows our model to account for the substantial wedge between the short and long-run elasticity of oil demand that appears to be a robust finding of the empirical literature.

We examine how an oil market specific shock (e.g., supply contraction) that raises the price of oil affects the external balance of an oil-importer such as the United States. The shock causes the oil component of the trade balance to deteriorate for a prolonged period, though the oil deficit eventually narrows as producers and households substitute away from oil. In addition, the non-oil component of the trade balance improves, in line with the empirical results of Kilian, Rebucci, and Spatafora (2007). The improvement in the non-oil component is attributable to a negative wealth effect on the oil importer relative to the oil exporter, which induces the former’s non-oil imports to contract, and its non-oil terms of trade to worsen. To satisfy intertemporal budget balance, the improvement in the non-oil balance must be large enough to offset the long-run deterioration in the oil balance, as well as to finance interest payments on the stock of debt accumulated along the transition path.

Several structural factors play a key role in determining the relative wealth effects across countries by influencing the magnitude of transfers from oil-importers to exporters, their persistence, and how they are discounted. We devote particular attention to the importance of the long-run price elasticity of oil demand, and use King’s (1990) approach for decomposing wealth and substitution effects to show how the wealth effects in each country depend on this parameter. Notably, a lower oil price elasticity of demand implies that an oil importer runs more persistent deficits on the oil component of its trade balance, while the oil-exporter experiences a greater revenue windfall. This requires an oil importer to generate a larger non-oil trade surplus, and experience a larger depreciation of its exchange rate. Similarly, the oil-importer experiences a larger real depreciation when it has a smaller endowment of oil, and when the steady state interest rate is higher.

We proceed to contrast the responses under our benchmark model with a sin-
ngle risk-free international bond to the case of complete markets. Under complete markets, an oil-importing country facing a rise in the oil price receives an insurance transfer that enables it to satisfy its intertemporal budget constraint without having to run a non-oil trade surplus. With the wealth effect symmetric across oil importers and exporters, there is no movement in the non-oil terms of trade. While the financial market arrangements we consider offer somewhat extreme perspectives, they suggest that enhanced financial risk-sharing would tend to dampen terms of trade movements in response to oil shocks relative to our benchmark model, while allowing oil importers to run larger and more protracted trade deficits.\(^3\)

We conclude by conducting sensitivity analysis to the particular type of oil price shock assumed, and to our assumption that wages and prices are fully flexible. The benchmark model posits that an adverse supply shock pushes up the oil price permanently by reducing the oil endowment of the foreign exporter enough to yield the desired price path. But the effects on the home oil-importing country are essentially unchanged under the alternative assumption that the oil price rises endogenously in response to an exogenous contraction in foreign oil supply, or if the oil price is driven by a higher foreign taste for oil. Finally, we show that although nominal rigidities in prices and wages affect the near-term path of domestic absorption, their inclusion has little consequence for the trade and non-oil terms of trade responses on which we focus.

The remainder of the paper is organized as follows. Section 2 describes our benchmark model, and Section 3 its calibration. Section 4 provides simulation results for our benchmark model, and sensitivity analysis. Section 5 considers alternative types of oil price shocks, and Section 6 assesses the role of nominal rigidities. Section 7 concludes.

\(^3\)Since the influential work of Cole and Obstfeld (1991), there has been considerable interest whether differences in financial market structure have pronounced effects. However, the focus has been almost exclusively on technology shocks. Among others, Erceg, Gust, and López-Salido (2007) found that complete and incomplete markets imply very similar macroeconomic responses to a technology shock when the trade price elasticity is around unity. However, a high trade price elasticity can generate substantial divergence across these alternative financial market structures under certain conditions on labor supply (different preferences over labor supply appear to account for why Baxter and Crucini (1995) found significant disparities, while Schmitt-Grohe and Uribe (2003) did not). Finally, Corsetti, Dedola, and Leduc (2008) showed that very low trade price elasticities can also imply pronounced differences across financial market structures.
2 Model Description

There are two countries, a home country (calibrated based on U.S. data) and a foreign country (rest-of-the-world). Because the structure of each country block is symmetric, we focus on the home country, although our calibration allows for differences in population size and in the per capita oil endowment. Each country specializes in the production of a final good that is an imperfect substitute for the final good produced in the other country. Production requires capital, labor, and oil. The consumption bundle entering the household utility function depends on consumption of the domestically-produced good, on imports of the foreign good, and oil. For expositional purposes, it is convenient to assume that this composite consumption bundle is produced by a competitive distribution sector with a productive structure that mirrors household preferences over the three goods. While asset markets are complete at the country level, we assume that asset markets are incomplete internationally. Finally, both the home and foreign country are endowed with a non-storable flow supply of oil each period.

2.1 Households

The utility functional of a typical member of household \( h \) is

\[
\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \left\{ \frac{1}{1-\sigma} C_{t+j} (h)^{1-\sigma} + \frac{\chi_0}{1-\chi} (1 - N_{t+j} (h))^{1-\chi} \right\},
\]

where the discount factor \( \beta \) satisfies \( 0 < \beta < 1 \). The period utility function depends on an individual’s current consumption \( C_t(h) \) and leisure \( 1 - N_t(h) \). To allow for population differences across countries, we assume that there are \( \zeta \) households in the home country, and normalize the household size to unity in the foreign country.

Each member of household \( h \) faces a flow budget constraint in period \( t \) which states that his combined expenditure on goods and on the net accumulation of financial assets must equal his disposable income:

\[
P_{C_t} C_t (h) + P_{I_t} I_t (h) + e_t P^{*}_{B_t} B_{F_{t+1}}(h) - e_t B_{F_t}(h) = W_t (h) N_t (h) + R_{K_t} K_t (h) + \Gamma_t (h) + T_t (h).
\]

Final consumption goods are purchased at the price \( P_{C_t} \), and final investment goods at the price \( P_{I_t} \). Investment in physical capital augments the per capita capital stock \( K_{t+1}(h) \) according to a linear transition law of the form:

\[
K_{t+1} (h) = (1 - \delta) K_t (h) + I_t (h),
\]
where $\delta$ is the depreciation rate of capital.

Households accumulate financial assets by purchasing state-contingent domestic bonds, and a non state-contingent foreign bond. Given the representative agent structure at the country level, we omit terms involving the former from the household’s budget constraint. The term $B_{Ft+1}(h)$ in the budget constraint represents the quantity of the non-state contingent bond purchased by a typical member of household $h$ at time $t$ that pays one unit of foreign currency in the subsequent period, $P_{Bt}$ is the foreign currency price of the bond, and $e_t$ is the exchange rate expressed in units of home currency per unit of foreign currency. To ensure that net foreign assets are stationary, we follow Turnovsky (1985) and assume there is an intermediation cost $\phi_{Bt}$ paid by households in the home country for purchases of foreign bonds. Specifically, the intermediation costs depend on the ratio of economy-wide holdings of net foreign assets to nominal output and are given by:

$$\phi_{Bt} = \exp\left(-\phi_b \left( \frac{e_t B_{Ft+1}}{P_{Bt} Y_t} \right) \right).$$

(4)

If the home economy has an overall net lender position, a household will earn a lower return on any holdings of foreign bonds. By contrast, if the economy has a net debtor position, a household will pay a higher return on any foreign debt.

Each member of household $h$ earns labor income $W_t(h) N_t(h)$ and capital income $R_{Kt} K_t(h)$. Each member also receives an aliquot share $\Gamma_t(h)$ of firm profits and of the oil endowment (discussed below), and receives net transfers of $T_t(h)$.

In every period $t$, household $h$ maximizes the utility functional (1) with respect to its consumption, labor supply, investment, (end-of-period) capital stock, and holdings of foreign bonds, subject to its budget constraint (2), and the transition equation for capital (3). In doing so, a household takes as given prices, wages, and net transfers.

2.2 Firms and Production

Each country produces a single distinct non-oil good. Focusing on the home country, this non-oil good is produced by perfectly competitive firms according to a constant-returns-to-scale technology. The representative firm’s technology can be characterized as a nested constant-elasticity of substitution (CES) specification of the form:

$$V_t = \left( \frac{\omega_k^{1+\rho_v}}{\omega_k^{1+\rho_v} K_t^{1+\rho_v} + L_t^{1+\rho_v}} \right)^{1+\rho_v},$$

(5)
Each producer utilizes capital and labor services, $K_t$ and $L_t$, to make a “value-added” input $V_t$. This composite input is combined with oil $OY_t$ to produce the domestic non-oil good $Y_t$. The factor $\varphi_{OY_t}$ reflects costs of adjusting the oil intensity of production, and is assumed to take the following quadratic form:

$$
\varphi_{OY_t} = \left[1 - \frac{\varphi_{oy}}{2} \left( \frac{OY_t}{V_t} \right)^2 \right],
$$

(7)

where $O_{Yt}^A$ and $V_{t-1}^A$ denote oil inputs and value added aggregated over all firms. From an aggregate perspective, this specification allows for oil use to respond quickly to gross domestic output, while potentially allowing for very slow adjustment to relative price changes. This form of adjustment cost implies a time-varying elasticity of substitution for oil, an important feature of putty-clay models such as in Atkeson and Kehoe (1999) and Wei (2003). In their setting, a large variety of types of capital goods are combined with energy in different fixed proportions. Thus the short-run elasticity of substitution for oil is low. In the long run, the elasticity is higher, as firms can invest in capital goods with different fixed energy intensities.

Given the presence of adjustment costs, the representative producer can be regarded as choosing a contingency plan for $K_t$, $L_t$, and $OY_t$ that minimizes the discounted expected cost of producing the domestic output good subject to equations (5)-(7). In solving this problem, the producer takes as given the rental price of capital $R_{Kt}$, the wage $W_t$, and the price of oil $P_{Ott}$. The representative firm sells its output to households and firms at a price $P_{Dt}$, which is the Lagrange multiplier from the cost-minimization problem.

**Production of Consumption and Investment Goods**

The consumption basket $C_t$ that enters the household’s budget constraint can be regarded as produced by perfectly competitive consumption distributors. These distributors purchase a non-oil consumption good $C_{Nt}$ (described below) and oil $OC_t$ as inputs in perfectly competitive input markets, and produce a composite consumption good according to a CES production function:

$$
C_t = \left(1 - \omega_{oc} \right)^{\rho_{oc}} \frac{\rho_{oc}}{1+\rho_{oc}} C_{Nt}^{\frac{1}{1+\rho_{oc}}} + \omega_{oc}^{\rho_{oc}} \left( \varphi_{OClt}OC_t \right)^{\frac{1}{1+\rho_{oc}}},
$$

(8)

where the quasi-share parameter $\omega_{oc}$ determines the importance of oil purchases in the household’s composite consumption bundle, and the parameter $\rho_{oc}$ determines...
the long-run price elasticity of demand for oil. The form of the production function mirrors the preferences of households over consumption of non-oil goods and oil. The term $\varphi_{OCt}$ captures costs of adjusting oil consumption at the household level, and is assumed to have the quadratic form:

$$\varphi_{OCt} = \left[ 1 - \frac{\varphi_{oc}}{2} \left( \frac{O_{ Ct}}{C_{Nt}} - 1 \right) \right]^2,$$

where $O_{ Ct-1}^A$ and $C_{Nt-1}^A$ denote oil consumption and non-oil consumption aggregated over all households, respectively.

Thus, households are similar to firms in that they adjust their oil demand slowly in response to changes in the relative price of oil. As in the case of firms, the presence of adjustment costs implies that the consumption distributors must solve a dynamic problem of choosing a contingency path for their inputs $C_{Nt}$ and $O_{ Ct}$ so as to minimize their discounted expected costs of producing the consumption bundle, taking as given input prices $P_{CNt}$ and $P_{Ot}$, respectively. The Lagrangian multiplier from this cost-minimization problem determines the price of the consumption bundle they charge to households, i.e., $P_{Ct}$ in the household’s budget constraint given in equation (2).

Similarly, it is also convenient to regard the non-oil consumption good $C_{Nt}$ and investment good $I_t$ as produced by perfectly competitive distributors. Both the domestically-produced non-oil good and the foreign non-oil good are utilized as inputs, though we allow for the proportion of each input to differ between non-oil consumption and investment goods. Thus, the production function for the non-oil consumption good $C_{Nt}$ is given by:

$$C_{Nt} = \left( (1 - \omega_{mc}) \frac{\rho_c}{1+\rho_c} C_{Dt}^{\frac{1}{1+\rho_c}} + \omega_{mc} \frac{\rho_c}{1+\rho_c} (M_{Ct})^{\frac{1}{1+\rho_c}} \right)^{1+\rho_c},$$

where $C_{Dt}$ denotes the quantity of domestically-produced non-oil goods used as inputs by the representative non-oil consumption distributor (purchased at a price of $P_{Dt}$), and $M_{Ct}$ denotes imports of the foreign non-oil good (purchased at a price of $P_{Mt}$). The Lagrangian multiplier from the cost-minimization problem for the distributors determines the price of the non-oil consumption good $P_{CNt}$.

Finally, the production function for investment goods is isomorphic to that given in equation (10), though allowing for possible differences in the import intensity of investment goods (determined by $\omega_{mi}$, akin to $\omega_{mc}$ in equation (10)), and the degree of substitutability between non-oil imports and domestically-produced goods in
producing investment goods (determined by $\rho_t$). The Lagrangian from the problem that investment distributors face determines the price of new investment goods $P_{It}$ that appears in the household’s budget constraint.\(^4\)

### 2.3 The Oil Market

Each period the home country is endowed with a constant exogenous supply of oil $Y_O$. In our benchmark analysis, the price of oil relative to the domestic non-oil good $\psi_t = \frac{P_m}{P_O}$ follows an exogenous stochastic process. With both domestic oil supply and the oil price determined exogenously, foreign oil production $Y^*_O_t$ adjusts endogenously to clear the world oil market:

$$Y_O + \frac{1}{\zeta} Y^*_O = O_{Yt} + O_{Ct} + \frac{1}{\zeta} O^*_Y + \frac{1}{\zeta} O^*_C.$$

Thus, the sum of the home and foreign oil production equals the sum of home and foreign oil consumption by firms and households. The term $\frac{1}{\zeta}$ accounts for population differences across countries. The relative price of oil is assumed to follow an autoregressive process of the form:

$$\log(\psi_t) = \rho \log(\psi_{t-1}) + \epsilon_t. \quad (12)$$

The persistence parameter $\rho$ is set arbitrarily close to one, so that the log of the relative price of oil effectively follows a random walk.

In Section 6, we explore a framework in which the oil price is determined endogenously to equate supply and demand in the world oil market. In this case, the foreign production level $Y^*_O_t$ in equation (11) is assumed to follow an exogenous stochastic process.

### 2.4 Fiscal Policy

We assume that a fixed share $g$ of the domestic non-oil good $Y_t$ is purchased by the government (but that the import content of government purchases is zero). Government purchases $G_t = gY_t$ have no direct effect on household utility. Given

\(^4\)As discussed in Erceg, Guerrieri, and Gust (2006), our trade specification implies that the activity variable driving (non-oil) import and export demand can be regarded as a weighted average of consumption and investment, with the latter receiving a large weight consistent with the high weight of investment in U.S. trade. The paper also provides empirical support in favor of this specification over a specification in which the real activity variable driving trade is total absorption.
the Ricardian structure of our model, we assume that net lump-sum transfers $T_t$ are adjusted each period to balance the government receipts and revenues, so that:

$$P_DtG_t + T_t = 0.$$  \hspace{1cm} (13)

### 2.5 Resource Constraints for Non-Oil Goods, and Net Foreign Assets

The resource constraint for the non-oil goods sector of the home economy can be written as:

$$Y_t = C_{Dt} + I_{Dt} + G_t + \frac{1}{\zeta}M^*_t,$$ \hspace{1cm} (14)

recalling that $M^*_t$ denotes the home country’s exports normalized by the foreign population size, and that the term $\zeta$ denotes the relative population size of the home country.

The evolution of net foreign assets can be expressed as:

$$e_{t}P^*_B B_{Ft+1} = e_{t}B_{Ft} + e_{t}P_{Mt} - P_{Mt}M_t + P_{Olt} (O_{Yt} + O_{Ct} - Y_{Olt}).$$ \hspace{1cm} (15)

This expression can be derived by combining the budget constraint for households, the government budget constraint, and the definition of firm profits.

### 3 Solution Method and Calibration

We solve the model by log-linearizing the equations around the model’s steady state. To obtain the reduced-form solution of the model, we use the numerical algorithm of Anderson and Moore (1985), which provides an efficient implementation of the method proposed by Blanchard and Kahn (1980) (see also Anderson (1997)).

The model is calibrated at a quarterly frequency. The parameter values for the home economy under our benchmark calibration are listed in Table 1. Parameters for the foreign economy are identical except for the trade share parameters which are determined by balanced trade given relative population sizes.

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5The steady state around which we linearize depends on the relative level of technology in each country, which we initialize to unity. We evaluated the robustness of our solution procedure by using a nonlinear Newton-Raphson algorithm that does not rely on linearization around an initial steady state, and found that the results were nearly identical to those reported.
The discount factor $\beta$ is 0.99. The parameter $\sigma$ in the subutility function over consumption is set equal to 1 (implying a logarithmic form). We set $\chi = 10$, implying a Frisch elasticity of labor supply of 0.2. The utility parameter $\chi_0$ is set so that employment comprises one-third of the household’s time endowment. The population size parameter $\zeta$ is set so that U.S. non-oil output comprises one fourth of world non-oil output.

The production function parameter $\rho_v$ is set to -2, implying an elasticity of substitution between capital and labor of 0.5. The depreciation rate of capital $\delta = 0.025$ is consistent with an annual depreciation rate of 10 percent. We set the government share of output to 18 percent, and the quasi-capital share parameter $\omega_k$ to 1.61, so that the investment share of output equals an empirically-realistic value of 20 percent.

Our calibration of $\omega_{oy}$ and $\omega_{oc}$ is determined by the overall oil share of output, and the end-use ratios of oil in consumption and production. Based on data from the Energy Information Administration of the U.S. Department of Energy, the overall oil share of the domestic economy is set to 3 percent, with one-third of total oil usage accounted for by households, and two-thirds by firms. The oil imports of the home country are set to 50 percent of total demand in the steady state, implying that half of oil demand is satisfied by domestic production. This estimate is based on 2003 data for the United States obtained from British Petroleum, and includes oil and natural gas.

Our choices for the parameters that determine the short- and long-run elasticity of substitution for oil ($\rho_{oy}$, $\rho_{oc}$, $\varphi_{oy}$ and $\varphi_{oc}$) are motivated by the regression results described in Appendix A. Based on our regression estimates, the parameter $\rho_{oy}$ is set to imply a long-run elasticity of substitution between oil and the other factors of production of 0.5, while the adjustment cost parameter $\varphi_{oy}$ implies that the half life of the response of oil demand to a permanent rise in the oil price is 10 years. As in most other studies, we find that the short-run elasticity of oil demand is very small relative to the long-run elasticity. Moreover, the implied time path of the price elasticity of demand for oil is within the wide range of estimates for energy’s own price elasticities reported in surveys such as Dahl and Sterner (1991) or Atkins.

\[^6^\text{In calibrating our model, we adopt a more inclusive interpretation of the oil sector that also incorporates natural gas. Our inclusion of natural gas reflects the close substitutability between oil and natural gas as energy inputs, and the high correlation between movements in oil prices and natural gas prices. Over the period 1970-2005, the correlation between crude oil prices and natural gas prices was 0.9 (using data from NYMEX).}^\]
of oil demand to be the same for households as for firms (so $\rho_{oc} = \rho_{oy}$ and $\varphi_{oc} = \varphi_{oy}$).

Turning to the parameters determining trade flows, the parameter $\omega_{mc}$ is chosen to match the estimated average share of imports in total U.S. consumption of about 6 percent using NIPA data, while the parameter $\omega_{mi}$ is chosen to match the average share of imports in total U.S. investment of about 43 percent. This calibration implies a non-oil-goods import-to-GDP ratio for the home country of about 12 percent. Given that trade is balanced in steady state, and that the oil import share for the home country is 1.5 percent of GDP, the goods export share is 13.5 percent of GDP. We assume that $\rho_c = \rho_i = 2$, consistent with a long-run price elasticity of demand for imported consumption and investment goods of 1.5.\(^8\)

### 4 Model Simulations

Figure 2 shows responses of the home oil-importing country (the United States) to a 20 percent permanent rise in the relative price of oil under our benchmark calibration.\(^9\) The oil price shock induces a progressive fall in home oil demand as both households and firms substitute away from this more costly input. With an oil elasticity of demand of one half, oil demand drops about 10 percent in the long-run.\(^10\)

The progressive decline in oil use has effects on gross (non-oil) output, the expenditure components, and the real interest rate that are qualitatively similar to those of a persistent decline in productivity growth. Thus, gross output declines gradually in response to lower oil use and a falling capital stock, while consumption contracts due to a reduction in household permanent income. Investment spending

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7 Some of the variation in the elasticity estimates depends on the specific energy aggregate being used. Cooper (2003) estimated the long-run price elasticity of the demand for crude oil – the same energy aggregate we examine – and found a very similar value of 0.45.

8 Hooper, Johnson, and Marquez (2000) estimated trade price elasticities using aggregate data for G-7 countries. They reported a long-run export price elasticity of 1.5 for the United States.

9 Although this is a substantial rise, Blanchard and Gali (2007) have documented how the oil price rose over 100 percent in logarithmic percentage terms on four occasions since the first OPEC shock in 1973.

10 While the fall in energy demand due to the direct effect of higher prices is amplified by an endogenous decline in output and consumption, the latter effect is small.
falls, reflecting that a lower path of oil usage pushes down the current and future marginal product of capital. The decline in the marginal productivity of capital induces a modest fall in domestic real interest rates.

Turning to the implications for the external sector, the rise in oil prices causes a sharp and immediate deterioration of the overall trade balance equal to about 0.2 percentage points of GDP. In the longer term, the overall trade deficit narrows and shifts into surplus roughly fifteen years after the occurrence of the shock.

The slow improvement in the overall trade balance (following the initial deficit) is mainly attributable to a gradual decline in the volume of oil imports as households and firms substitute away from higher-priced oil. However, this substitution effect eventually has a powerful restraining effect on oil imports, in part because the home country (the United States) produces half of the oil it consumes in the pre-shock steady state. With domestic oil production unaffected by the shock, lower oil demand is borne entirely by imports, implying that the long-run fall in oil import demand in percentage terms (about 20 percent) is around twice as large as the decline in overall oil demand.

The deterioration in the overall trade balance is partially offset by an improvement in the home country’s non-oil trade balance, consistent with the empirical results of Kilian, Rebucci, and Spatafora (2007). The improvement in the non-oil component is attributable to a negative wealth effect on the home country relative to the foreign oil-exporter. It is helpful to consider the wealth effect as having two components. One component reflects that oil is a more expensive input into production and consumption, which tends to depress consumption demand in both the home and foreign country. A second component reflects a transfer of purchasing power to the country with the relatively larger endowment of oil. The latter component exacerbates the negative wealth effect on the oil importer, and creates a pronounced disparity in the wealth effects between the oil-importing and oil-exporting country.

The decline in the relative wealth of the home country is reflected in a higher present value of deficits in the oil component of its trade balance. To satisfy intertemporal budget balance, the improvement in the home non-oil balance must be large enough to offset any long-run deterioration in its oil balance due to the shock, as well as to finance interest payments on the stock of debt accumulated along the transition path. The requisite improvement in home real net exports is achieved by a decline in home consumption relative to foreign, as well as through a deterioration of the home country’s non-oil terms of trade. The non-oil terms of
trade deteriorates due to home bias in consumption (since the transfer of purchasing power to the foreign economy depresses the relative price of home goods), with the consumption-based real exchange rate declining in parallel.

From a quantitative perspective, the adjustment of the non-oil balance plays a modest role in contributing to the eventual improvement in the overall trade balance, and that the real exchange rate only experiences a small depreciation of 0.4 percent. This reflects that the long-run deterioration of the oil balance is quite small under our benchmark calibration for the United States, given our estimate of a fairly high long-run price elasticity of oil demand of 0.5, and that the United States is a substantial producer of oil (which greatly magnifies the response of import volumes to a price change). As a result, oil shocks have very large and persistent effects on the overall trade deficit until households and firms substitute away from this higher-priced input.

4.1 The Oil Elasticity of Substitution

Several structural parameters that play a key role in determining the relative wealth effects across countries by influencing the magnitude of transfers from oil-importers to exporters, their persistence, and how they are discounted. We begin by considering sensitivity to the long-run price elasticity of demand for oil. Figure 3 contrasts the responses under our benchmark calibration to the 20 percent oil price hike with those derived under two alternative calibrations of this long-run elasticity. One alternative imposes a long-run elasticity of unity, consistent with a Cobb-Douglas production function over the factor inputs. A second alternative imposes a long-run price elasticity of demand elasticity of 0.10, close to a Leontief specification. In each case, the adjustment cost parameter on oil is changed so that the half life of adjustment to a permanent shock remains 10 years, while all other parameters are unchanged. Our consideration of a wide range of values for the long-run price elasticity of oil demand reflects our finding of a wide band of uncertainty surrounding its estimate. Moreover, the oil price elasticity may differ substantially across countries, since specific taxes on oil damp the effective long-run elasticity of oil demand with respect to the pre-tax price.

Under the near-Leontief specification, the oil price rise causes a more persistent

\[^{11}\text{The depreciation of the real exchange rate would be even smaller if the elasticity of non-oil imports and exports were higher than the 1.5 value assumed in our benchmark calibration}\]
deterioration in the oil component of the trade balance than under our benchmark; in particular, with oil demand falling only a few percent in the long-run, the oil deficit remains around 0.3 percent point of GDP even after 30 years. Accordingly, the home country experiences a larger fall in its relative wealth; and intertemporal budget balance requires a bigger improvement in the non-oil trade balance, and a larger deterioration of the terms of trade. Interestingly, these offsets imply that the overall trade deficit rises by less in response to the same oil price rise.

Under the other alternative with a price elasticity of demand of unity, oil imports fall so much that the home country’s oil balance shifts into surplus after about a decade. With the long-run oil balance improving, the non-oil balance actually deteriorates, and the terms of trade improve slightly.

4.2 Complete vs. Incomplete Markets

In the benchmark model with incomplete markets, oil price shocks have substantial wealth effects that diverge across countries. It is helpful to contrast these results with an alternative financial structure of complete markets in order to emphasize the pivotal role that such cross-country changes in the distribution of relative wealth play in driving the dynamics of the terms of trade and non-oil trade balance.

Figure 4 contrasts responses to the permanent oil price shock in the benchmark model with incomplete markets to responses derived from a variant that allows for complete financial markets. The results are strikingly different: in particular, the non-oil terms of trade remain at their steady-state level with complete markets, and the non-oil trade balance is essentially unaffected.

These disparities reflect that ownership of the profit flow associated with oil production is effectively shared across countries in the complete markets case through insurance transfers. Although oil price hikes still have a contractionary effect on consumption, the effect is mitigated in the home country and the non-oil terms of trade remain essentially unchanged. While the oil-importing country runs an overall trade deficit under complete markets, the insurance payments offset the higher nominal oil expenditures: these transfers eliminate the need to accrue a surplus on its non-oil balance.

As a corollary, structural factors such as the oil price elasticity can influence the magnitude of the non-oil trade balance response only through the differential wealth effects across countries. However, the size of these differentials hinges on the complete market structure. With these differential wealth effects virtually eliminated
under complete markets, variations in these structural factors have no effect on the non-oil terms of trade or non-oil balance. This is illustrated in Figure 4, which also shows the complete markets response for the case of a low long-run substitution elasticity between oil and other inputs of 0.1, compared with 0.5 in our benchmark. Under complete markets, the response of the non-oil terms of trade and the non-oil trade balance are indistinguishable for these very different substitution elasticities. This contrasts sharply with the large divergence in the incomplete markets case that was highlighted in our discussion of Figure 3. Thus, the assumption of complete financial markets prevents the interaction between the oil and non-oil components of the trade balance shown by Kilian, Rebugue, and Spatafora (2007).

4.3 Wealth and Substitution Effects

In the two previous sections, we have argued that our model can replicate the empirical observations about the non-oil trade balance if the wealth effects for the two countries are sufficiently different in response to the shock. Both in the model with complete markets and the model with an oil price elasticity of one, we fail to generate an improvement in the non-oil trade balance subsequent to an oil price increase, since the wealth effects across countries are virtually identical.

To show more precisely how these wealth effects differ across parameterizations and financial market structure we employ King’s (1990) “Hicksian” method for decomposing the consumption and labor supply responses into (i) a wealth effect, (ii) a real wage effect, and (iii) a real interest rate effect. To compute the wealth effects, we first find the change in discounted lifetime utility due to the oil price shock. As preferences are time-separable, the wealth effects on consumption and labor supply are given by the constant consumption and labor profiles that match the computed change in utility while wages and interest rates are held constant at their steady state values. The real wage effect is the part of the overall response in consumption (or labor) that is due to changes in the real wage alone, keeping utility at its steady state level. The real interest rate effect is computed analogously.

In Figure 5 we plot the wealth and substitution effects for consumption in the home and foreign country for our benchmark calibration, as well as for the model with complete markets, and the model with a low oil price elasticity of 0.1 and incomplete financial markets. Figure 6 shows the corresponding decomposition for
the labor supply response under the same three cases.\textsuperscript{12}

The disparity in wealth effects between the two countries are influenced by two basic considerations: \textit{i}) the oil-importing home country faces a higher oil bill for its imports; \textit{ii}) the foreign country faces a smaller oil endowment given the way we implement the oil price shock.

Under the benchmark calibration (solid line), the home country experiences a large negative wealth effect relative to the foreign country. In the home country, the wealth effect reduces consumption, and increases labor supply, whereas there is almost no wealth effect on these variables in the foreign country.\textsuperscript{13} If international financial markets are complete (dashed line), the home country is insured against the oil price increase and receives transfer payments. Both countries experience a negative wealth effect of similar impact on consumption and labor. With a low oil price elasticity, a given oil price hike has a more pronounced contractionary effect on the home country. This is reflected in the larger wealth effects on consumption and labor under an oil price elasticity of 0.1 (dotted line) relative to our benchmark calibration. In the foreign country, the oil price shock has a large positive wealth effect that pushes up consumption and leads to a substantial drop in the labor supply.

For completeness, we also report the substitution effects on consumption and labor due to changes in the real wage and the real interest rate. While these effects are important in explaining the response of consumption, they differ little across countries. In all three scenarios, real interest rates and wages fall in both countries. The real wage decline (not shown) and leads households to substitute from consumption towards leisure (and away from labor).\textsuperscript{14} Lower interest rates imply a lower price of current consumption and leisure relative to future consumption and leisure. Therefore, the interest rate substitution effect is positive on consumption and negative on labor.

Overall, the analysis confirms that it is differences in wealth effects between the two countries that explain most of the differences in the consumption and labor

\textsuperscript{12}We chose to omit the case of an oil price elasticity of one with incomplete markets since the quantitative results are very similar to the case with an oil price elasticity of 0.5 and complete markets.

\textsuperscript{13}The absence of a wealth effect in the foreign country does not imply, that consumption and labor are unaffected by the oil shock. However, these variables are impacted through the substitution effects caused by a lower real wage and a higher real interest rate.

\textsuperscript{14}The wage substitution effect on consumption is constant since our utility function is additive separable in consumption and leisure, and time separable.
response between countries and across scenarios.

4.4 Other Important Factors

In light of our discussion of relative wealth effects, it is straightforward to see that the local oil endowment and the discount factor can have a first-order influence on the response of trade flows to oil shocks. Ceteris paribus, the lower the oil endowment of the oil-importing country, the larger is the increase in the oil bill subsequent to the shock, implying a larger difference in the wealth effects between countries. Accordingly, as can be seen in Figure 7, when the oil-importing country has no oil endowment, an oil shock produces a more pronounced deterioration of terms of trade and a bigger improvement of the non-oil trade balance relative to the benchmark calibration.\footnote{The empirical analysis of Kilian (2006) seems consistent with these implications. In particular, Kilian found that an exogenous oil price increase would lead to a depreciation relative to the dollar for Italy, France, Germany, and Japan, all countries whose level of per capita domestic oil production is lower than for the United States. Moreover, he found the Canadian dollar appreciated, which is also in line with our model given that Canada is a major energy exporter.}

Changes in the discount factor $\beta$ primarily influence the wealth effect in the foreign country. As the home country reduces its oil imports over time in response to the price hike a low discount factor implies that the foreign country places a larger weight on periods when oil revenues are relatively high. The lower the discount factor, the larger is the (positive) wealth effect in the foreign country, which amplifies the differences in the wealth effect between the two countries. Consequently, as shown in Figure 8, a lower discount factor is associated with a sharper deterioration of the non-oil terms of trade for the home country and a stronger improvement of the non-oil trade balance.

5 Oil Supply and Demand Shocks

Thus far, the relative price of oil has been determined by an AR(1) process, with foreign oil production responding endogenously to meet world demand. We show that the same channels for trade balance adjustment are at play when the oil price responds endogenously to shocks either to the quantity of oil supplied by the foreign country, or to foreign oil demand in response to a taste-shock.
Figure 9 shows responses to an adverse supply shock that induces a gradual reduction in the foreign flow endowment of oil $Y_{Ot}^*$. In this framework, the foreign endowment is assumed to follow an AR(1) in the growth rate with an autoregressive parameter of 0.9, and the innovation is scaled so that the relative price of oil rises 20 percent above steady state at its peak under our benchmark calibration. Although the peak effect is identical to that analyzed earlier, the figure shows that the oil price declines considerably over the long simulation horizon. The gradual decline in the oil price is due to falling world oil demand as households and firms substitute away from oil.

Focusing on the benchmark calibration, it is clear from Figure 9 that the qualitative effects of the oil supply shock are identical to those derived under an exogenous shock to the oil price (recalling Figure 2). The higher oil price shifts the trade balance of the oil-importer into persistent deficit. This deficit must be offset by an improvement in the non-oil balance, which is achieved through a deterioration in the non-oil terms of trade. The notable difference between the responses in Figure 9 and those in Figure 2 is that the former are quantitatively smaller, reflecting that the oil price hike is less persistent, and the wealth transfer to the oil-importer correspondingly smaller.

Key structural parameters affect the response of the non-oil terms of trade and the trade balance in the same way under an endogenous price response (figures 3-8). The divergence in responses across alternative calibrations of the elasticity of substitution parameter and the domestic production share parameter tends to be even larger than in simulations with an exogenous oil price. To illustrate this, Figure 9 compares the effects of the oil quantity shock under our benchmark calibration in which the oil elasticity of demand is set to 0.5 with alternatives in which the elasticity is set equal to unity and 0.1 just as in Figure 3. Given that the fall in world oil supply is identical across simulations, the oil price shows a larger and more persistent increase for low values of the long-run oil price elasticity of demand. Accordingly, the terms of trade deteriorate by more and the non-oil balance improves by more if the oil price elasticity of demand is low, which mirrors the qualitative pattern in Figure 3. However, because the oil price response is now much larger under the low-elasticity calibration, the magnitude of the non-oil terms of trade deterioration and non-oil trade balance response is greatly amplified. By comparison, under a high elasticity of substitution the oil price remains only slightly above its steady state in the long run, so that the terms of trade and non-oil balance remain nearly
We next compare the effects of an oil price rise generated by the supply shock with a similar-sized price increase induced by a demand shock that is specific to the oil market. To model the latter, we modify the foreign consumption demand equation to allow for a preference shock $\mu_{ot}^{*}$ to the foreign demand for oil:

$$C_{t}^{*} = \left(1 - \omega_{ct}^{*}\right)^{1/1+\rho_{oc}} C_{nt}^{*} + \frac{1}{1+\rho_{oc}} \left(\frac{\omega_{ct}^{*}}{\mu_{ot}^{*}} \frac{O_{ct}^{*}}{\mu_{ot}^{*}} \right)^{1+\rho_{oc}}.$$  \hspace{1cm} (16)

Thus, a rise in $\mu_{ot}^{*}$ raises the marginal productivity of oil abroad, and raises household oil demand at constant relative prices.$^{16}$ The demand shock follows an autoregressive process in the growth rate with an autoregressive parameter of 0.88.

Figure 10 compares the responses of the home country to the supply and the demand shock in the oil market. The latter is scaled so that the peak response of the oil price is roughly the same as for the supply shock. The two shocks have comparable effects on the trade balance and non-oil terms of trade, as well as on other domestic variables. Thus, either shock causes the oil component of the trade balance to shift into persistent deficit, which in turn induces the non-oil terms of trade to worsen.

Our results suggest that only the path of the oil price is relevant to the home country, provided that the oil price hike is generated by a foreign supply or demand disturbance that is specific to the oil market. From a practical perspective, it is immaterial to the United States whether oil prices rise because of a supply contraction in the Middle East, or because of cold weather in China, so long as the oil price responds commensurately.

In interpreting our results, two caveats are important. First, given that the demand and supply shocks were designed to elicit similar oil price paths, our analysis should not be taken to imply that reasonably calibrated oil-specific demand and supply shocks generate similar oil price responses. Clearly, these shocks may have different effects on the dynamic response of the oil price, depending on the size of underlying innovations, and on the time path of the elasticities of the supply and demand for oil.$^{17}$ The second caveat is that there are other types of shocks that

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$^{16}$ The increase in the marginal product reflects that oil and non-oil consumption goods are complements in the household consumption bundle under our benchmark calibration.

$^{17}$ It is still of interest to understand the transmission process from various oil demand and supply shocks to oil prices. For example, colder weather in China would presumably have smaller and shorter-lived effects on oil prices than a rise in oil demand associated with a greater use of motorized vehicles.
affect the world oil market primarily through their effects on aggregate expenditure. Oil price increases caused by different shocks could lead to very different responses of the home economy. For example, a rise in foreign productivity growth raises oil demand through its stimulative effect on foreign absorption and GDP, which in turn raises the demand for home country exports. This export stimulus is disconnected from the oil market and would remain operative even if the oil share of the world economy declined toward zero.

6 Nominal Rigidities

This section examines the sensitivity of our results to the inclusion of nominal rigidities in both price- and wage-setting, as well as real rigidities in both consumption and investment, that have become standard in the literature on dynamic New-Keynesian models. First, we account for stickiness in the aggregate price of the domestically produced non-oil good $P_{Dt}$ by assuming that it is produced by a continuum of monopolistically-competitive firms. The domestically produced non-oil good in equation (6) effectively serves as the factor input to the monopolistic producers. Thus, all producers have the same marginal cost. These monopolistically-competitive firms set prices in their domestic currency in Calvo-style contracts (“producer currency pricing” in the export market), with a mean contract duration of four quarters, and full indexation to past prices as in Christiano, Eichenbaum, and Evans (2005). Consumption habits and investment adjustment costs curb excess sensitivity of consumption and investment to interest rate movements. Second, following Erceg, Henderson, and Levin (2000), wages are set in Calvo-style staggered contracts by a continuum of households with differentiated labor inputs. The mean duration of wage contracts is four quarters and there is full indexation to past wage inflation. Finally, monetary policy is determined by an interest rate rule that responds to inflation only.

As seen in Figure 11, the response of the trade balance and its components in the model with the addition of habits in consumption and adjustment costs in investment (the solid lines) are nearly identical to the responses for the benchmark calibration in Figure 2. Moreover, the responses with nominal rigidities under a rule that responds to core inflation are virtually identical to that in the model with flexible prices and wages. This similarity reflects that the rule chosen is nearly optimal, as discussed by Bodenstein, Erceg, and Guerrieri (2007). Interestingly, the
trade balance and the terms of trade response is little varied when using a more aggressive rule that responds to headline inflation and leads to a larger contraction of GDP. This similarity arises because oil demand is mainly driven by the large change in the relative price of energy. Given the similar response of the oil component of the trade balance across the models, the required adjustment in the non-oil balance and non-oil terms of trade turns out to be similar. The requisite non-oil terms of trade adjustment can occur even in the model with nominal rigidities because the real exchange rate can adjust flexibly.

7 Conclusion

Oil shocks that lead to an increase in the price of oil are associated with protracted oil deficits, but the effects on the overall trade balance are mitigated by the improvement in the non-oil goods balance. This interaction between the oil and non-oil balances hinges on two model features. First, the oil price elasticity of demand needs to be sufficiently low. Second, international financial markets need to be incomplete. With these features, the oil price increase induces sufficiently different wealth effects between countries that lead to an improvement of the non-oil trade balance for the oil-importing country.
References


Table 1: Benchmark Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Used to Determine</th>
<th>Parameter</th>
<th>Used to Determine</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta = 0.99$</td>
<td>discount factor</td>
<td>$\chi = 10$</td>
<td>labor supply elasticity$^a$</td>
</tr>
<tr>
<td>$\sigma = 1$</td>
<td>intertemporal consumption elasticity</td>
<td>$\omega_{oc} = 0.018$</td>
<td>weight of oil in consumption</td>
</tr>
<tr>
<td>$\rho_{oc} = -2$</td>
<td>oil elasticity in consumption (0.5)</td>
<td>$\varphi_{oc} = 142$</td>
<td>oil adj. costs in consumption</td>
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<tr>
<td>$\delta = 0.025$</td>
<td>depreciation rate of capital</td>
<td>$\rho_v = -2$</td>
<td>K-L sub. elasticity (0.5)</td>
</tr>
<tr>
<td>$\omega_k = 1.61$</td>
<td>parameter on K in value added</td>
<td>$\omega_{ogy} = 0.024$</td>
<td>weight of oil in production</td>
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<tr>
<td>$\rho_{ogy} = -2$</td>
<td>value-added oil sub. elasticity (0.5)</td>
<td>$\varphi_{ogy} = 142$</td>
<td>oil adj. costs in production</td>
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<tr>
<td>$\rho_c = -1$</td>
<td>consumption import sub. elasticity (1.5)</td>
<td>$\omega_{mc} = 0.063$</td>
<td>weight of imports in non-oil consumption</td>
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<tr>
<td>$\rho_i = -1$</td>
<td>investment import sub. elasticity (1.5)</td>
<td>$\omega_{mi} = 0.43$</td>
<td>weight of imports in investment</td>
</tr>
</tbody>
</table>

$^a$ The Frisch elasticity is $2/\chi = 0.2$. 

25
Figure 1: The U.S. Trade Balance and Some Key indicators for Oil Trade

Sources: Bureau of Economic Analysis (NIPA) and Bureau of Labor Statistics.
Figure 2: A 20% Increase in the Price of Oil

Home Response
Figure 3: A 20% Rise in the Oil Price

Alternative Price Elasticities of Oil Demand (Home Response)
Figure 4: 20% Rise in the Oil Price

Complete and Incomplete Markets (Home Response)
Figure 5: Hicks Decomposition of Home and Foreign Consumption Response to a 20% Rise in the Oil Price

- **Wealth Effect on Home Consumption**
- **Wealth Effect on Foreign Consumption**
- **Real Wage Effect on Home Consumption**
- **Real Wage Effect on Foreign Consumption**
- **Real Interest Rate Effect on Home Consumption**
- **Real Interest Rate Effect on Foreign Consumption**
- **Overall Response of Home Consumption**
- **Overall Response of Foreign Consumption**

**Legend:**
- **Benchmark**
- **Complete Markets**
- **Oil Price Elasticity = 0.1**

30
Figure 6: Hicks Decomposition of Home and Foreign Labor Response to a 20% Rise in the Oil Price.
Figure 7: A 20% Rise in the Oil Price

Alternative Oil Endowments (Home Response)

Oil Demand

Benchmark
No Domestic Oil Production

Oil Imports

Non–Oil Terms of Trade

Oil Balance (GDP share)

Non–Oil Balance (GDP share)

Overall Trade Balance (GDP share)

Benchmark
No Domestic Oil Production
Figure 8: A 20% Rise in the Oil Price

Alternative Discount Factors (Home Response)
Figure 9: Contraction in Foreign Oil Supply (Home Response)

- Oil Demand
- Oil Price Elasticity = 0.5 (benchmark)
- Oil Price Elasticity = 1
- Oil Price Elasticity = 0.1

- Non-Oil Terms of Trade

- Oil Balance (GDP share)

- Non-Oil Balance (GDP share)

- Overall Trade Balance (GDP share)

Legend:
- Oil Price Elasticity = 0.5 (benchmark)
- Oil Price Elasticity = 1
- Oil Price Elasticity = 0.1
Figure 10: Comparing a Contraction in Foreign Oil Supply with an Expansion in Foreign Oil Demand (Home Response)
Figure 11: A 20% Increase in the Price of Oil
Flexible vs. Sticky Prices (Home Response)
A Appendix: Estimation of the Oil Demand Equation

The log-linearized behavioral equation determining aggregate oil demand can be expressed as:

\[ \hat{O}_t = \varepsilon_a \left( \frac{O_Y}{O} \hat{Y}_t + \frac{O_C}{O} \hat{C}_t \right) - \frac{\varepsilon_o}{1 + \varepsilon_o \varphi_o} \left( \frac{O_Y}{O} \hat{P}_{O,t} - \frac{O_Y}{O} \hat{P}_{Dt} - \frac{O_C}{O} \hat{P}_{Ct} \right) \]

\[ - \frac{\varepsilon_o \varphi_o}{1 + \varepsilon_o \varphi_o} \left( \frac{O_Y}{O} \hat{Y}_{t-1} + \frac{O_C}{O} \hat{C}_{t-1} - \hat{O}_{t-1} \right) + u_t. \]

(17)

In equation (17), a “hat” denotes a variable’s percentage deviation from its steady state value. Steady state values are distinguished by the omission of a time subscript. To derive the equation above, we imposed the restrictions \( \varepsilon_o = \frac{1 + \rho_{oy}}{\rho_{oy}} = \frac{1 + \rho_{oc}}{\rho_{oc}} \) and \( \varphi_o = \varphi_{oy} = \varphi_{oc} \), by which the price elasticity of oil demand is equalized across the two end uses in our model at each point in time. The parameter \( \varepsilon_o \) is the absolute value of the long-run price elasticity of oil demand. The variable \( u_t \) reflects a stochastic shock to oil demand, as discussed in Section 6. Our model imposes that the coefficient \( \varepsilon_a \) on contemporaneous activity \( \left( \frac{O_Y}{O} \hat{Y}_t + \frac{O_C}{O} \hat{C}_t \right) \) equals one, but we estimate it as a free parameter as a test of our specification.

To control for endogeneity, and guard the estimates against possible model misspecification, our econometric approach follows the limited information maximum likelihood method described in Pagan (1979) and Fukac and Pagan (2006). As a statistical model, we use a three equation VAR(4) that includes oil demand, the activity measure, and the relative price measure.\(^{18} \) The demand equation (17) replaces the oil equation in that VAR system. This system is estimated using maximum likelihood.

As a measure of oil demand, we take total petroleum consumption (product supplied) from the Energy Information Administration. We construct the activity measure as a weighted average of consumption and GDP from the National Income

\(^{18}\)We chose a lag length of 4 to avoid serial correlation in the residual.
and Product Accounts (NIPA).\(^1\)\(^9\) For the relative price measure, we use the crude oil producer price index series from the Bureau of Labor Statistics, and the consumption and GDP deflators from NIPA. We run all regression equations using log linearly detrended quarterly data. The estimation sample covers the period from the second quarter of 1948 to the fourth quarter of 2005.

The regression results for the oil demand equation are summarized in Table 2. The estimated elasticity for the activity measure is not significantly different from 1, which is the value imposed by our theoretical model. The oil price elasticity is estimated at 0.28. Our benchmark calibration value of 0.5 is within one standard deviation. The estimated adjustment cost parameter \(\varphi_o\) is 139.2, which implies a half life of 7 years for the response of oil demand to a permanent price increase. Our benchmark half life of 10 years lies within one standard deviation of the estimate of \(\varphi_o\).

The Durbin Watson statistics suggests serial correlation in the regression residuals of the oil demand equation. To address the potential mispecification, we modify the adjustment cost function for oil demand in production (equations 7) as follows:

\[
\varphi_{OY_t} = \left[ 1 - \frac{\varphi_{oy1}}{2} \left( \frac{O_{Yt}}{V_{t-1}} - 1 \right)^2 - \frac{\varphi_{oy2}}{2} \left( \frac{O_{Yt-1}}{V_{t-2}} - 1 \right)^2 \right].
\] (18)

We change the adjustment cost function for oil demand in consumption (equation 9) analogously.

Upon log-linearizing, the aggregate oil demand equation now takes the form:

\[
\dot{O}_t = \varepsilon_a \left( \frac{O_Y}{O} \dot{Y}_t + \frac{O_C}{O} \dot{C}_t \right) - \frac{\varepsilon_o}{1 + \varepsilon_o (\varphi_{o1} + \varphi_{o2})} \left( \dot{P}_{O,t} - \dot{P}_{D,t} \right) +
\]

\[
- \frac{\varepsilon_o \varphi_{o1}}{1 + \varepsilon_o (\varphi_{o1} + \varphi_{o2})} \left( \frac{O_Y}{O} \dot{Y}_{t-1} + \frac{O_C}{O} \dot{C}_{t-1} - \dot{O}_{t-1} \right)
\].

\(^1\)\(^9\)The weights reflect the share of total oil usage devoted to consumption and production under our benchmark calibration (of 1/3 and 2/3, respectively. Gross output is the theoretically preferred production measure implied by our model. However, because deriving a gross output measure requires estimating the share of oil imports directed towards production, we used GDP in our benchmark specification. As a robustness check, we constructed an alternative proxy for gross output, and found little difference from the results reported below.
\[-\frac{\varepsilon_o \varphi_o}{1 + \varepsilon_a (\varphi_o + \varphi_o)} \left( \frac{O_y}{O} \hat{Y}_{t-2} + \frac{O_C}{O} \hat{C}_{t-2} - \hat{O}_{t-2} \right). \tag{20} \]

Following the same estimation procedure as described above we obtain estimates summarized in Table 3.

This alternative regression specification reduces the residual autocorrelation, as indicated by a lower Durbin-Watson statistic. The coefficient estimates $\varepsilon_a$ and $\varepsilon_o$ are not significantly different from those in the simpler specification. Again, we fail to reject the restriction imposed by our theoretical model that $\varepsilon_a$ is 1. The estimates for the adjustment cost parameters $\varphi_o$ and $\varphi_o$ imply a half life of 10 years for the response of oil demand to a permanent price increase, as in our benchmark calibration. Given the short length of the time series and the slow adjustment of oil demand, it is difficult to estimate the long-run elasticity precisely. Nonetheless, the regression does very well at capturing the variation in oil demand, as suggested by the high $R^2$ statistic.

Table 2: Regression results for baseline oil demand equation*

<table>
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<tr>
<th>Coefficient</th>
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<th>Prob.</th>
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<td>$\varepsilon_a$</td>
<td>1.05</td>
<td>0.05</td>
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<tr>
<td>$\varepsilon_o$</td>
<td>0.28</td>
<td>0.27</td>
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<tr>
<td>$\varphi_o$</td>
<td>139.2</td>
<td>95.0</td>
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<td>$R^2 = 0.98$</td>
<td>Durbin-Watson stat. = 2.40</td>
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* See equation (17).

Table 3: Regression results for augmented oil demand equation*

<table>
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<td>0.05</td>
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<tr>
<td>$\varepsilon_o$</td>
<td>0.45</td>
<td>0.42</td>
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<tr>
<td>$\varphi_{o1}$</td>
<td>81.9</td>
<td>40.3</td>
</tr>
<tr>
<td>$\varphi_{o2}$</td>
<td>21.3</td>
<td>11.6</td>
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<tr>
<td>$R^2 = 0.98$</td>
<td>Durbin-Watson stat. = 2.05</td>
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* See equation (19).