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Limited Liability and the Trade-off between Risk and Incentives*

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Several empirical findings have challenged the traditional trade-off between risk and incentives. By combining risk aversion and limited liability in a standard principal-agent model the empirical puzzle on the positive relationship between risk and incentives can be explained.

Key words: limited liability; piece rates; risk aversion; risk-incentives trade-off.

JEL classification: D01; D82; J3; M5.

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1 Introduction

In the well-known principal-agent model with a risk neutral principal and a risk averse agent we face the typical negative relationship between exogenous risk and incentives: If risk is large and, hence, the risk premium (or risk costs) from high-powered incentives would be high the principal will optimally choose low incentives for the agent. However, if risk is small so that risk costs from induced incentives are rather low, the principal will prefer high-powered incentives.

Prendergast (2002a) refers to several empirical studies which point to a *positive* relation between risk and incentives and therefore challenge the traditional view. Consequently, he offers alternative explanations for this empirical puzzle (Prendergast 2002a, 2002b). In his modeling, he assumes that the agent is risk neutral in order to abstract from the classical trade-off. However, this assumption may be crucial. For example, if in Prendergast (2002a) the agent were risk averse and high risk makes the principal prefer output-based to input-based contracts, the well-known traditional trade-off would apply again. Another explanation for the empirical puzzle is introduced by Raith (2003). There, the positive relationship between risk and incentives comes as a by-product. Raith assumes that agents are risk averse so that incentives indeed imply risk costs. However, since agents face a binding participation constraint and principals always realize zero profits due to market competition, neither the agent nor the principal (but society) has to bear the risk costs from inducing incentives. Hence, less extreme competition may lead to different findings concerning the trade-off. Finally, Wright (2004) and Serfes (2005) independently develop a matching approach that can explain the puzzle. If competition makes less risk averse agents match with more risky principals, the outcome may be a positive relation between risks and incentives.

However, in this paper we offer an alternative explanation for the empirical puzzle which does neither need risk neutral agents nor market competition as a driving force. We only combine two standard contracting problems – risk aversion and limited liability – which should be present in many circumstances so that this approach seems to be the most natural explanation for a possibly positive relation between risk and incentives. The theoretical results show that if the agent earns a positive rent in optimum and if his cost-of-effort function is not to steep, higher exogenous risk will lead to stronger incentives. In this situation, the principal strictly gains from increased incentives leading to higher output since he has not to pay for the additional incentives which only reduce the agent's rent. However, if in optimum effort is not very sensitive to incentives the traditional trade-off will prevail.

Note that we restrict the set of possible contracts to linear incentive schemes. This restriction is introduced for two reasons. First, most of the empirical literature that challenges the traditional trade-off is based on linear incentives schemes. Second, all of the cited papers that have dealt with the given topic so far use a linear incentive scheme. Thus, adopting the assumption on linear payment makes our findings directly comparable to the previous results.

2 The Model

We consider a typical principal-agent relationship between a risk neutral principal P and a strictly risk averse agent A who has the utility function $u(I)$ with $u'(I) > 0$, $u''(I) < 0$ and I denoting A's income. A produces the output $q = e + \varepsilon_r$ for P where e stands for A's chosen effort and ε_r for exogenously given noise with subscript r indicating risk. Let ε_r be distributed over $[-\bar{\varepsilon}_r, \bar{\varepsilon}_r]$ according to the density function $f_r(\varepsilon_r)$ with mean $E[\varepsilon_r] = 0$ and variance σ_r^2. In the following, we will distinguish two probability distributions $r = L, H$ with $\bar{\varepsilon}_L < \bar{\varepsilon}_H$ and $f_L(\varepsilon_L)$ exhibiting second-order stochastic dominance over $f_H(\varepsilon_H)$ which implies $\sigma_L^2 < \sigma_H^2$ (Hadar and Russel 1971, Theorem 3) so that distribution $f_H(\varepsilon_H)$ is more risky than $f_L(\varepsilon_L)$. In other words, we can transform $f_L(\varepsilon_L)$ into $f_H(\varepsilon_H)$ by shifting probability mass from the mean to the enlarged tails. There is the usual hidden-action problem: P observes the verifiable output q but neither e nor ε_r. P also knows A's utility function and the probability distribution. A's cost-of-effort function is given by $c(e)$ with $c'(e), c''(e) > 0$ and $c(0) = 0$. We restrict the set of possible contracts to linear payment schemes $w = \alpha + \beta q$. Hence, A's income is given by $I = w - c(e) = \alpha + \beta q - c(e)$. It is assumed that A is protected by limited liability: P has to guarantee A a minimum payment so that $w \geq w_{\min}$. A's reservation utility is given by $\bar{u}$ with $\bar{u} = u(\bar{w})$. The timing of the model is the usual one: First, P offers A a contract (α, β).

Then, A accepts or rejects. Given that he has accepted P's offer, A then chooses e. Finally, ε_r is realized and payments are made to P and A.

3 Optimal Linear Contract

P maximizes expected net profits $(1-\beta)\,e-\alpha$ subject to the incentive constraint, the individual-rationality constraint and the limited-liability constraint of the agent. A's incentive constraint is given by

$$e^* \in \arg\max_e \left\{ \int_{-\bar{\varepsilon}_r}^{\bar{\varepsilon}_r} u\left(\alpha+\beta\left(e+\varepsilon_r\right)-c\left(e\right)\right) f_r\left(\varepsilon_r\right) d\varepsilon_r \right\}$$

which, by using Jensen's Inequality, can be rewritten as

$$e^* \in \arg\max_e \left\{ u\left(\alpha+\beta e-c\left(e\right)-\rho_r\left(\beta\right)\right)\right\}$$

with $CE=\alpha+\beta e-c\left(e\right)-\rho_r\left(\beta\right)$ being the certainty equivalent and $\rho_r\left(\beta\right)>0$ the corresponding risk premium with $\rho_r'\left(\beta\right)>0$ and r indicating the underlying probability distribution. The first-order condition $u'\left(CE\right)\left(\beta-c'\left(e\right)\right)=0$ yields

$$e^* = h\left(\beta\right) \tag{IC}$$

with $h\left(\cdot\right):=c'^{-1}\left(\cdot\right)$ denoting the inverse of the marginal cost function which is monotonically increasing because of the convex cost function.[1]

The individual-rationality constraint, which guarantees A's participation,

$$\int_{-\bar{\varepsilon}_r}^{\bar{\varepsilon}_r} u\left(\alpha+\beta\left(e+\varepsilon_r\right)-c\left(e\right)\right) f_r\left(\varepsilon_r\right) d\varepsilon_r \geq \bar{u},$$

[1] Note that the second-order condition $u''\left(CE\right)\left(\beta-c'\left(e\right)\right)^2-c''\left(e\right)u'\left(CE\right)<0$ is always satisfied.

can be simplified to

$$\alpha + \beta e - c(e) - \rho_r(\beta) \geq \bar{w}. \tag{IR}$$

Finally, the limited-liability constraint has to ensure A his minimum wage $w_{\min}$ even in the worst case with the lowest realization of ε_r:

$$\alpha + \beta(e - \bar{\varepsilon}_r) \geq w_{\min}. \tag{LL}$$

To sum up, P maximizes expected net profits subject to (IC), (IR) and (LL).

Inserting (IC) into P's objective function, into (IR) and into (LL), the principal's optimization problem can be described by the Lagrangean

$$\begin{aligned} L(\alpha, \beta) &= (1-\beta) h(\beta) - \alpha + \lambda_1 [\alpha + \beta h(\beta) - c(h(\beta)) - \rho_r(\beta) - \bar{w}] \\ &\quad + \lambda_2 [\alpha + \beta (h(\beta) - \bar{\varepsilon}_r) - w_{\min}] \end{aligned}$$

with $\lambda_1, \lambda_2 \geq 0$ denoting the respective Lagrange multipliers. Differentiation with respect to α and β yields that in optimum we must have

$$\lambda_1 + \lambda_2 = 1 \quad \text{and} \tag{1}$$

$$\begin{aligned} -h(\beta) + (1-\beta) h'(\beta) + \lambda_1 [h(\beta) + \beta h'(\beta) - c'(h(\beta)) h'(\beta) - \rho_r'(\beta)] \\ + \lambda_2 [h(\beta) + \beta h'(\beta) - \bar{\varepsilon}_r] &= 0 \end{aligned} \tag{2}$$

According to condition (1), in optimum either the individual-rationality constraint is binding ($\lambda_1 = 1$ and $\lambda_2 = 0$) or the limited-liability constraint ($\lambda_1 = 0$ and $\lambda_2 = 1$) which is also intuitively plausible since P does not want to waste his money.

First, consider the case of binding (IR). Substituting $\lambda_1 = 1$ and $\lambda_2 = 0$ into (2) gives

$$h'(\beta) = \rho_r'(\beta) + c'(h(\beta))\, h'(\beta).$$

Hence, if A does not earn a positive rent, P will choose optimal incentives which maximize total welfare so that induced marginal returns equal the sum of marginal risk costs and marginal costs of effort. Second, if A earns a positive rent and (LL) is binding, $\lambda_1 = 0$ and $\lambda_2 = 1$ imply

$$h'(\beta) = \bar{\varepsilon}_r \tag{3}$$

which leads to the following result:

Proposition 1 *If, in optimum, A earns a positive rent optimal incentives, β^*, will be described by $h'(\beta^*) = \bar{\varepsilon}_r$, otherwise by $h'(\beta^*) = \rho_r'(\beta^*) + c'(h(\beta^*))\, h'(\beta^*)$.*

Proposition 1 shows that on the one hand we may have the well-known solution for risk averse agents (who are not wealth-constrained) where the principal maximizes total welfare so that the usual negative trade-off between risk and incentives should apply. On the other hand, agents may earn positive rents so that the binding limited-liability constraint directly determines the optimal level of incentives. Let $\beta^*(\sigma_r^2)$ denote the optimal piece-rate dependent on the risk of the given probability distribution $r = L, H$. From (3), we immediately obtain the following corollary:

Corollary *If A earns a positive rent in optimum and $h(\beta)$ is convex, then $\beta^*(\sigma_H^2) > \beta^*(\sigma_L^2)$ so that risk and incentives will be positively related.*

The intuition for the interesting finding of the corollary is the following: First, note that in case of a positive rent P does not have to pay for additional effort costs and additional risk costs implied by higher incentives. These costs are fully borne by the agent since his positive rent is decreased. Hence, given a positive rent the usual trade-off between risk and incentives is overridden.

However, as the corollary shows, P is not always interested in increasing the piece rate when $\bar{\varepsilon}_r$ rises. P will only enhance incentives, if effort strongly reacts to an increased piece rate, i.e. if $h(\cdot)$ is convex or, in other words, if the cost function is not too steep (for example, if $c(e) = e^{1.5}$ in case of a polynomial cost function). If risk is shifted to the tails so that $\bar{\varepsilon}_r$ increases, P has to compensate A for this increase according to the (LL) condition. (LL) shows that on the one hand P can decrease β in order to give the increase of $\bar{\varepsilon}_r$ a lower weight.[2] On the other hand, P can counterbalance the increase in $\bar{\varepsilon}_r$ by an increase of effort e induced by a higher piece rate according to (IC). Which of these alternatives is the better one from P's perspective, crucially depends on the shape of $h(\cdot)$. If e^* is very sensitive to incentives, P will optimally choose a higher piece rate. If effort does not react very much to increased incentives in the optimum since the cost function is very steep, P will prefer to decrease the piece rate when $\bar{\varepsilon}_r$ rises.

Finally, recall that plugging $\lambda_1 = 0$ and $\lambda_2 = 1$ into Eq. (2) yields

$$(1-\beta)\, h'(\beta) + \beta h'(\beta) - \bar{\varepsilon}_r = 0,$$

[2]Note that both α and $w_{\min}$ can be negative.

which points to a further aspect. The first term shows that increasing the piece rate β also decreases P's share in produced output. Hence, it will only be optimal for him to increase the piece rate, if his decreased share is overcompensated by the increase of effort. To sum up, we need two conditions for a positive relation between risk and incentives in a principal-agent model – the agent must earn a positive rent so that the principal does not have to pay for higher incentives, and effort must be rather sensitive to incentives.

References

Hadar, J. and W.R. Russel, 1971, Stochastic dominance and diversification, Journal of Economic Theory 3, 288-305.

Prendergast, C., 2002a, The tenuous trade-off between risk and incentives, Journal of Political Economy 110, 1071-1102.

Prendergast, C., 2002b, Uncertainty and incentives, Journal of Labor Economics 20, S115-S137.

Raith, M., 2003, Competition, risk, and managerial incentives, American Economic Review 93, 1425-1436.

Serfes, K., 2005, Risk sharing vs. incentives: contract design under two-sided heterogeneity, Economics Letters 88, 343-349.

Wright, D.J., 2004, The risk and incentives trade-off in the presence of heterogeneous managers, Journal of Economics 83, 209-223.