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as an incentive device to avert  
relocation

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# Carbon leakage: Grandfathering as an incentive device to avert relocation

Robert C. Schmidt \*

## **Abstract:**

Emission allowances are often distributed for free in an early phase of a cap-and-trade scheme (grandfathering) to reduce adverse effects on the profitability of firms. If the grandfathering scheme is phased out over time, firms may nevertheless relocate to countries with a lower carbon price once the competitive disadvantage of their home industry becomes sufficiently high. We show that this is not necessarily the case. A temporary grandfathering policy can be a sufficient instrument to avert relocation in the long run, even if immediate relocation would be profitable in the absence of grandfathering. A necessary condition for this is that the permit price triggers investments in low-carbon technologies or abatement capital.

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# 1. Introduction

Suppose there are two countries, A and B. Country A introduces a cap-and-trade scheme, while greenhouse gas emissions in country B remain free of charge. Consider the location choice of a firm initially located in country A. If the carbon price in A (induced by the cap-and-trade scheme) is sufficiently high, the firm may decide to relocate to country B. However, if (part of) the emission allowances are distributed to the firm for free, relocation may become unprofitable, at least for a certain period of time. This holds if the amount of allowances allocated to the firm for free is sufficiently high to compensate the firm for the competitive disadvantage of its home industry caused by the carbon price.

The goal of this paper is to investigate under what conditions the firm's location decision may be affected by the free allocation of emission allowances (grandfathering) also in the long run, assuming that grandfathering is only a temporary policy option and must be phased out over time. The main contribution of this paper is to identify conditions under which temporary grandfathering can permanently avert relocation, and how policies that try to achieve this goal in a cost-efficient way should be designed.

A central aspect is the role of *sunk fixed costs* of investments in energy-saving equipment or low-carbon technologies. The idea is as follows. If a sufficient amount of allowances is allocated to the firm for free, it will continue to produce in country A for a certain period of time. During this period, it faces the permit price in this country. This makes it profitable for the firm to build up some "abatement capital" that lowers the firm's expenditure on certificates or allows the firm to sell (part of) the grandfathered allowances as long as it continues to produce in A. The optimal investment in abatement capital is increasing in the length of the period during which the firm plans to stay. Conversely, *given* a rise in the abatement capital stock of the firm, the optimal length of this period increases. Under some conditions, the strength of this positive feedback effect can make temporary grandfathering a sufficient policy instrument to permanently affect the firm's location choice. As shown in this paper, a necessary condition for this is that the fixed installation costs of the firm's abatement capital stock are sunk and, hence, can not be recovered when the firm relocates.

Another crucial aspect concerns the *timing* of the grandfathering policy. In order to avert relocation in the long run, an initially high rate of grandfathered allowances that declines rapidly is generally a less effective tool than a policy that entails a lower rate initially and that declines less rapidly. For a given stock of abatement capital, a firm relocates when the share of grandfathered allowances falls below a certain threshold. Therefore, in the former case, firms tend to "free-ride" the initially generous grandfathering policy, but do not invest a lot in

abatement capital and relocate after a short period of time. Hence, in order to avert relocation permanently, grandfathering must not be phased-out too quickly.

The third crucial aspect highlighted in this paper concerns the *observability* of the firm's output or location choice on the optimal design of a grandfathering policy. In general, grandfathering policies should be made contingent on the firm's location if this is possible, and follow an exogenous, pre-defined path instead of being updated to the firm's emission levels over time. This assures that no artificial incentives are created for the firm to raise its emissions in order to be allocated more allowances in the future (see Harrison and Radov, 2002). Furthermore, location-dependent policies lead to a maximum punishment for relocation (assuming that the grandfathering rate, then, drops to zero). However, in practice, firms may be able to shift only partially to other countries, while leaving e.g. their headquarters or final good assembly etc. in country A. This makes the "location" of the firm an irrelevant indicator of its productive activity in the home country. Therefore, policy measures have been proposed where the free allocation of allowances is conditioned on the firm's output, such as benchmarking or Best Available Technology (BAT) policies (Ahman and Zetterberg, 2002). However, these policies require a reliable measure for the firm's output, that in many cases does not exist when firms may relocate partially to other countries (e.g. by outsourcing). Hence, policies that are contingent on the firm's output are often not feasible. We, therefore, analyze whether second-best policies exist that are contingent only on the firm's emissions (that are easier to monitor), rather than location or output. To tackle this issue, we first analyze policies that are, by assumption, linear in the firm's emission level. Hence, if the firm emits twice as much CO<sub>2</sub>, then it is allocated twice as many certificates for free. It is shown that linear policies can be sufficient to avert relocation. However, due to the artificial incentives they create to raise emissions, the range of parameter values where this is possible is more narrow than in the case where policies are contingent on the firm's location. An alternative policy approach is to use non-monotonic allocation schemes where a fixed emissions target is defined<sup>1</sup> for the firm by the regulator, and the amount of allowances allocated to the firm for free decreases *both* when it produces higher or lower emissions than under its reference level. This removes the artificial incentives to raise emissions in country A, and yet maintains the punishment for shifting production to B. As shown in this paper, such policies can sometimes be used to implement the first-best solution when the firm's location and output are not observable to the policy-maker. However, a necessary condition for this is that the firm does not find it profitable to relocate its production partially to country

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<sup>1</sup> E.g. using data on the firm's past emissions and final output production.

B, e.g. due to complementarities in production. Otherwise, the firm may achieve its reference emission level without investing in low-carbon technologies or abatement capital.

The idea that grandfathering can be used as an incentive device to avert relocation in the long run via a lock-in effect of investments in low-carbon technologies or abatement capital, appears to be novel in the literature. However, the model introduced in this paper is related to contributions from the literature with a different focus. E.g., Petrakis and Xepapadeas (2003) analyze the location decision of a monopolist when a carbon tax is introduced in its home country. The authors compare a situation where the regulator sets the tax first and then the monopolist invests in abatement capital with a situation where the tax is chosen after the investment is undertaken. They refer to the former case as time-inconsistent, and to the latter as time-consistent, because in the former case, the regulator has *ex post*, when the investment costs incurred by the monopolist are sunk, an incentive to alter the tax rate. Hence, the authors highlight effects of commitment power of the regulator not to deviate from an announced policy. In contrast to this, we abstract from time-inconsistency issues, and assume that the regulator can credibly commit to a grandfathering policy.

Hepburn et al. (2006) analyze in a static Cournot framework how large the grandfathering rate should be in order to achieve profit neutrality under a cap-and-trade scheme, relative to business-as-usual. The authors demonstrate that the grandfathering rate required for profit neutrality depends, among other things, on the degree of asymmetry in the market split. Hence, a firm with a larger market share may require a different rate of grandfathered allowances (as a percentage of its emissions) than a firm with a smaller market share. In this paper, we do not use profit neutrality as a criterion for policy design, and instead analyze whether grandfathering schemes can be used as an incentive device to avert relocation.

The remainder of this paper is organized as follows. Section 2 presents the model and the results. Section 2.1 highlights the role of sunk fixed costs for the effectiveness of a grandfathering scheme to avert relocation. Section 2.2. demonstrates the importance of timing, and Section 2.3 focuses on the observability of the firm's location. Section 3 concludes. All proofs are relegated to the Appendix.

## 2. Model & Results

Suppose, a cap-and-trade scheme is implemented in country A at time 0, and the resulting carbon price is perfectly predictable and remains constant forever. Let  $a$  be the firm's investment in abatement capital, and let  $\frac{\alpha}{2}a^2$  be the associated fixed cost. Suppose, any investments in abatement capital occur at time  $t = 0$ . This is plausible, because the resulting

cost savings then accrue over the entire time interval during which the firm remains in country A.<sup>2</sup> Furthermore, assuming continuous time, let  $z_t$  be the flow of certificates allocated to the firm for free if it remains in A through time  $t$ . The value of the flow of grandfathered permits at time  $t$  is  $p_z z_t$ , where  $p_z$  is the constant price of certificates in country A (from  $t = 0$  onwards). Let  $T$  be the point in time when the firm relocates from A to B, and note that  $T = 0$  implies immediate relocation, while  $T \rightarrow \infty$  implies no relocation.<sup>3</sup> The discounted

value of grandfathered permits through time  $T$  is given by  $GF_T \equiv \int_0^T p_z z_t e^{-\delta t} dt$ , where  $\delta$  is the

discount rate for future profits. Throughout the paper, we assume that the regulator announces and credibly commits to a grandfathering policy before the firm chooses  $a$  and  $T$ .

Let  $\Pi_T$  be the maximized present value of the firm's profit, given that it plans to relocate from A to B at time  $T$ . Hence, all other variables of the firm (including the investment in abatement capital  $a$ ) are chosen optimally given  $T$ . Assuming that the fixed investment costs in abatement capital are sunk,  $\Pi_T$  can be written as follows:

$$\Pi_T = \int_0^T \pi_{A,t}(p_z, a_T^*) e^{-\delta t} dt + \int_T^\infty \pi_{B,t} e^{-\delta t} dt - \frac{\alpha}{2} (a_T^*)^2 - e^{-\delta T} F + GF_T \quad (1)$$

, where  $\pi_{A,t}$  ( $\pi_{B,t}$ ) denotes the firm's profit flow in A (B) at time  $t$ ,  $F$  is a fixed relocation cost, and  $a_T^*$  the optimized value of  $a$ , given that the firm relocates at  $T$ . Throughout the paper, we assume for simplicity that the market structure and other external conditions (e.g. input and output prices) do not change over time, and that all other choice variables of the firm (in addition to  $a$ ) can be adjusted instantaneously at no additional cost. Therefore,  $\pi_{A,t}$  and  $\pi_{B,t}$  are constants, and the time subscript can be omitted.

## 2.1 The role of sunk fixed costs

In the following, we analyze the role of the fixed installation costs of abatement capital for the effectiveness of a grandfathering policy to avert relocation. Only when they are *sunk*, temporary grandfathering can be a sufficient instrument to avert relocation permanently.

<sup>2</sup> Hence, the firm has no incentive to delay the investment. Depreciation of the abatement capital stock is assumed away for simplicity. The main results in this paper are not affected by this assumption.

<sup>3</sup> Hence, the firm either relocates its entire production or stays in A. In most of the cases we analyze in this paper, this follows directly from the incentives the firm faces: either production is more profitable in A or in B, so the firm will either relocate its entire production or stay in A. Only in Section 2.3 where non-linear grandfathering schemes are discussed, partial relocation is ruled out by assumption.

**Proposition 1:**

When the fixed installation costs of abatement capital are sunk, a temporary grandfathering policy can be a sufficient instrument to avert relocation permanently if it holds that:

$$\Pi_{\infty} - GF_{\infty} \leq \Pi_0 \leq \Pi_{\infty} - GF_{\infty} + \frac{\alpha}{2} (a_{\infty}^*)^2 \quad (2)$$

When the fixed costs are not sunk, grandfathering that is completely phased out in finite time, can not affect the firm's location choice in the long run.

The intuition behind Proposition 1 is as follows. Temporary grandfathering can permanently affect the firm's location choice by inducing investments in abatement capital that can not be recovered when relocating. If the induced investments (due to the higher carbon price in country A) are sufficiently large, then the competitive disadvantage in A may be offset, so the relocation option becomes unprofitable even when the grandfathering scheme terminates. However, if the fixed investment costs in abatement capital are not sunk, then this lock-in mechanism becomes ineffective.

To understand the intuition behind condition (2), note that if the left inequality is violated, then the option to stay permanently in A is more profitable than to relocate even in the absence of grandfathering. Therefore, grandfathering has no effect upon the firm's location choice. If the right inequality is violated, relocation becomes profitable as soon as grandfathering terminates, even if the firm has built up an abatement capital stock so large that it would be optimal for a permanent stay in A ( $a_{\infty}^*$ ). In this case, grandfathering that terminates in finite time can never avert relocation permanently. Note, that (2) is a necessary condition. If it is fulfilled, this means that a temporary grandfathering policy *can* be found that averts relocation in the long run. For sufficiency, further conditions are required. First of all, the implied transfers to the firm must be sufficiently high to compensate it for the competitive disadvantage in country A. Furthermore, the timing of the grandfathering policy is crucial. And finally, it matters which indicators the policy can be conditioned on. These issues are elaborated in the following subsections.

**2.2 The role of timing**

In the following, it is assumed that  $z_t$  follows an exogenous pre-defined path, and that grandfathering can be made contingent on the firm's location. Hence, a grandfathering policy is effectively equivalent to a *subsidy* that is contingent on the firm's location. As soon as the firm relocates from A to B, the transfers implied by the grandfathering policy terminate.

Since we assume that the market structure and other external conditions do not change over time (so for  $p_z$  and  $a$  fixed,  $\pi_A$  and  $\pi_B$  are constant), the expression in (1) simplifies to:

$$\Pi_T = \frac{1 - e^{-\delta T}}{\delta} \pi_A(p_z, a) + \frac{e^{-\delta T}}{\delta} \pi_B - \frac{\alpha}{2} a^2 - F e^{-\delta T} + \int_0^T p_z z_t e^{-\delta t} dt \quad (3)$$

When the maximization over  $a$  is carried out, the following first-order condition obtains:

$$\frac{1 - e^{-\delta T}}{\delta} \frac{\partial \pi_A}{\partial a} = \alpha \cdot a \quad (4)$$

Eq. (4) implicitly defines an optimal abatement choice  $a_T^*$  for any  $T$ .

In the maximization over  $T$ , the following first-order condition obtains:<sup>4</sup>

$$p_z z_T = \pi_B - \pi_A(p_z, a_T^*) - \delta F \quad (5)$$

This optimality condition says that the benefit of a marginal rise in the relocation time  $T$ , given by the value of the flow of grandfathered permits at time  $T$ , is equal to the marginal cost, which is the difference between profit flows in B and A, corrected for the benefit of delayed relocation costs.

In the following, we will investigate what is the minimum length of the interval during which grandfathering must occur to have a permanent effect upon the firm's location choice, and what is the minimum initial level of the subsidy, assuming a non-increasing path. In order to derive specific results, it is useful to impose more structure on the model. Suppose, the profit flow in A,  $\pi_A(p_z, a)$ , can be decomposed as follows:

$$\pi_A(p_z, a) = \pi_A(p_z) + p_z a \quad (6)$$

Underlying this decomposition is the assumption that the firm's optimal choice of all other variables (besides  $a$  and  $T$ ) is *independent* of the abatement capital stock  $a$ , and that the actual abatement flow at any point in time is equal to  $a$ .<sup>5</sup> The firm's choice of  $a$ , thus, only affects the profit flow through savings in its expenditure on certificates, equal to  $p_z a$ . An example for this type of abatement are investments in heat insulation of buildings belonging to the firm. Fixed investments at a certain point in time yield a constant flow of reduced emissions in the future – irrespective of the output of the firm. Other types of abatement capital may, however, affect the firm's optimal output quantity (or other choice variables). Think, e.g., of investments in more energy-efficient machines. The lower the energy demand per unit of output produced, the higher the optimal output quantity. This can *amplify* the

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<sup>4</sup> Using the envelope theorem.

<sup>5</sup> This requires that there exists a linear relationship between the abatement capital stock and the abatement possibilities. Via a rescaling of the units of the pollutant, it is then always possible to assume that the level of the abatement flow is equal to the abatement capital stock.

effects that drive the results of this paper, but is not crucial to derive them. In the following, we, therefore, focus on the analytically simpler case shown in (6).

Using (6), (4) yields the following expression for the optimal choice of  $a$  (given  $T$ ):

$$a_T^* = \frac{p_z}{\alpha\delta}(1 - e^{-\delta T}) \quad (7)$$

This shows that the longer the firm plans to stay in A, the more it invests in abatement capital.

For an infinite stay, the optimal abatement level is given by:  $a_\infty^* = p_z / \alpha\delta$ .

Using (6), the first-order condition for  $T$ , (5), simplifies to:

$$p_z z_T = \Delta\pi - p_z a_T^* \quad (8)$$

, where  $\Delta\pi \equiv \pi_B - \pi_A(p_z) - \delta F$  is defined for an ease of notation.

These results can be used to characterize the parameter range in which it is possible to find a temporary grandfathering policy that permanently averts relocation. Using (2) (from Proposition 1), (3), (6), and (7), we obtain the following condition:

$$\frac{p_z^2}{2\alpha\delta} \leq \Delta\pi \leq \frac{p_z^2}{\alpha\delta} \quad (9)$$

Note, that  $p_z^2 / 2\alpha\delta^2$  is the loss of profit in A (due to the introduction of the cap-and-trade scheme) that is *avoided* by an optimal adjustment of the choice variable  $a$  to the carbon price  $p_z$  (in (9), it is converted into a flow via multiplication by  $\delta$ ). The left-hand side of (9),

which can be rewritten as  $\frac{\pi_A(p_z)}{\delta} + \frac{p_z^2}{2\alpha\delta^2} \leq \frac{\pi_B}{\delta} - F$ , thus, says that in the *absence* of grandfathering, it is less profitable to stay permanently in A than to relocate immediately.

In the following, let  $\tau$  be the point in time when grandfathering is completely phased out. We are ready to state the following result:

**Proposition 2:**

Any non-increasing grandfathering policy  $z_t$  that is sufficient to avert relocation in the long

run, has a minimum duration of  $\tau^{\min} = \frac{1}{\delta} \log\left(\frac{p_z^2 / 2\alpha\delta}{p_z^2 / \alpha\delta - \Delta\pi}\right)$ , and entails a minimum initial

grandfathering rate of  $z_0^{\min} = \frac{1}{p_z} \left(\Delta\pi - \frac{p_z^2}{2\alpha\delta}\right)$ .

The intuition behind Proposition 2 is simple. A grandfathering policy that averts relocation in the long run must entail a sufficiently high rate of grandfathered permits in order to

compensate the firm for the competitive disadvantage of its home industry induced by the carbon price  $p_z$ . And it must not be phased-out too quickly, for otherwise, the firm “free-rides” the initially generous grandfathering policy, but does not invest a lot in abatement capital and relocates in finite time.

Note, that if the left-hand side of (9) is fulfilled with equality, then  $\tau^{\min}$  is zero. Hence, the minimum duration of the grandfathering subsidy approaches zero when the option to stay permanently in A becomes more profitable even in the absence of grandfathering. Conversely, if the right-hand side of (9) is fulfilled with equality,  $\tau^{\min}$  becomes infinite. Hence, the subsidy must be maintained forever to avert relocation if the lock-in effect of the sunk fixed costs is not sufficiently strong to avert relocation.

*Grandfathering phased out at a constant rate:*

We have seen that an effective grandfathering policy must be sufficiently high and not be phased out too quickly to avert relocation in the long run. In the following, let us illustrate these findings for a specific grandfathering policy, namely one where the rate of grandfathered allowances declines (by assumption) at a constant rate  $\varphi$ , hence:  $z_t \equiv z_0 e^{-\varphi t}$ .<sup>6</sup>

Under this assumption, we obtain the following expression for the minimum rate of grandfathered allowances at time zero required to avert relocation:<sup>7</sup>

$$z_0^{\min} = \frac{\delta + \varphi}{p_z \delta} \left( \Delta\pi - \frac{p_z^2}{2\alpha\delta} \right) \quad (10)$$

Note, that there exists a linear relation between  $z_0^{\min}$  and the phase-out rate of the subsidy  $\varphi$ .

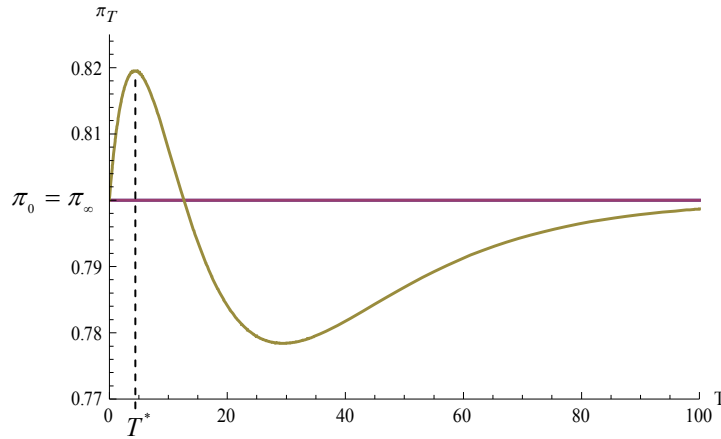
In the following, we will derive a condition that assures that the grandfathering subsidy is not phased out too quickly. Before we go to the details, let us provide a graphical intuition for this problem. Figure 1 shows the profit as a function of the relocation time  $T$  (to derive  $\Pi_T$ , use (3), (6), (7), and  $z_t = z_0 e^{-\varphi t}$ ). The parameters are chosen such that  $z_0 = z_0^{\min}$  (see (10)) holds, hence, the maximized profit when the firm stays forever in A coincides with the profit when it immediately relocates:  $\Pi_0 = \Pi_\infty$  (indicated by the horizontal line). This parameter choice is convenient, because it helps to isolate effects related to the *timing* of grandfathering, while the present value of the implied transfers (for  $T \rightarrow \infty$ ) is held constant when  $\varphi$  is changed.

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<sup>6</sup>  $\tau$  is assumed to be infinity.

<sup>7</sup> To derive it, follow the same steps as shown in the Proof of Proposition 2.

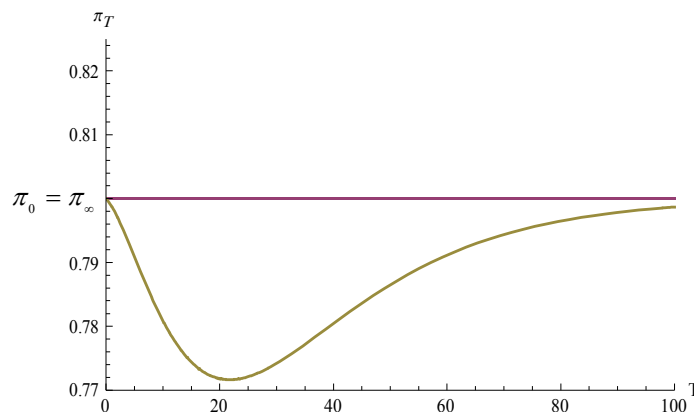
**Figure 1:** Profit as a function of relocation time  $T$  (plotted for  $\varphi = 0.3$ ,  $\delta = 0.05$ ,  $\Delta\pi = 0.75$ ,  $p_z = 0.2$ ,  $z_0 = 8.75$ ,  $\alpha = 0.8$ )



As Figure 1 illustrates, when the subsidy is phased out quickly (large  $\varphi$ ), the firm's profit reaches a maximum (located at  $T^*$ ) in finite time. The firm, thus, benefits from the initially generous grandfathering subsidy in A, and then relocates. Furthermore, to understand the intuition behind the presence of a local *minimum* in the profit function (see Figure 1), recall that the firm's optimal investment in abatement capital is larger the longer it plans to stay in country A (see (7)), because the cost-savings on certificates then accumulate over a longer period of time. Therefore, the avoided losses of profit (compared to business-as-usual) due to the optimal adjustment of the variable  $a$  are increasing over time, so in the absence of grandfathering,  $\Pi_T$  always increases in  $T$  and approaches  $\Pi_\infty$  from below.

The situation is different when the grandfathering rate is reduced less quickly (lower  $\varphi$ ), as illustrated in Figure 2. Once more, the parameters were chosen such that  $\Pi_0 = \Pi_\infty$  holds, and the *present value* of the grandfathering transfers to the firm are the same as before. However, the lower value of  $\varphi$  implies a lower  $z_0$  (all other parameters are as in Figure 1).

**Figure 2:** Profit as a function of relocation time  $T$  (plotted for  $\varphi = 0.1$ ,  $\delta = 0.05$ ,  $\Delta\pi = 0.75$ ,  $p_z = 0.2$ ,  $z_0 = 3.75$ ,  $\alpha = 0.8$ )



As Figure 2 illustrates, the local maximum disappears when grandfathering is phased out less quickly. The profit first declines in the relocation time  $T$ , and then increases again. Asymptotically, it reaches the same value as for  $T = 0$ .<sup>8</sup>

The comparison of Figure 1 and 2 illustrates the importance of timing for the effectiveness of a grandfathering policy. Holding the present value of the grandfathering subsidy constant (given an infinite stay in A), a rapidly declining grandfathering rate is an *ineffective* tool to avert relocation permanently, while a grandfathering rate that is phased out less quickly can be effective. A rapidly declining grandfathering rate does not trigger sufficiently large investments in abatement capital to render the stay-option profitable in the long run.

Let us now analyze the problem formally. The firm chooses the abatement  $a$  and the relocation time  $T$  optimally, given the grandfathering policy  $(z_0, \varphi)$  announced by the regulator. The optimal choice of  $a$  is given by (7) and depends on the grandfathering policy only via  $T$ . Using  $z_t = z_0 e^{-\varphi t}$  and (7), the first-order condition for  $T$ , (8), becomes:

$$p_z z_0 e^{-\varphi T} = \Delta\pi - \frac{p_z^2}{\alpha\delta} (1 - e^{-\delta T}) \quad (11)$$

It can be shown that this equation has at most two real-valued solutions in the non-negative range for  $T$  (see the Proof of Proposition 3). If two solutions exist, one of them is a minimum of the profit function  $\Pi_T$ , and the other one is a maximum (see Figure 1). However, a closed-form solution for the optimal relocation time  $T^*$  can not generally be obtained.<sup>9</sup> Nevertheless, we can derive an expression for the highest-possible phase-out rate of the grandfathering policy, consistent with averted relocation in the long run. To this end, let us assume that the regulator's goal is to design a policy that averts relocation with a minimum of transfers to the firm (in present value). Therefore, the regulator always chooses the lowest value for  $z_0$  ( $z_0^{\min}$ , see (10)) such that  $\Pi_0 = \Pi_\infty$  holds. Under this assumption, the following result obtains:

**Proposition 3:**

If the regulator is constrained to use policies that are phased out at a constant rate  $\varphi$ , and for any given  $\varphi$ , chooses the lowest  $z_0$  ( $z_0^{\min}$ ) that renders the permanent stay-option as

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<sup>8</sup> To make the stay-option strictly dominate the relocation option, the regulator in A should set the initial grandfathering rate  $z_0$  (at least) marginally higher (given  $\varphi$ ), or  $\varphi$  marginally lower (holding  $z_0$  fixed).

<sup>9</sup> Only for some parameter values, this is possible, e.g. when  $\varphi = 2\delta$ .

profitable as the option to relocate immediately ( $\Pi_0 = \Pi_\infty$ ), the largest possible phase-out rate consistent with averted relocation is given by:  $\varphi^{\max} = \frac{\delta}{2\alpha\delta\Delta\pi / p_z^2 - 1}$ .

Hence, if the regulator tries to avert relocation at minimal costs, using a policy that is phased out at a constant rate and as rapidly as possible, the grandfathering scheme  $(z_0^{\min}, \varphi^{\max})$  is implemented. Let us now turn to the question of an *optimal* policy design. To this end, we drop the regulator's constraint to use policies that decline at a constant rate.

*Optimal policy:*

In order to derive an optimal policy, we need to make an assumption about the regulator's preferences over different policies. Otherwise, a unique policy can not be determined. A plausible assumption appears to be that the regulator discounts future payments to the firm at a (slightly) lower rate than the firm discounts future profits.<sup>10</sup> Hence, let  $\rho$  be the regulator's discount rate, and suppose that  $\delta > \rho \geq 0$  holds. The following result obtains:

**Proposition 4:**

If grandfathering is conditioned on the firm's location and phased out when the firm relocates, the optimal rate of grandfathered allowances is given by:  $z_t = \frac{1}{p_z} \left( \Delta\pi - \frac{p_z^2}{\alpha\delta} (1 - e^{-\delta t}) \right)$  for  $t \in [0, \tau^{\min}]$ , and  $z_t = 0$  afterwards.

According to Proposition 4, the optimal policy requires that at time  $t = 0$ , the grandfathering subsidy compensates the entire lack of competitiveness in A (the initial flow of transfers to the firm,  $p_z z_0$ , is given by  $\Delta\pi$ ). The longer the firm continues to produce in A, the longer it faces the carbon price  $p_z$ . This induces investments in abatement capital, that (partially) offset the competitive disadvantage. Therefore, the amount of grandfathered allowances required to avert relocation declines. At time  $t = \tau^{\min}$ , the firm has built up enough abatement capital to render the option to stay permanently in A as profitable as the option to relocate. At

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<sup>10</sup> Although borrowing and lending are not explicitly modeled here, it is clear that since firms face the risk of bankruptcy, a risk premium is charged, which implies that the market interest rate at which profits are discounted, is higher than the risk-free discount rate of government expenditure.

this point, the optimal rate of grandfathered allowances *discontinuously* drops to zero. (Note, that for  $t = \tau^{\min}$ , the expression for  $z_t$  shown in Proposition 4 is strictly greater than zero.)

Figure 3 illustrates the firm's profit as a function of  $T$  under the optimal policy.

**Figure 3:** Profit as a function of  $T$  (plotted for  $\delta = 0.05$ ,  $\Delta\pi = 0.04$ ,  $p_z = 0.1$ ,  $\alpha = 4$ )

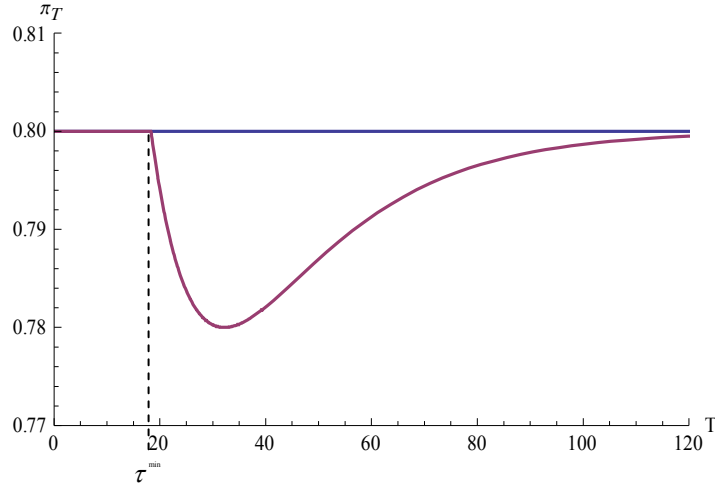


Figure 3 shows that under the optimal policy, the firm is indifferent between staying and relocating at each point in the interval  $T \in [0, \tau^{\min}]$ . From  $\tau^{\min}$  onwards, the option to stay permanently in A dominates the option to relocate, so grandfathering is no longer required.

Let us briefly compare the optimal policy with the policy discussed earlier, characterized by a constant phase-out rate  $\varphi$ . Both policies start with a transfer rate of  $p_z z_0 = \Delta\pi$ . However, the transfer rate under the optimal policy does not decline as rapidly at the beginning, but drops to zero at  $\tau^{\min}$ , whereas under the other policy it approaches zero for  $t \rightarrow \infty$ . Note, that for  $\delta = \rho$ , the regulator is *indifferent* between the two policies. In this case, both are optimal, because they induce the same amount of transfers to the firm when evaluated at the discount rate  $\delta$ . When  $\rho < \delta$ , the government is more patient than the firm and, thus, more concerned about payments in the distant future.

### 2.3 The role of observability of the firm's location

In the previous subsection it was assumed that the grandfathering policy can be conditioned on the firm's location. In practice, however, this may not always be possible because firms may be able to shift partially to foreign countries, and leave e.g. their headquarters in the country of origin. In this case, the "location" of a firm becomes irrelevant, and it may be difficult to design a grandfathering scheme that terminates when the firm relocates most of its productive activity to a foreign country. Nevertheless, second best policies may be found that

are contingent on the firm's emissions that are easier to verify than location or output. In this case, the problem arises that a grandfathering policy that gives the firm an incentive to continue to produce in country A, may also generate incentives to raise the emissions. This undermines the firm's incentives to build up abatement capital and may, therefore, make it difficult for the policy maker to design a policy that averts relocation in the long run.

Note, that if the allocation of free allowances can only be conditioned on the firm's emissions and not on its location or output, then the regulator will *never* adopt grandfathering policies where (part of) the allowances are distributed to the firm independently of its emissions. To see this, suppose to the contrary that allowances are allocated to the firm for free, independently of its emissions. Since the induced transfers do not depend on the firm's location or output, they do not create any incentive to continue to produce in A. Hence, whenever the location or output choice is not verifiable by the policy maker, the allocation of free allowances will be made contingent on the firm's emissions (assuming that this is the only verifiable indicator of the firm's productive activity in A).

In the following, we investigate under what conditions grandfathering schemes that are contingent only on the firm's emissions, may be an effective tool to avert relocation. We focus on two specific types of policy: 1. policies where the free allocation of allowances is linear in the firm's emissions, and 2. non-linear schemes where the allocation of allowances decreases both when the firm emits more or less than its reference level (set by the government). As before, we assume that the firm makes all investments in abatement capital at once, hence, these investments can not be "stretched" over time. Under mild conditions, it is, then, always optimal for the firm to do these upfront investments at  $t = 0$ .<sup>11</sup>

#### *Linear schemes:*

Consider a grandfathering scheme that, at any point in time, allocates an amount of free allowances to the firm that is proportional to its emissions. The firm's emissions can be expressed as its baseline emissions (under a carbon price of zero) minus the abatement  $a$ . Via a rescaling of the units of emissions, it is possible to set the baseline emission level to one. Hence, the firm's constant<sup>12</sup> emission flow when it produces in country A is given by:  $1 - a$ , and the amount of allowances allocated to the firm at a given point in time is, by assumption, proportional to this. To derive analytical results, we must specify how the grandfathering rate

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<sup>11</sup> Situations where firms can not stretch investments over time may be characterized by additional fixed costs that are incurred if the investment is not effected at once. Otherwise, firms may sometimes delay some of their investments in abatement capital if the grandfathering policy initially provides strong incentives to generate high emissions, and these incentives are declining over time. Our approach is chosen for tractability.

<sup>12</sup> The flow of emissions is constant because we assume that all investments in abatement capital occur at time 0.

changes over time. For simplicity, we once more focus on policies with a constant phase-out rate  $\varphi$ , hence:  $z_t \equiv z_0 e^{-\varphi t} (1-a)$  (see Section 2.2).<sup>13</sup>

Under the above assumptions, we obtain the following present value of transfers to the firm, given that it relocates at time  $T$ :  $GF_T = \int_0^T p_z z_0 (1-a) e^{-(\delta+\varphi)t} dt$ . The flow of transfers terminates at  $T$  because the firm's emissions in A then, drop to zero.<sup>14</sup> Assuming constant profit flows in A and in B, and an additive relation between  $\pi_A(p_z, a)$  and  $a$  as defined in (6), after evaluating the integrals, (1) yields the following profit function  $\Pi_T$ :

$$\Pi_T = \frac{1-e^{-\delta T}}{\delta} (\pi_A(p_z) + p_z a) + \frac{e^{-\delta T}}{\delta} \pi_B - \frac{\alpha}{2} a^2 - F e^{-\delta T} + p_z z_0 (1-a) \frac{1-e^{-(\delta+\varphi)T}}{\delta+\varphi} \quad (12)$$

The maximization over  $a$  yields the first-order condition:

$$a_T^* = \frac{p_z}{\alpha \delta} (1-e^{-\delta T}) - \frac{p_z z_0}{\alpha (\delta+\varphi)} (1-e^{-(\delta+\varphi)T}) \quad (13)$$

It is straight-forward to show that this is increasing in  $T$  if  $z_0 \leq 1$ , hence, if the initial allocation of free allowances does not exceed the firm's baseline emissions. In the following we will assume that this holds.

The comparison of (13) and (7) shows that for any  $T > 0$  and  $z_0 > 0$ , the above policy induces lower investments in abatement capital than in the case where grandfathering is contingent on the firm's location (see Section 2.2). The distortion is captured by the second term in (13), that is increasing in  $z_0$ . This highlights a trade-off that the regulator faces in the design of a policy: if relocation is profitable in the absence of grandfathering, then  $z_0$  must be sufficiently large to avert relocation. However,  $z_0$  may not be chosen too large, for otherwise, the firm's incentives to invest in abatement capital vanish. This narrows the range of parameter values for which it is possible to find a policy sufficient to avert relocation in the long run. Before we proceed to characterize this range of parameters, let us derive an expression for the minimum rate of grandfathered allowances at time zero required to avert relocation (to derive it, use (12) and (13) and set  $\Pi_0 = \Pi_\infty$ ):

<sup>13</sup> Alternatively, one could derive an optimal scheme as in Proposition 4 (with the restriction that it must be linear in the abatement  $a$ ), but this becomes complicated because the profit-maximizing choice of  $a$  depends not only on  $T$  (as in Section 2.2) but also on  $z_t$ . This results in a differential equation. For tractability, we focus on the simpler scheme with a constant phase-out rate  $\varphi$ . This is sufficient to demonstrate the main results.

<sup>14</sup> It can be shown that partial relocation is never profitable under this type of grandfathering scheme.

$$z_0^{\min} = \frac{\delta + \varphi}{p_z} \left( \sqrt{\frac{2\alpha}{\delta} \left( \Delta\pi - p_z + \frac{\alpha\delta}{2} \right)} - \alpha + \frac{p_z}{\delta} \right) \quad (14)$$

**Proposition 5:**

A grandfathering scheme where the rate of allowances allocated to the firm for free increases linearly in its emissions, can be a sufficient policy instrument to avert relocation in the long run, but the range of parameter values where this is possible is more narrow than in the case where grandfathering is contingent on the firm's location. If the regulator chooses the lowest value of the initial grandfathering rate that renders the permanent stay-option as profitable as the option to relocate immediately ( $z_0 = z_0^{\min}$ , so  $\Pi_0 = \Pi_\infty$ ), the range is given by:

$$\frac{p_z^2}{2\alpha\delta} \leq \Delta\pi \leq \frac{p_z^2}{\alpha\delta} - p_z \left( \sqrt{1 + \frac{p_z^2}{\alpha^2\delta^2}} - 1 \right) \quad (15)$$

According to Proposition 5, schemes where the free allocation of allowances increases linearly in the firm's emissions, are a less effective tool to avert relocation than schemes that are conditioned on the firm's location. They induce artificial incentives to raise emissions, which makes it harder or impossible to design a grandfathering scheme that permanently averts relocation. A comparison of (9) and (15) shows that the upper boundary of the range of parameter values for which grandfathering can avert relocation is lower than in the case where the policy can be conditioned on the firm's location<sup>15</sup>, while the lower boundary is the same.

*Non-linear schemes:*

Section 2.2 showed that, when the policy maker can condition the flow of grandfathered allowances on the location of the firm, a policy that assigns an exogenously declining rate of free allowances can be used to implement the optimal solution, which requires that the firm is induced to stay permanently in A with a minimum of transfers. If grandfathering can not be conditioned on the location of the firm, the regulator can use the firm's emissions as a proxy for its productive activity in A. As shown above, linear schemes can sometimes be used as a second-best instrument to avert relocation, but they distort the firm's investments in abatement capital. Under certain conditions, non-monotonic allocation schemes are sufficient to implement the *first-best solution*. However, this requires that partial relocation is *never* a profitable strategy for the firm. This may e.g. be the case when there are strong

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<sup>15</sup> It can be shown that this restriction is especially relevant for smaller values of  $\alpha$ , while for increasing values of  $\alpha$ , the upper boundaries converge in the two cases.

complementarities in the firm's production, or when the relocation cost  $F$  is sufficiently high, and represents a fixed set-up cost that is incurred independently of the amount of production being relocated.

**Proposition 6:**

If partial relocation is never a profitable strategy of the firm, then non-monotonic allocation schemes of free allowances, that punish deviations from the firm's emission level under the optimal long-run investment in abatement capital  $a_\infty^*$ , are a sufficient policy instrument to implement the first-best solution.

Proposition 6 indicates that non-linear schemes that prescribe a target rate of emissions to a firm and punish deviations towards higher or lower emissions, may be a useful tool for policy makers if the firm's output or location are not verifiable. However, this requires that the firm can not relocate partially in order to lower its emissions without investing in abatement capital. The firm's reference emissions level set by the regulator must be sufficiently low to induce investments in abatement capital. A punishment of emissions above the firm's reference level can be justified by the firm's usage of an inefficient abatement technology, while emissions below the reference level can be interpreted as an indicator of declining productive activity in country A.

### **3. Discussion / Conclusion**

Grandfathering is often seen as a device to shield firms from adverse effects on their profitability in countries that adopt ambitious climate protection policies, such as a cap-and-trade schemes with a sufficiently low cap. Critics may argue that grandfathering schemes are an indirect subsidy to large emitters, and award firms with high emissions in the past. They may argue that if firms threaten to relocate to countries with less ambitious climate protection policies, then grandfathering will only postpone relocation, but can not avert relocation in the long run unless the transfers to the firm are maintained indefinitely. The results in this paper indicate that such criticism is not generally justified. It has been shown that grandfathering *can* be an effective policy instrument to avert relocation in the long run, even in situations where immediate relocation would be profitable in the absence of grandfathering. A necessary condition for this is that there remains scope for investments in low-carbon technologies or abatement capital, and that these investments can generate a sufficient amount of abatement without reducing the output of the firm. In order to induce the firm to undertake these

investments, the regulator should design a grandfathering scheme that awards the firm for maintaining its production in the home country for a certain period of time. If the carbon price that the firm faces during this period is sufficiently high, this will induce the investments that ultimately render the option to stay more profitable than the option to relocate. Grandfathering can, then, be phased out without triggering relocation.

There are a number of aspects that may indicate whether, for a particular industry, grandfathering may be a useful device to avert relocation. E.g., the industry should be exposed to intense international competition, and be characterized by high emissions per unit of output to make relocation to countries with a lower carbon price a credible threat. Furthermore, as pointed out above, there must remain enough scope for investments in abatement capital. If most abatement possibilities have already been exploited, then the lock-in effect highlighted in this paper becomes ineffective.

It was shown in this paper that the effectiveness of grandfathering as an incentive device to avert relocation depends, among other things, on the observability of the firm's location or output choice. If firms can relocate part of their production to other countries to reduce emissions domestically without lowering final output production, and partial relocation is hard to verify by the regulator, then second-best grandfathering schemes (e.g. linear schemes) can sometimes be designed that are contingent on the firm's emissions.

However, readers should keep in mind that grandfathering can *never* be a sufficient instrument to completely rule out carbon leakage. It can only be used to reduce carbon leakage effects in certain industries, in particular when technological progress or investments in abatement capital can reduce emissions to such an extent that relocation becomes unprofitable despite differences in carbon prices across countries. Hence, although grandfathering can sometimes be a useful policy device, governments should favor instruments that are targeted more directly at the *causes* of carbon leakage whenever this is possible. In particular, a carbon tax, combined with border-tax-adjustments can (in theory) eliminate carbon leakage completely. This would allow countries that are willing to implement ambitious climate policies to maintain their international competitiveness, even when other countries refuse to implement comparable carbon prices (Stiglitz, 2006). Furthermore, the literature offers a variety of arguments why taxes are superior to cap-and-trade schemes when applied to the problem of global warming (see e.g. Newell and Pizer, 2003). Hence, a uniform carbon tax in countries that are willing to combat climate change in a serious manner, combined with border-tax-adjustments to eliminate competitive disadvantages vis-à-vis countries that refuse to implement the tax, may be seen as a superior

policy approach compared to cap-and-trade schemes, even when the latter are combined with grandfathering to reduce some of the most adverse effects of carbon leakage.

Future research may analyze the robustness of the results presented in this paper with respect to uncertainty about abatement costs, future carbon prices or output. In this paper, we assumed for simplicity that all of these were deterministic and common knowledge (for the firm and the regulator). In the light of uncertainty, the effectiveness of grandfathering policies to avert relocation may be reduced, and the impact on the optimal policy design should be analyzed.

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## Appendix:

### Proof of Proposition 1:

To show the first part of the Proposition, note that  $\Pi_{\infty} - GF_{\infty}$  is the firm's maximized profit if it stays permanently in country A, in the absence (or net) of grandfathering transfers. Hence, if the left inequality in (2) is violated, the stay-option is more profitable anyway, even in the absence of grandfathering, so the implementation of a grandfathering policy has no effect upon the firm's location choice. The right inequality requires that the firm's long-run profit in A in the absence of grandfathering exceeds the profit when immediately relocating, given that the optimal long-run level of  $a$  ( $a_{\infty}^*$ ) has already been implemented, hence, when the fixed installation costs  $\frac{\alpha}{2}(a_{\infty}^*)^2$  are neglected. If this inequality is violated, then relocation becomes profitable when a grandfathering policy terminates after an arbitrarily long period of time, hence, even when the optimal long-run level of  $a$  has (almost) been implemented. In this case, a temporary grandfathering policy can never avert relocation permanently.

To show the second part of the Proposition, reset time to zero at the point when grandfathering terminates, and let  $\Pi_T$  be the *continuation* profit from that point onwards. Suppose, an abatement capital stock of  $a$  has previously been implemented. Now the permanent stay-option leads to a continuation profit of:  $\Pi_\infty = \int_0^\infty \pi_A(p_z, a)e^{-\delta t} dt$ , and immediate relocation<sup>16</sup> to a profit of:  $\Pi_0 = \int_0^\infty \pi_B e^{-\delta t} dt + \frac{\alpha}{2} a^2 - F$ , given that the fixed costs previously incurred are not sunk. The permanent stay-option is more profitable if  $\Pi_0 < \Pi_\infty$ , which yields:

$$\int_0^\infty \pi_B e^{-\delta t} dt - F < \int_0^\infty \pi_A(p_z, a)e^{-\delta t} dt - \frac{\alpha}{2} a^2 \quad (16)$$

However, the relevant parameter space is restricted to values where the left-hand side of (2) is fulfilled, which yields (using (1)):

$$\int_0^\infty \pi_B e^{-\delta t} dt - F \geq \int_0^\infty \pi_A(p_z, a_\infty^*)e^{-\delta t} dt - \frac{\alpha}{2} (a_\infty^*)^2 \quad (17)$$

By the definition of  $a_\infty^*$ , it holds that  $\int_0^\infty \pi_A(p_z, a_\infty^*)e^{-\delta t} dt - \frac{\alpha}{2} (a_\infty^*)^2 \geq \int_0^\infty \pi_A(p_z, a)e^{-\delta t} dt - \frac{\alpha}{2} a^2$  for any  $a \neq a_\infty^*$ . Therefore, (16) and (17) can not be fulfilled simultaneously. *Q.E.D.*

### Proof of Proposition 2:

A necessary condition for the firm to stay permanently in A is:  $\Pi_{T=\tau} \leq \Pi_{T \rightarrow \infty}$ . If it is violated, the firm relocates to B at time  $\tau$  (or earlier), irrespective of the value of subsidy payments it receives in A until  $\tau$ . The minimum duration of grandfathering is, thus, obtained when the condition holds with equality. Using (3) and (6), this yields (note, that  $GF_\tau = GF_\infty$  holds since grandfathering terminates at  $\tau$ ):

$$\frac{1-e^{-\delta\tau}}{\delta} (\pi_A(p_z) + p_z a_\tau^*) + \frac{e^{-\delta\tau}}{\delta} \pi_B - \frac{\alpha}{2} (a_\tau^*)^2 - F e^{-\delta\tau} = \frac{1}{\delta} \pi_A(p_z) + p_z a_\infty^* - \frac{\alpha}{2} (a_\infty^*)^2 \quad (18)$$

Using (7) to replace  $a_\tau^*$  and  $a_\infty^*$  in (18), we obtain (after rearranging) the expression for the minimum duration of the grandfathering subsidy  $\tau^{\min}$  shown in the Proposition.

To derive the minimum initial level of a non-increasing grandfathering policy, assume a constant rate of grandfathered allowances chosen such that  $\Pi_0 = \Pi_\infty$ . Using (3), (6), and (7),

<sup>16</sup> Since grandfathering has been phased out, the firm will either relocate immediately or stay permanently in A.

we obtain:  $\Pi_0 = \frac{\pi_B}{\delta} - F$  and  $\Pi_\infty = \frac{\pi_A(p_z)}{\delta} + \frac{p_z^2}{2\alpha\delta} + \frac{p_z z}{\delta}$ . Using  $\Delta\pi \equiv \pi_B - \pi_A(p_z) - \delta F$  and solving for  $z$ , we obtain the expression for  $z_0^{\min}$  shown in the Proposition. *Q.E.D.*

### **Proof of Proposition 3:**

Suppose, the largest possible phase-out rate  $\varphi^{\max}$  consistent with averted relocation is greater than  $\delta$ . Hence, we restrict our attention to situations where  $\varphi > \delta$  holds. Below, we will show that under this assumption, we find an expression for  $\varphi^{\max}$  greater than  $\delta$ , so the above assumption is fulfilled.

We first show that at most two extrema of the function  $\Pi_T$  exist in the non-negative range for  $T$ , and if so, then the one with the lower value of  $T$  is always a local maximum. Consider the first-order condition (11). Using the derivation of this condition, it is easy to show that  $d\Pi_T / dT > 0$  holds whenever the left-hand side (LHS) of (11) is greater than the right-hand side (RHS). Plotted over  $T$ , both LHS and RHS are exponentially declining functions, but since  $\varphi > \delta$ , LHS is declining more rapidly. The intersection point of LHS with the vertical axis is  $p_z z_0$ , hence, two extrema exist in the non-negative range for  $T$  if and only if  $z_0$  is sufficiently large (note, that by the right inequality in (9), RHS converges to a negative value for  $T \rightarrow \infty$  while LHS converges to 0; this assures that the right intersection point *always* exists). To see that the left intersection point is always a local maximum (if it exists), it suffices to note that LHS > RHS at  $T = 0$ , so  $d\Pi_T / dT > 0$ , and that LHS < RHS for values of  $T$  in between the two extrema.

Given the finding that  $\Pi_T$  can have at most two local extrema, and if so, the left one (located at a lower value of  $T$ ) is always a maximum, a sufficient condition for the non-existence of the local maximum is that  $\left. \frac{d\Pi_T}{dT} \right|_{T=0} \leq 0$ . Hence, to derive the largest possible value of  $\varphi$  ( $\varphi^{\max}$ ) consistent with averted relocation, replace  $z_0$  by  $z_0^{\min}$  in (11) (using (10)), and set  $T = 0$ . Solve for  $\varphi$  to obtain the expression for  $\varphi^{\max}$  given in the Proposition. Using (9), it is easy to show that  $\varphi^{\max}$  is greater than  $\delta$ . This completes the proof. *Q.E.D.*

### **Proof of Proposition 4:**

The regulator's problem is to  $\min_{\{z_t, \tau\}} \int_0^{\tau} p_z z_t e^{-\rho t} dt$ , s.t.  $\max_{T \in [0, \infty)} \{\Pi_T\} \leq \Pi_\infty$ . For  $\delta > \rho$ , the optimal policy entails  $\Pi_T = \Pi_\infty \quad \forall T \in [0, \tau^{\min}]$  (the value of  $\tau^{\min}$  is defined in Proposition 2). Hence, the first-order condition for  $T$ , (5), must be fulfilled  $\forall T \in [0, \tau^{\min}]$ . Using (6) and (7), this yields the expression for  $z_t$  shown in Proposition 4.

*Proof by contradiction.* Suppose that the regulator deviates from this policy, by reducing  $z_t$  at some  $t_1 \in (0, \tau)$  by some  $\Delta z_{t_1}$  for a marginal unit of time  $dt$ . To assure that  $\Pi_0 = \Pi_\infty$  remains fulfilled (hence, immediate relocation does not become profitable at  $t = 0$ ), the regulator must raise  $z_t$  at some  $t_2 \in (0, \infty)$  by some  $\Delta z_{t_2}$ . Suppose  $t_2 > t_1$ . Then it must hold that  $\Delta z_{t_2} > \Delta z_{t_1}$  to assure that  $\Pi_0 = \Pi_\infty$ , because the firm's profit is changed by  $-p_z \Delta z_{t_1} e^{-\delta t_1} dt + p_z \Delta z_{t_2} e^{-\delta t_2} dt$ . The regulator's discounted expenditure changes by:  $p_z \Delta z_{t_1} e^{-\rho t_1} dt - p_z \Delta z_{t_2} e^{-\rho t_2} dt$ . Since  $\delta > \rho$ , this is strictly greater than zero, so the deviation from the original policy is not profitable. Now suppose  $t_2 < t_1$ . Since the original policy entails  $\Pi_T = \Pi_\infty \quad \forall T \in [0, \tau^{\min}]$ , the new policy entails  $\Pi_{t_2+dt} > \Pi_\infty$ , hence, relocation in finite time becomes profitable, which violates the regulator's maximization constraint. *Q.E.D.*

### **Proof of Proposition 5:**

The lower bound to the range of parameter values for which it is possible to design a policy to avert relocation is identical to the one in the case where the rate of grandfathered allowances is contingent on the firm's location (see Section 2.2), because in both cases, it is defined by the condition that the profit of staying permanently in A in the absence of a grandfathering scheme, is larger than the profit of relocating immediately. Hence, it is given by the left inequality in (9). The (theoretical) upper bound of the parameter range where it is possible to find a grandfathering scheme that is phased out over time but permanently averts relocation is defined by the right-hand side in (2), where  $a_\infty^*$  is understood to be the optimal choice of  $a$  for  $T \rightarrow \infty$ , in the absence of a grandfathering policy. This theoretical upper bound can be reached with grandfathering policies that terminate when the firm relocates, hence, if the firm's location is observable. However, when this is not possible, and a policy is implemented where the rate of grandfathered allowances increases linearly in the firm's emissions, the theoretical upper bound can no longer be reached. Under the constraint of linear policies, the upper bound is defined by the right inequality in (2), given that  $a_\infty^*$  is understood to be the

optimal choice of  $a$  for  $T \rightarrow \infty$ , given the grandfathering policy, since the firm's optimal investment in abatement capital depends on the grandfathering policy. Using (12) and (13), this yields the following modified condition:

$$\frac{p_z^2}{2\alpha\delta} \leq \Delta\pi \leq \frac{p_z^2}{\alpha\delta} - \frac{p_z^2 z_0}{\alpha(\delta + \varphi)} \quad (19)$$

The right-hand side in (19) depends on the policy parameters  $z_0$  and  $\varphi$ , but these are chosen by the regulator, depending on the other parameters of the model. It is not obvious whether there exists a non-empty set of parameters for which it is possible to choose  $z_0$  and  $\varphi$  such that relocation becomes unprofitable, *and* the right-hand side of (19) is fulfilled. To show that such parameter values exist, assume that (for a given value of  $\varphi$ ) the regulator chooses the lowest possible value of  $z_0$  such that  $\Pi_0 = \Pi_\infty$  holds, hence  $z_0 = z_0^{\min}$  (from (14)). Inserting  $z_0^{\min}$  in (19),  $(\delta + \varphi)$  cancels out, so that the resulting expression is independent of both policy parameters ( $z_0$  and  $\varphi$ ). Since the right-hand side of the resulting inequality contains the expression  $\Delta\pi$ , solve this condition (assuming that it holds with equality) for  $\Delta\pi$ , and identify the relevant solution. Following this approach, the right-hand side of (15) is obtained. To complete the proof, it is straight-forward to verify that parameter values exist that fulfill both sides of (15). *Q.E.D.*

### **Proof of Proposition 6:**

Suppose the firm's baseline emissions (optimal emissions in the absence of the cap-and-trade scheme) are equal to one. Hence, if the firm implements an abatement capital stock of  $a$ , its actual emissions are  $1 - a$ . Since the firm can not use partial relocation as a strategy to manipulate its emissions, all the regulator needs to do to avert relocation is to define a scheme that punishes the firm for deviations from the optimal long-run investment in  $a$  given no relocation:  $a_\infty^* = p_z / \alpha\delta$  (see (7)). E.g., consider the following scheme:

$$z_t = z_0 e^{-\varphi t} \left( 1 - |a - a_\infty^*| \right) \quad (20)$$

A deviation by the firm from the reference level towards higher or lower emissions leads to a reduction in its allocation of free allowances. Therefore, any scheme with a sufficiently high  $z_0$  and a sufficiently low  $\varphi$  will induce the firm to implement  $a_\infty^*$ , and (given that (9) is fulfilled) will avert relocation. To show that a non-linear policy can avert relocation with a minimum of transfers to the firm, note that the punishment for deviations from  $a_\infty^*$  can be

made arbitrarily high, e.g. by replacing the absolute value  $|a - a_\infty^*|$  in (20) by a step function where the rate of grandfathered allowances is zero unless  $a = a_\infty^*$ . *Q.E.D.*