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Abstract

Consider a two-product firm that decides on the quality of each product. Product quality is unknown to consumers. If the firm sells both products under the same brand name, consumers adjust their beliefs about quality subject to the performance of *both* products. We show that if the probability that low quality will be detected is in an intermediate range, the firm produces high quality under umbrella branding whereas it would sell low quality in the absence of umbrella branding. Hence, umbrella branding mitigates the moral hazard problem. We also find that umbrella branding survives in asymmetric markets and that even unprofitable products may be used to stabilize the umbrella brand. However, umbrella branding does not necessarily imply high quality; the firm may choose low-quality products with positive probability.

Keywords: Umbrella branding, reputation transfer, signaling, experience goods.

JEL-Classification: L14, L15, M37, D82.

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1 Introduction

Umbrella branding is a standard business practice for products with experience good attributes. The main reason why umbrella branding works is that consumers make inferences from the characteristics observed in one product to the characteristics of others. Perhaps most important is that consumers can draw inferences from experience about the quality of a product sold under the same umbrella brand. For instance, if a consumer has a negative experience with a product, she may be less inclined to buy another product of the same brand. A firm can thus try to link the expected quality of one product to the customers' experience with another product.[1] If this is the case, an umbrella brand carries information. Or, as Richard Branson, founder of Virgin, puts it, "consumers understand that all the values that apply to one product – good service, style, quality, value and fair dealing – apply to others" (Time Magazine, June 24, 1996, cited by Andersson, 2002). Umbrella branding can increase the scope of a firm and be an incentive to provide high quality. Our paper explores this link.

A high quality product is assumed to satisfy the needs of consumers, whereas a low quality product breaks down or does not work properly with a certain probability. Hence, after experiencing the product consumers only imperfectly observe the quality of the product. This is the feature of many products such as cosmetics and many services such as maintenance or financial services. Also, for goods such as consumer electronics and household durables an important aspect is their reliability. Here, an intertemporal link may not be provided through repeat purchase but through word-of-mouth.

In this paper we provide a parsimonious framework for the study of umbrella branding. The features of our model are the following: a firm simultaneously decides about the product quality of its two products and the use of umbrella branding, where it is assumed that umbrella branding is associated with a higher cost. A product is sold for two periods. After the first period, there is a positive probability that consumers will detect low quality. Consumers have box demand. We analyze those equilibria in which the firm absorbs all the expected surplus. The basic insight is that a firm has a stronger incentive to provide high quality under umbrella branding because there is a positive probability that a deviation will be punished in the second period, not just with respect to the product one has the bad experience with, but also with respect to the other.

When selecting the Pareto-dominant equilibrium, our model gives the following predictions: a firm chooses low quality and no umbrella branding for sufficiently high

[1]Such a strategy requires the firm to be able to judge the product quality before the product is launched on the market. It also requires that at least a share of the consumers of one product must also be potential consumers of the other products sold under the same umbrella brand. Otherwise, umbrella branding becomes meaningless (provided that one consumer does not punish a firm because of the experience of another consumer). For an exploration of this aspect see Cabral (2001); see also Section 5.

costs of quality provision and sufficiently low detection probabilities; it chooses high quality and no umbrella branding for sufficiently low costs of quality provision and sufficiently high detection probabilities; and it chooses high quality and umbrella branding for an intermediate range of costs and detection probabilities. Clearly, umbrella branding can only play a role when the firm is vulnerable to quality defections. Umbrella branding then provides a safeguard to consumers, since a defection can be more severely punished.

We also show that, on a range of parameter values, there are asymmetric pure-strategy equilibria in our symmetric set-up. In addition to the equilibria in which the quality choice is a pure strategy, we characterize all mixed-strategy equilibria. A particular type of equilibrium takes the form that product quality is positively correlated under umbrella branding. We thus obtain a rich set of equilibria with different qualitative features.

Our framework also allows us to analyze cost and value asymmetries and differences in detection probabilities. Focusing on the Pareto-dominant equilibrium, our analysis reveals that pronounced asymmetries between products may hinder the use of umbrella branding for transmitting information to consumers. However, umbrella branding with high quality may be used in circumstances in which one low-quality and one high-quality product would be introduced under independent selling. We also show that the firm may want to sell one of its products below costs to stabilize the umbrella brand, and quality is overprovided from a social point of view.

We then analyze a more general symmetric setting in which the probability that consumers detect *high* quality is also positive. If the detection probability for high quality is larger than that for low quality, our earlier results are confirmed. If the detection probability of high quality is larger than that for low quality, then mixed strategy equilibria, in which the umbrella is used, do not exist. However, also in this case there are pure-strategy equilibria in which umbrella branding leads to high quality.

Literature Review. Umbrella branding has received a lot of interest in recent years, both in the marketing and the industrial organization literature. Here, we discuss the most related empirical and theoretical studies.[2]

Recent experimental and empirical work in the marketing literature shows that the signaling argument of umbrella branding is broadly consistent with the data. The marketing literature on brand stretching and umbrella branding is concerned with the sources of success and failure of these marketing instruments. There are a number of papers presenting experimental evidence about when umbrella branding works. With respect to the general mechanism at work, Aaker and Keller (1990) find

[2]We only discuss work in which umbrella branding is seen as a marketing instrument to solve problems of asymmetric information. For a different view, according to which a consumer's utility increases if a brand is better known, see e. g. Pepall and Richards (2002).

experimental evidence that the perceived quality of one good affects the expected quality of another one.[3]

While most work considers hypothetical brand stretching, some authors have analyzed actual extensions. For instance, Reddy, Holak, and Bhat (1994) analyze brand stretching for cigarettes with annual data over a period of more than 20 years. Their findings suggest that even with cannibalization, brand stretching can be profitable. Erdem (1998) uses panel data for two oral hygiene products, toothpaste and toothbrush, in which some of the two products share the same brand name in both product categories. Her regression results can be interpreted as follows: consumers are uncertain about quality levels, and experience does not provide perfect information. Consumers' expected product qualities are highly correlated if products are sold under an umbrella brand.[4] Using scanner data for yoghurts and detergents, Balachander and Ghose (2003) find reciprocal spillover effects between two brands under the same umbrella. They maintain that the reason is that umbrella-branded products benefit one another because of economies of information. As they point out, one mechanism leading to such economies of information is the one in Wernerfelt (1988), which we discuss next.

The industrial organization literature on umbrella branding is part of a rich body of work on the firm as a bearer of reputation – for a review see Bar-Isaac (2004). The first theoretical analysis of umbrella branding was by Wernerfelt (1988). Wernerfelt considers an adverse selection environment in which a firm with an old product uses its brand for a new product. After the old product is sold in the first period, consumers learn its quality with some probability. They can then use this information, together with the decision whether the firm uses umbrella branding, to form their beliefs about the new product's quality. However, since in Wernerfelt's model the qualities of the products are not related, the information about the old product's quality does not affect the beliefs about the new product's quality. After the new product is sold in the second period, consumers learn its quality with some probability. In this setting, Wernerfelt shows that umbrella branding is used by the firm when its two products are of high quality; that is, umbrella branding is a signal of high quality. As stated by Wernerfelt, umbrella branding invites consumers to pool their experience with the two products to infer the quality of both.[5] The key mechanism supporting the informative role of umbrella branding is that a false signal –

[3]Related work considers image spillovers. One explanation for image spillovers is the information value of brands. For instance, Sullivan (1990) finds empirical evidence for image spillovers in the automobile market. Experimental work concerning brand stretching includes that of Keller and Aaker (1992).

[4]Erdem and Sun (2002) extend the analysis to find evidence of spillover effects for umbrella brands generated through advertising and sales promotions.

[5]Montgomery and Wernerfelt (1992) consider umbrella branding of firms operating in a competitive market with adverse selection. In contrast to the rest of the literature, they obtain that products which are sold under an umbrella brand have less than average quality. However, in their model, umbrella branding has a different function: it is risk reducing as the quality-variance of products sold under an umbrella brand is lower.

that is, the use of umbrella branding when one of the two products is of low quality – implies an even less attractive probability distribution over types than no signal at all. A critical assumption is that umbrella branding requires an upfront cost. By contrast, we do not require such money burning.

In related work, Cabral (2000) considers a market in which firms are active in three periods. A firm's products are of a given quality, which is only known to the firm. Consumers receive a noisy signal about the firm's quality; the realization of the signal is public information. The firm sells product 1 in the first and third period. In the second period it has to decide whether it sells product 2 under the same brand name. If not, product 2 is indistinguishable from a product by a new firm. The use of brand stretching depends on the signal at the end of period 1 and on the quality of the firm. Cabral shows the existence of a semi-separating equilibrium in which the probability of brand stretching is positive in the quality of the products. Several effects explain the non-neutrality of brand stretching: a firm's reputation, derived from the experience with product 1 in the first period, affects the willingness to pay in future periods – Cabral calls this the direct reputation effect. The experience derived after consuming product 2 influences the willingness to pay for product 1 in the third periods – Cabral calls this the feedback reputation effect. In addition, to the extent that the decision to use umbrella branding depends on the true quality of the products, there is also a signalling effect.

Choi (1998) considers a monopoly set-up in which each product is sold in two subsequent periods. For a single product, the monopolist faces the same price-signaling problem as in Milgrom and Roberts (1986). In addition, in each period a new product may be launched under the umbrella brand. In his model, adverse selection means that quality is not controlled by the firm. Apart from setting the price for each good, the firm only has to decide whether or not to use the umbrella brand. The use of an umbrella brand then allows the firm to distort its price less than under separate selling. The umbrella brand is protected because the inclusion of a low-quality product would trigger the loss of brand capital; that is, consumers would no longer trust the brand in the future. This part of the argument is similar to Klein and Leffler (1981) and Shapiro (1983).

While in the papers by Wernerfelt, Cabral, and Choi product quality cannot be controlled by the firm, umbrella branding may affect the incentives to provide quality. Andersson (2002) analyzes a model with this feature. In an infinite horizon model, a firm chooses the product quality of its two products in each period. If it has only one product, the analysis is simply a restatement of Klein and Leffler (1981). In this case, the firm can sustain the reputation, although it may have to distort its price. Whenever the firm has to distort the price, umbrella branding can relax the incentive constraint such that the price distortion above the full-information monopoly price is less severe. This is favorable to consumers and the firm alike. Such an equilibrium is supported by the consumers' belief that a deviation from producing two-high quality products implies that both products will be of low quality in the future. Overall, Andersson shows that the upper bound for profits is shifted outward by considering the possibility of umbrella branding. However, in his model the firm always chooses high quality.

In a similar vein, Cabral (2001) also considers the incentives for umbrella branding under moral hazard in an infinite horizon model. In Cabral (2001) a high-quality product also runs the risk of breaking down. There is some positive probability that this will be punished by the consumers. The argument here is related to the analysis by Green and Porter (1984) in the context of collusion between firms. If the firm uses umbrella branding consumers can interpret the functioning of both products. For high values of the discount factor, consumers interpret the break-down of one of the two product as "bad luck" and therefore do not change their beliefs. Only when both products break down does the reputation of the brand suffer; that is, reputation breaks down with some positive probability. For lower discount factors, consumer beliefs are more sensitive to the products' breakdowns: a break-down of both products completely destroys the reputation of the brand, whereas the break-down of one product is enough to cause the reputation of the firm to suffer.

We depart from the infinite horizon setup to focus on a basic mechanism behind umbrella branding. We construct a simple model which allows us to explicitly analyze the pricing decision in combination with the branding decision: we characterize the set of perfect Bayesian equilibria. Our model allows us to separate umbrella branding and the incentives to provide quality from other issues such as price signalling. Following Andersson (2002) and Cabral (2001) we consider a moral hazard environment. Hence, the firm decides the quality of the products. With respect to the information received by the consumers, our model follows Wernerfelt (1988). In addition, following the work in adverse selection environments, the quality of a product is determined for all the periods in which the product is sold; in other words, the firm is committed to a certain quality level. Umbrella branding is studied in a set-up that is symmetric with respect to the timing of the two products; that is, consumers receive information about the quality of the products at the same time – the only other symmetric models in the above list are those offered by Andersson (2002) and Cabral (2001). This allows us to focus on the scope of a brand and to separate this analysis from dynamic considerations of brand extension.

The plan of the paper is as follows. In Section 2, we present the basic model. In Section 3, we show that for a set of parameters, a firm which sells a single product or two products under different brands provides low quality, whereas a firm that sells both products using umbrella branding provides high quality. Furthermore, in Section 4.1, we introduce asymmetries with respect to costs and detection probabilities. Here, we show that a firm may decide to sell below cost. We also explain that umbrella branding is not viable if the asymmetries are sufficiently strong. In Section 4.2, we analyze the firm's quality decision under different information structures with respect to quality revelation. We show that our general insights about the use of umbrella branding still holds in this more general setting. Section 5 concludes.

2 The Model

Consider a market in which a firm produces and sells two experience goods for two periods. The firm decides for each product whether to produce low or high quality, $q_i \in \{\mathrm{L}, \mathrm{H}\}$. We denote with $\lambda_{q_1 q_2}$ the probability to choose quality q_1 for product 1 and quality q_2 for product 2. The same production technology is used for both periods, hence quality remains the same over time. Producing one unit of high quality for each period entails a total cost of $c_1 = c_2 = c$. High-quality products never break down, whereas low-quality products break down with probability $\delta_1 = \delta_2 = \delta$. The underlying random variables are independently distributed across time and products. The consumers' willingness to pay for a product that does not break down is $v_1 = v_2 = v$ in each period. For a product that breaks, the willingness to pay is $\underline{v}_1 = \underline{v}_2 = \underline{v}$. Without loss of generality, normalize $\delta\, \underline{v} + (1-\delta)\, v = 0$, so that the willingness to pay for a low quality product is exactly zero. Hence, we set $\underline{v} := -v\,(1-\delta)/\delta$. Furthermore, production costs of a high-quality product are independent from the quality of the other product. Similarly, consumer valuations for one high quality product are independent of the quality of the other product. Hence in a world without asymmetric information, there are no demand or supply-side economies of scope.

Consumers of mass 1 demand up to one unit of each product in each period. The indirect utility for high quality product $i = 1, 2$ in period $t = 1, 2$ is $u_i^t = v - p_i^t$, where p_i^t is the price to be paid. For low quality, u_i^t is equal to $-p_i^t$, and if consumers do not buy, $u_i^t = 0$. A consumer derives utility $u_1^1 + u_2^1 + u_1^2 + u_2^2$. If both products are sold in both periods, the firm's profits (gross of any costs of umbrella branding) are $p_1^1 + p_2^1 + p_1^2 + p_2^2 - C(q_1) - C(q_2)$, where $C(q_i)$ is equal to c if high quality is provided; otherwise it is zero. Hence under perfect information, if $2\,v \geq c$, the firm chooses $q_i = \mathrm{H}$ and $p_i^t = v$ for both products and both periods, its profits are $4\,v - 2\,c$. If $2\,v < c$, then $q_i = \mathrm{L}$ and $p_i^t = 0$, and profits are zero.

The Multi-Stage Game We analyze the following three-period game, solving for perfect Bayesian equilibria.[6]

- $t = 0$: the firm decides whether to use umbrella branding. Then it chooses the qualities of both products, i. e. probabilities λ_{HH}, λ_{HL}, λ_{LH}, $\lambda_{\mathrm{LL}} \geq 0$ with $\lambda_{\mathrm{HH}} + \lambda_{\mathrm{HL}} + \lambda_{\mathrm{LH}} + \lambda_{\mathrm{LL}} \leq 1$.
- $t = 1$: Consumers observe whether the firm uses umbrella branding, but they do not observe the quality of any of the products. The firm makes "take it or leave it" offers p_1^1 and p_2^1 for products 1 and 2 to each consumer. Consumers form beliefs and accept both, one or neither of the two offers.

[6]We characterize pure-strategy as well as mixed-strategy equilibria. We consider those equilibria in which the firm extracts the full expected surplus (see below) and provide a full characterization of this restricted equilibrium set. Mixed-strategy equilibria are understood to involve some real mixing between quality.

- $t = 2$: If the product is of low quality, consumers detect quality with probability $\delta_1 = \delta_2 = \delta$ for each product. The firm makes take-it-or-leave-it offers p_1^2 and p_2^2 for products 1 and 2 to each consumer. Consumers update beliefs and accept both, one or neither of the two offers.

Asymmetric Information. The firm's choice of quality cannot immediately be observed by the consumers. Hence, in the first period they lack any hard information about product quality. In the second period, if a product is of low quality, they receive a negative realization with probability δ and thus detect low quality before making a second purchase. They use this information to update their beliefs in period 2. Our construction can be motivated as follows. Consumers may have to regularly replace a product, that is, in periods 1 and 2. The firm, due to technological choices, is committed to offer the same product in both periods. Then δ represents the detection probability of low quality. If consumers realize that the product is not working properly or is of low quality they adjust expectations accordingly. Even if consumers do not buy the product in both periods, we can apply our model. Suppose there are new consumers in the second period who rely on word-of-mouth communication. In this case, the effective detection probability depends on the detection probability by period-1 consumers and the diffusion of information among consumers. In our analysis we restrict consumer beliefs: they are assumed not to use prices in their belief formation. Effectively, the firm is able to extract the full expected surplus from consumers. This allows us to focus on the quality dimension in the use of umbrella branding and amounts to the equilibrium selection which is most favorable for the firm.

Perfect Information and First Best. As a benchmark for later results, it is useful to consider the first-best. In this case, products should be of high quality whenever $2\,v > c$. Since the firm absorbs all of the surplus, it implements the first-best allocation under perfect information. In other words, we have constructed the model such that any deviations from the first-best are due to the asymmetric information problem faced by the firm and not due to its market power.

3 Product Choice and Umbrella Branding

In this section we characterize perfect Bayesian equilibria of the game described above. We proceed in two steps. First we consider the one-product case. This corresponds to a situation in which umbrella branding cannot be used or in which consumers ignore any potential information about product quality which umbrella branding might contain. Second, we consider the two-product case. Here, the firm has the possibility to link the consumers' experience with one product to the consumers' beliefs about the product quality of the other product.

3.1 The One-Product Case

Consider a situation in which, from observing the quality of one product, consumers do not make inferences about the quality of the other products. One reason may be that consumers believe that types and production decisions with regard to both products are independent. Another possible reason is that there are two non-intersecting groups of consumers that consume each of the products and that do not communicate. In this case, both products can be treated separately and we simply denote with λ firm's production decision, i. e. the probability that the product under consideration is of high quality.

In $t = 1$, the consumer's expected value of the product is simply $\lambda\, v$. If the product does not work well in $t = 1$, then the expected value drops to zero in $t = 2$. If the product works well, then consumers update their beliefs about quality. They believe that the firm produces high quality with probability $\lambda/(\lambda + (1 - \lambda)\,(1 - \delta))$, hence the expected value of the product rises to

$$\frac{\lambda}{\lambda + (1 - \lambda)\,(1 - \delta)}\, v.$$

Because the firm makes take-it-or-leave-it offers, it can set the price equal to the expected valuation of the consumers. Hence, if it decides to produce high quality, expected profits amount to

$$\Pi_{\mathrm{H}} = \lambda\, v + \frac{\lambda}{\lambda + (1 - \lambda)\,(1 - \delta)}\, v - c.$$

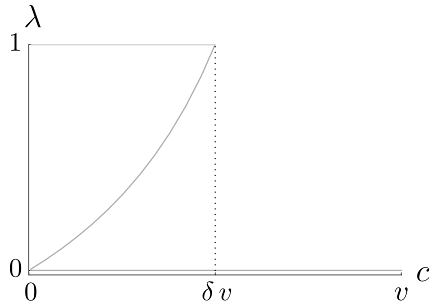
If it produces low quality, expected profits are

$$\Pi_{\mathrm{L}} = \lambda\, v + (1 - \delta)\, \frac{\lambda}{\lambda + (1 - \lambda)\,(1 - \delta)}\, v.$$

Proposition 1 (Equilibrium, One-Product Case) *If the cost of high quality is less than $\delta\, v$, there are three equilibria: an equilibrium in which the firm chooses high quality; an equilibrium in which the firm chooses low quality; and an equilibrium in which the firm chooses high quality with probability $\frac{c}{v-c}\,\frac{1-\delta}{\delta}$ and low quality with the remaining probability. If the cost of high quality is greater than $\delta\, v$, there is a unique equilibrium. In this equilibrium, the firm chooses low quality.*

Proof: First, we check under which parameter constellations we obtain pure-strategy equilibria. If the consumers believe that quality is low ($\lambda = 0$), then expected profits are $\Pi_{\mathrm{H}} = 0 \cdot v + 0 \cdot v - c = -c$ for the high quality choice, and $\Pi_{\mathrm{L}} = 0$ for low quality. As a result, there is always an equilibrium with $\lambda = 0$. If the consumers believe that quality is high ($\lambda = 1$), then $\Pi_{\mathrm{H}} = v + v - c = 2\, v - c$ and $\Pi_{\mathrm{L}} = v + (1 - \delta)\, v = (2 - \delta)\, v$, hence $\Pi_{\mathrm{H}} \geq \Pi_{\mathrm{L}}$ iff $c \leq \delta\, v$.

Figure 1: Equilibrium Correspondence for λ



Second, we check under which conditions we obtain non-degenerate, mixed strategy equilibria. In a mixed strategy equilibrium, expected profits for both alternatives must be equal,

$$\begin{aligned} & \Pi_{\mathrm{H}} = \Pi_{\mathrm{L}} \\ \iff & c = \delta \, \frac{\lambda}{\lambda + (1-\lambda)\,(1-\delta)} \, v \\ \iff & \lambda = \lambda^* = \frac{c}{v-c} \, \frac{1-\delta}{\delta}. \end{aligned} \tag{1}$$

Clearly, $\lambda^* \geq 0$ whenever $c \geq 0$, and $\lambda^* \leq 1$ whenever $c \leq \delta\, v$. ■

To sum up, we get the following equilibrium correspondence for λ,

$$\lambda = \begin{cases} 0 & : \; c \in [0;\; \infty), \\ \frac{c}{v-c} \cdot \frac{1-\delta}{\delta} & : \; c \in [0;\; \delta\, v], \\ 1 & : \; c \in [0;\; \delta\, v]. \end{cases}$$

The equilibrium correspondence is plotted in Figure 1. Let us take a closer look at the mixed-strategy equilibrium. There, the average quality rises with falling detection probability δ, rising costs c, and falling valuation differential v. To gain a better understanding of these properties, note that in mixed-strategy equilibrium, the firm is indifferent between high and low quality. Then if the costs of producing high quality c rises (or if v or δ fall), producing high quality becomes relatively unattractive. In a mixed strategy equilibrium, this must be levelled out by readjusting λ to make the production of high quality attractive again, i. e. by raising λ. The mixed-strategy equilibrium is not stable: If consumers believe that the probability of high quality is only slightly higher than λ^*, the firm is inclined to produce only high quality.

Among the different types of equilibria, we may want to select those which Pareto-dominate the others. Note that if consumers do not obtain any surplus, the firm's profit is equal to the total surplus $2\,\lambda\, v - \lambda\, c$, which increases with rising λ provided that $c < 2\, v$.

Remark 1 (Pareto-Dominance) *For $c < 2\,v$ and any equilibria with $\lambda' \geq \lambda''$, the equilibrium with λ' (weakly) Pareto-dominates that with λ''.*

We observe that for $c < \delta\, v$ only the equilibrium with high quality Pareto-dominates the other equilibria, i. e. it is Pareto-dominant. Selecting the Pareto-dominant equilibrium implies that, for small and large costs c, the first-best is implemented: for $c < \delta\, v$ the firm chooses the socially optimal high quality; for $c > 2\,v$ the firm chooses the socially optimal low quality. For an intermediate range $\delta\, v < c < 2\,v$, the firm chooses low quality, although the first-best would be to provide high quality. Hence due to moral hazard, quality is socially underprovided.

3.2 The Two-Products Case

In order to correlate beliefs about product quality across products, the consumers must at least know that the firm produces two products. In the model, this is achieved by putting the products under the same umbrella brand. The firm now has four different options. It can produce both products in high quality (HH), product 1 in high quality and product 2 in low quality (HL), the reverse (LH), and, finally, both products in low quality (LL).

Let us start by discussing the pure-strategy equilibrium in which $\lambda_{\mathrm{HH}} = 1$. These are the beliefs that are most favorable to support umbrella branding. We are interested in the question for which parameter constellation the provision of high quality can be supported as an equilibrium outcome. Clearly, if $\lambda_{\mathrm{HH}} = 1$, low quality can never be observed along the equilibrium path, and off-equilibrium beliefs need to be specified. If beliefs are correlated across products, consumers may believe that if one product defaults, the other product is also of low quality. In this case, prices drop to zero in the second period. Depending on the production choices, expected profits amount to

$$\begin{aligned}
\Pi_{\mathrm{HH}} &= 2\,v + 2\,v - 2\,c,\\
\Pi_{\mathrm{HL}} = \Pi_{\mathrm{LH}} &= 2\,v + 2\,(1-\delta)\,v - c,\\
\Pi_{\mathrm{LL}} &= 2\,v + 2\,(1-\delta)^2\,v.
\end{aligned}$$

We see that $\Pi_{\mathrm{HH}} \geq \Pi_{\mathrm{LL}}$ iff $c \leq \big(1-(1-\delta)^2\big)\,v = \delta\,(2-\delta)\,v$. Furthermore, $\Pi_{\mathrm{HH}} \geq \Pi_{\mathrm{HL}}$ iff $c \leq 2\,\delta\,v$, which is implied by $c \leq \delta\,(2-\delta)\,v$. To sum up, an equilibrium with $\lambda_{\mathrm{HH}} = 1$ can be supported for $c \leq \delta\,(2-\delta)\,v$. Under uncorrelated beliefs (without the umbrella), it could be supported only for $c \leq \delta\,v$.[7]

[7] If c is sufficiently small, other off-equilibrium beliefs, which also support $\lambda_{\mathrm{HH}} = 1$, can be found.

Remark 2 (Umbrella Branding) *If beliefs can be correlated across products, the region of parameter constellations where high quality provision ($\lambda_{\mathrm{HH}} = 1$) can be supported as an equilibrium action expands. Hence, umbrella branding can mitigate the moral hazard problem.*

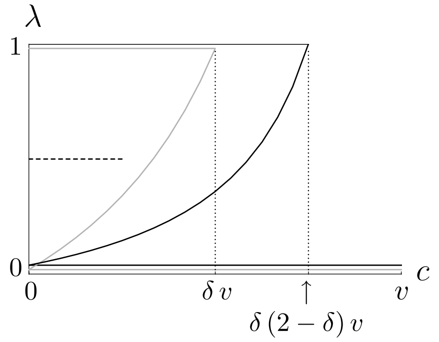
Now consider pure-strategy equilibria in which only low quality is chosen, i. e. $\lambda_{\mathrm{LL}} = 1$. Here, consumers believe that if one product is detected to be of low quality, the other product is perceived to be of low quality, too. Expected profits are then $\Pi_{\mathrm{HH}} = -2\,c$, $\Pi_{\mathrm{HL}} = \Pi_{\mathrm{LH}} = -c$ and $\Pi_{\mathrm{LL}} = 0$. As a result, it is always optimal to provide low quality, and an equilibrium with $\lambda_{\mathrm{LL}} = 1$ can be supported for every parameter constellation with $c > 0$. Note that this equilibrium action can also be supported by uncorrelated beliefs.

Next, we characterize perfect Bayesian equilibria with symmetric beliefs, i. e. $\lambda_{\mathrm{HL}} = \lambda_{\mathrm{LH}}$. It is useful to distinguish between two kinds of equilibria. First, the firm can mix between all four options, in which case we must have $\Pi_{\mathrm{HH}} = \Pi_{\mathrm{HL}} = \Pi_{\mathrm{LH}} = \Pi_{\mathrm{LL}}$ in equilibrium. Second, the firm can mix between less than four options. In this case, only the profits pertaining to options that are chosen with positive probability need to be equal, the expected profits for the other options must be equal or lower. As an example, for $\lambda_{\mathrm{HL}} = \lambda_{\mathrm{LH}} = 0$ and $\lambda_{\mathrm{HH}}, \lambda_{\mathrm{LL}} \in (0;\ 1)$, the only necessary conditions are $\Pi_{\mathrm{HH}} = \Pi_{\mathrm{LL}}$, and additionally $\Pi_{\mathrm{LH}}, \Pi_{\mathrm{HL}} \leq \Pi_{\mathrm{HH}}$. The following proposition (proof in the Appendix) shows that consumer beliefs can be positively or negatively correlated across products, or uncorrelated as in the case of independent products. Equilibria with positively correlated beliefs can support the provision of low quality and, for certain parameters, the provision of high quality, as has been demonstrated above. In addition, an equilibrium with mixed strategies may exist.

Proposition 2 (Equilibrium, Two-Product Case) *There are three types of equilibria:*

- *Equilibria with positively correlated beliefs where consumers believe that both products are of equal quality, i. e. $\lambda_{\mathrm{HL}} = \lambda_{\mathrm{LH}} = 0$. For $c < \delta\,(2-\delta)\,v$, there are three such equilibria (one mixed, one with $\lambda_{\mathrm{HH}} = 1$, one with $\lambda_{\mathrm{LL}} = 1$). For $c > \delta\,(2-\delta)\,v$, there is only one (with $\lambda_{\mathrm{LL}} = 1$).*
- *Equilibria with uncorrelated beliefs where consumers believe that the qualities of products are unrelated, i. e. $\lambda_{\mathrm{HH}}\,\lambda_{\mathrm{LL}} = \lambda_{\mathrm{LH}}\,\lambda_{\mathrm{HL}}$. For $c < \delta\,v$, there are three such equilibria (one mixed, one with $\lambda_{\mathrm{HH}} = 1$, one with $\lambda_{\mathrm{LL}} = 1$). For $c > \delta\,v$, there is only one (with $\lambda_{\mathrm{LL}} = 1$).*
- *Equilibria with negatively correlated beliefs where consumers believe that products are of contrasting quality, i. e. $\lambda_{\mathrm{HH}} = \lambda_{\mathrm{LL}} = 0$. For $c < \delta^2\,v$, there is one such equilibrium (with $\lambda_{\mathrm{LH}} = \lambda_{\mathrm{HL}} = 1/2$). For $c > \delta^2\,v$, there is none.*

Figure 2: Equilibrium Correspondence for λ, Multiple Products



The black graph stands for positively correlated equilibria, the gray graph for uncorrelated equilibria (cf. Figure 1); the dashed line stands for negatively correlated equilibria.

Quality choices in the different equilibria are represented in Figure 2. In the area between $\delta\, v$ and $\delta\,(2-\delta)\,v$, the firm produces high quality only if it puts the two products under the same umbrella brand.

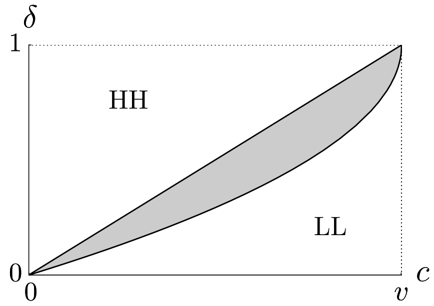
Next, we characterize Pareto-dominant equilibria. Note that the insight of Remark 1 still holds in the two-product case: the equilibrium with the highest probability of high quality provision is Pareto-dominant. The Pareto-dominant equilibrium yields high quality only ($\lambda_{\mathrm{HH}} = 1$) for $c \leq \delta\,(2-\delta)\,v$, otherwise low quality only ($\lambda_{\mathrm{LL}} = 1$). Selecting the Pareto-dominant equilibrium, umbrella branding does not improve upon independent selling for $c < \delta\, v$. Umbrella branding does improve upon independent selling if $\delta\, v < c < \delta\,(2-\delta)\,v$. If $c > \delta\,(2-\delta)\,v$, low quality is provided even under correlated beliefs, hence umbrella branding is neutral to quality provision. These three regions are represented in Figure 3.

When umbrella branding leads to an improvement, the firm gains $4\,v - 2\,c$ with the umbrella, compared to zero gains without the umbrella. Hence the firm will choose the umbrella brand whenever benefits exceed costs, i. e. whenever $2\,v > c$. To sum up, the umbrella is used whenever $\delta\, v < c < \delta\,(2-\delta)\,v$. Thus, umbrella branding is chosen for intermediate values of detection probability δ, cost differential c, or value differential v. In particular, for any given $0 < \delta < 1$ and v, there is a range of parameters c such that the umbrella is chosen. We summarize our above findings by the following proposition.

Proposition 3 (Scope of Umbrella Branding) *Suppose that the firm and consumers coordinate on Pareto-dominant equilibria. Then umbrella branding is chosen if inequality $\delta < c/v < \delta\,(2-\delta)$ holds.*

Proposition 3 relies on the selection of the Pareto-dominant equilibrium. Alternatively, we can introduce a cost b for the use of umbrella branding, which is incurred

Figure 3: Dominant Equilibria for Multiple Products



In the gray region, the umbrella brand is built and products are of high quality. In the white regions, products are either of high quality even without the umbrella (HH), or of low quality (LL).

in period $t = 0$, and apply a forward induction argument by van Damme (1989) to select among pure-strategy equilibria. Recall that if $\delta\, v < c < \delta\,(2 - \delta)\, v$, there is a unique perfect Bayesian equilibrium when products are sold independently, whereby the firm chooses low quality for both products. In the equilibrium with uncorrelated beliefs, the firm cannot improve. Hence the fact that the firm chooses umbrella branding and spends $b > 0$ should be interpreted by consumers to mean that $\lambda_{\mathrm{HH}} > 0$. This implies $\lambda_{\mathrm{HH}} = 1$ in the unique *pure-strategy* equilibrium that satisfies forward induction.

The forward induction argument by van Damme (1989) can also be used in a slightly modified game to always select the high-quality equilibrium (when it exists). Consider our game with the modification that the firm first commits to prices of the goods and whether to use umbrella branding and second chooses the quality of its products. Consumers then can make inferences from prices and umbrella branding decision about the intended quality choice. Consumers interpret high prices as the firm intending to play the equilibrium with high quality, provided that this can be supported as an equilibrium outcome. Consumers always interpret low prices as the firm intending to play the equilibrium with low quality. Hence, the low quality equilibrium does not satisfy forward induction for small c independent of the umbrella branding decision, and the firm chooses high quality in any equilibrium that satisfies forward induction. Since umbrella branding is costly, the firm does not use the umbrella. For intermediate values of c, the firm chooses high quality and uses umbrella branding in the unique equilibrium that satisfies forward induction.

4 Extensions

4.1 Asymmetric Products

In Proposition 3 we have shown that, when products are symmetric, umbrella branding is beneficial for medium detection probability δ, medium cost differential c and medium value differential v. In this subsection we characterize equilibria when products are not symmetric; that is, we consider products that have differing detection probabilities δ_1 and δ_2, differing cost differentials c_1 and c_2 and differing value differentials v_1 and v_2. To keep the analysis simple, we consider only positively correlated and uncorrelated equilibria.

The characterization of uncorrelated equilibria can be taken from Proposition 1 by simply substituting variables. In the Pareto-dominant equilibrium, product 1 is made in high quality iff $c_1 \leq \delta_1 v_1$, and product 2 is made in high quality iff $c_2 \leq \delta_2 v_2$. In addition to this equilibrium, there are mixed strategy equilibria and equilibria in which only low quality is produced.

The characterization of positively correlated equilibria, i. e. equilibria with $\lambda_{\text{HL}} = \lambda_{\text{LH}} = 0$, is more involved. Expected profits are then

$$\begin{aligned}
\Pi_{\text{HH}} &= (v_1+v_2)\,\lambda_{\text{HH}} + (v_1+v_2)\,\frac{\lambda_{\text{HH}}}{\lambda_{\text{HH}} + (1-\delta_1)\,(1-\delta_2)\,\lambda_{\text{LL}}} - (c_1+c_2),\\
\Pi_{\text{HL}} &= (v_1+v_2)\,\lambda_{\text{HH}} + (v_1+v_2)\,(1-\delta_2)\,\frac{\lambda_{\text{HH}}}{\lambda_{\text{HH}} + (1-\delta_1)\,(1-\delta_2)\,\lambda_{\text{LL}}} - c_1,\\
\Pi_{\text{LH}} &= (v_1+v_2)\,\lambda_{\text{HH}} + (v_1+v_2)\,(1-\delta_1)\,\frac{\lambda_{\text{HH}}}{\lambda_{\text{HH}} + (1-\delta_1)\,(1-\delta_2)\,\lambda_{\text{LL}}} - c_2, \quad \text{and}\\
\Pi_{\text{LL}} &= (v_1+v_2)\,\lambda_{\text{HH}} + (v_1+v_2)\,(1-\delta_1)\,(1-\delta_2)\,\frac{\lambda_{\text{HH}}}{\lambda_{\text{HH}} + (1-\delta_1)\,(1-\delta_2)\,\lambda_{\text{LL}}},
\end{aligned}$$

with $\lambda_{\text{LL}} = 1 - \lambda_{\text{HH}}$. In the following, we focus on Pareto-dominant equilibria. This always is a pure-strategy equilibrium with either $\lambda_{\text{HH}} = 1$ or $\lambda_{\text{LL}} = 1$. For $\lambda_{\text{HH}} = 1$, profits are $\Pi_{\text{HH}} = 2\,(v_1+v_2) - (c_1+c_2)$, $\Pi_{\text{HL}} = (2-\delta_2)\,(v_1+v_2) - c_1$, $\Pi_{\text{LH}} = (2-\delta_1)\,(v_1+v_2) - c_2$, and $\Pi_{\text{LL}} = (2-\delta_1-\delta_2+\delta_1\,\delta_2)\,(v_1+v_2)$. If the following three conditions hold, there is no incentive to deviate from $\lambda_{\text{HH}} = 1$,

$$\Pi_{\text{HH}} \geq \Pi_{\text{HL}}, \quad \text{i. e.,} \quad \delta_2 \geq \frac{c_2}{v_1+v_2}, \tag{2}$$

$$\Pi_{\text{HH}} \geq \Pi_{\text{LH}}, \quad \text{i. e.,} \quad \delta_1 \geq \frac{c_1}{v_1+v_2}, \tag{3}$$

$$\Pi_{\text{HH}} \geq \Pi_{\text{LL}}, \quad \text{i. e.,} \quad \delta_1+\delta_2-\delta_1\,\delta_2 \geq \frac{c_1+c_2}{v_1+v_2}. \tag{4}$$

If each of these three conditions hold, only high quality is provided under umbrella branding. As before, the brand is only used if the firm benefits from the umbrella.

Without an umbrella, both products are made in high quality iff $\delta_1 \geq c_1/v_1$ *and* $\delta_2 \geq c_2/v_2$. As a result, the umbrella can be supported in equilibrium, and it increases profits (gross of the costs of umbrella branding b) if inequalities (2), (3) and (4) hold and if $\delta_1 < c_1/v_1$ *or* $\delta_2 < c_2/v_2$. This result is further illustrated by Figure 4.

In the *white* region with LL, both δ_1 and δ_2 are small, and the firm produces low quality independent of umbrella branding. In the *white* region with HH, both δ_1 and δ_2 are large, and the firm produces high quality independent from umbrella branding. In both areas, umbrella branding is irrelevant and thus not used.

In the two *hatched* areas, detection probabilities are so asymmetric that, without umbrella branding, one product is made in high quality, the other in low quality. However, there is no positively correlated equilibrium with $\lambda_{\text{HL}} = \lambda_{\text{LH}} = 0$. Therefore, umbrella branding, i. e. the firm's announcement to either make both products in high or in low quality, is not credible. Consumers understand that the firm will choose different qualities for its products. In this sense, pronounced asymmetries between products may hinder the use of umbrella branding for transmitting information to consumers (see also below).

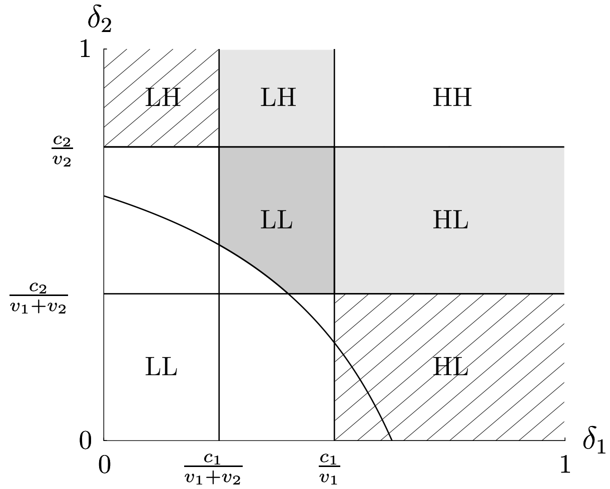
In the *dark gray* region, both detection probabilities are so low that without umbrella branding, both products are made in low quality. With umbrella branding, both products are made in high quality. Hence umbrella branding solves a moral hazard problem for each product. This situation corresponds to the one characterized in Proposition 3.

In the two *light gray* areas, detection probabilities are sufficiently asymmetric such that, without umbrella branding, one product is made in high quality, the other in low quality. With umbrella branding, both products are made in high quality. For one product, there is no moral hazard problem; and for the other product, umbrella branding solves the moral hazard problem.

Remark 3 (Umbrella Branding and Asymmetric Products) *If probabilities are sufficiently asymmetric such that under independent selling one of the products is of high quality and the other of low quality, umbrella branding may solve the moral hazard problem for the latter product.*

A simple formal argument for the use of umbrella branding in our asymmetric setup is the following (corresponds to dark and light gray area). Assume that the sum of value differentials exceeds the sum of cost differentials, $v_1 + v_2 > c_1 + c_2$. Then for some detection probabilities δ_1 and δ_2, the firm makes use of umbrella branding. Then because $v_1 + v_2 > c_1 + c_2$, we must have at least $v_1 > c_1$ or $v_2 > c_2$. Assume without loss of generality that $v_1 > c_1$. Then $\delta_1 = c_1/(v_1 + v_2)$ and $\delta_2 = 1$ fulfill inequalities (2), (3) and (4). Shifting δ_1 slightly upwards and δ_2 slightly downwards makes the inequalities strict. Because $\delta_1 < c_1/v_1$, at least product 1 would not be

Figure 4: Dominant Equilibria for Asymmetric Products



Parameters are $v_1 = v_2 = 1$, $c_1 = 1/2$ and $c_2 = 3/4$. Letters stand for quality choices in the absence of the umbrella. Left of the c_1/v_1-line, product 1 is of low quality without the umbrella. Below the c_2/v_2-line, product 2 is of low quality without the umbrella. In the gray areas, the umbrella is used. Left of the $c_1/(v_1+v_2)$-line, condition (2) is violated. Below the $c_2/(v_1+v_2)$-line, condition (3) is violated. To the south-west of the concave curve, condition (4) is violated. In light gray areas, the umbrella leads to a quality shift for one product. In the dark gray area, is leads to a quality shift for both.

produced in high quality without the umbrella. If $c_2 > v_2$, product 2 would also be of low quality without the umbrella. Therefore, the firm spends the (small) branding costs and raises the price of at least product 1.

Interestingly, an equilibrium with umbrella branding may even exist if the cost of high quality for one product exceeds its value, i. e. $2v_i - c_i < 0$. In this case, the firm sells one of its product at a loss. The umbrella is needed to credibly commit to the high quality of the other product. Put differently, producing high instead of low quality leads to a positive spill-over to the other product, which overcompensates the loss made from selling the product at a loss.

Proposition 4 (Below-Cost Pricing) *There are parameter constellations such that, in the Pareto-dominant equilibrium, the firm uses umbrella branding for two high-quality products and sells one of the products below costs.*

Proof by example: Consider the following parameter constellation for product 1: $v_1 = 2$ and $c_1 = 5$. Hence $2v_1 < c_1$, and the firm can sell a high-quality product only at a loss. Suppose parameters for product 2 are $v_2 = 10$ and $c_2 = 5$. Consequently,

$c_2/v_2 = 1/2$ and $c_2/(v_1+v_2) = 5/12$. Note that under independent selling the firm chooses low quality for each product for any $\delta_1 \in [0,1]$ and $\delta_2 < 1/2$. The dark grey area from Figure 4 corresponds to all (δ_1, δ_2) with $\delta_1 > 1/2$, $\delta_2 \in (5/12, 1/2)$, and $\delta_1 + \delta_2 - \delta_1\delta_2 \geq (c_1+c_2)/(v_1+v_2) = 5/6$. For instance, detection probabilities $\delta_1 = 5/6$ and $\delta_2 \in (5/12, 1/2)$ satisfy these conditions. Since the previous analysis was made under the assumption that $2v_i > c_i$ for $i = 1, 2$, we still have to check that umbrella branding leads to higher profits. In the equilibrium with umbrella branding, profits are $2\,v_1 + 2\,v_2 - c_1 - c_2 - b = 14 - b > 0$, provided that $b < 14$. This proves the claim. ■

From a social point of view, excessive quality is provided for the subsidized product, whereas quality is first-best only for the other product within the umbrella brand. As a result, compared to the first best, umbrella branding may lead to a social *overprovision* of quality. If $c_2 > 2\,v_2$ and $c_1 < v_1 - v_2$, then it is socially optimal to provide high quality for product 1, but low quality for product 2. Still, there are detection probabilities such that the firm chooses to use the umbrella and produce two high-quality products.

Having argued that products must be sufficiently symmetric in order to be put under the same brand, one may ask for the dimension in which products may not differ too much. We make a couple of observations. First, detection probabilities may differ. If the probability of quality detection δ is zero, a product is a pure credence good. In the other extreme, if the probability of quality detection δ is one, then a product is a pure experience good.

Then Figure 4 shows that products that are very much like credence goods and products that are much like experience goods cannot be put under the same brand. Second, products may differ in costs c and valuations v. If, for example, $v_1 \gg v_2$ and $c_1 \gg c_2$, the ratios c_1/v_1 and c_2/v_2 may still be of comparable size. Call product 1 the large product, product 2 the small one. As can be seen from Figure 4, subsuming both products under one brand adds little to the range where the large product is produced of high quality, but increases the area where the small product is produced of high quality significantly.

4.2 The Structure of Quality Revelation

In the above model, we have assumed that low quality is revealed with probability δ. In this section we analyze the way in which the results depend on the structure of quality revelation. Assume that *high* quality is revealed with probability δ_{H}, and *low* quality is revealed with probability δ_{L}. We show that equilibria with uncorrelated beliefs always exist, and that in the case $\delta_{\mathrm{H}} < \delta_{\mathrm{L}}$, the behavior of all equilibria is similar to the case $\delta_{\mathrm{H}} = 0$. We then characterize the set of all equilibria with positively correlated beliefs.

Uncorrelated Equilibria. Because of the independence of product lines in the uncorrelated equilibrium, we look at one product only. Again, be λ the probability to produce high quality. Then the expected value of each product in $t = 1$ is $\lambda\, v$. If high quality is detected, it rises to v in $t = 2$. If low quality is detected, it drops to zero in $t = 2$. If quality remains unrevealed, the expected value is adjusted to

$$v\, \frac{(1-\delta_{\mathrm{H}})\,\lambda}{(1-\delta_{\mathrm{H}})\,\lambda + (1-\delta_{\mathrm{L}})\,(1-\lambda)}.$$

For $\delta_{\mathrm{H}} > \delta_{\mathrm{L}}$, the expected value adjusts downwards, whereas for $\delta_{\mathrm{H}} < \delta_{\mathrm{L}}$, it adjusts upwards. Depending on the quality choice, expected profits are

$$\Pi_{\mathrm{H}} = v\,\lambda + \delta_{\mathrm{H}}\, v + (1-\delta_{\mathrm{H}})\, v\, \frac{(1-\delta_{\mathrm{H}})\,\lambda}{(1-\delta_{\mathrm{H}})\,\lambda + (1-\delta_{\mathrm{L}})\,(1-\lambda)} - c \quad \text{or}$$
$$\Pi_{\mathrm{L}} = v\,\lambda + (1-\delta_{\mathrm{L}})\, v\, \frac{(1-\delta_{\mathrm{H}})\,\lambda}{(1-\delta_{\mathrm{H}})\,\lambda + (1-\delta_{\mathrm{L}})\,(1-\lambda)}.$$

In a mixed strategy equilibrium, $\Pi_{\mathrm{H}} = \Pi_{\mathrm{L}}$, hence we calculate

$$\lambda = \lambda^* = \frac{c - \delta_{\mathrm{H}}\, v}{v - c}\, \frac{1-\delta_{\mathrm{L}}}{\delta_{\mathrm{L}} - \delta_{\mathrm{H}}}.$$

Clearly, this equation reduces to (1) for $\delta_{\mathrm{H}} = 0$. As in the previous section, there are pure-strategy equilibria. If consumers believe that quality is low ($\lambda^* = 0$), then for $c \geq \delta_{\mathrm{H}}\, v$, the firm produces low quality. If consumers believe that quality is high ($\lambda^* = 1$), then for $c \leq \delta_{\mathrm{L}}\, v$, only high quality is produced. Summing up, the complete correspondence (which is even a function if $\delta_{\mathrm{H}} > \delta_{\mathrm{L}}$, see Figure 5 below) for uncorrelated equilibria is

$$\lambda^* = \begin{cases} 0 & : \quad c \in [0;\ \delta_{\mathrm{H}}\, v], \\ \frac{1-\delta_{\mathrm{L}}}{\delta_{\mathrm{L}}-\delta_{\mathrm{H}}} \cdot \frac{\delta_{\mathrm{H}}\, v - c}{v - c} & : \quad c \text{ between } \delta_{\mathrm{L}}\, v \text{ and } \delta_{\mathrm{H}}\, v, \\ 1 & : \quad c \in [\delta_{\mathrm{L}}\, v;\ \infty). \end{cases}$$

Positively Correlated Equilibria. Let λ_{HH}, λ_{HL}, λ_{LH} and λ_{LL} be defined as above. We only analyze equilibria in which $\lambda_{\mathrm{HL}} = \lambda_{\mathrm{LH}} = 0$, hence consumers believe that both products are made of the same quality. Then expected profits amount to

$$\Pi_{\mathrm{HH}} = 2\, v\, \lambda_{\mathrm{HH}} + 2\, v\, \delta_{\mathrm{H}}\,(2-\delta_{\mathrm{H}}) + 2\, v\,(1-\delta_{\mathrm{H}})^2\, \frac{\lambda_{\mathrm{HH}}\,(1-\delta_{\mathrm{H}})^2}{\lambda_{\mathrm{HH}}\,(1-\delta_{\mathrm{H}})^2 + \lambda_{\mathrm{LL}}\,(1-\delta_{\mathrm{L}})^2} - 2\, c,$$
$$\Pi_{\mathrm{HL}} = 2\, v\, \lambda_{\mathrm{HH}} + 2\, v\, \delta_{\mathrm{H}} + v\, \delta_{\mathrm{H}}\, \delta_{\mathrm{L}} + 2\, v\, \delta_{\mathrm{H}}\,(1-\delta_{\mathrm{L}})$$
$$+ 2\, v\,(1-\delta_{\mathrm{H}})\,(1-\delta_{\mathrm{L}}) \frac{\lambda_{\mathrm{HH}}\,(1-\delta_{\mathrm{H}})^2}{\lambda_{\mathrm{HH}}\,(1-\delta_{\mathrm{H}})^2 + \lambda_{\mathrm{LL}}\,(1-\delta_{\mathrm{L}})^2} - c,$$
$$\Pi_{\mathrm{LL}} = 2\, v\, \lambda_{\mathrm{HH}} + 2\, v\,(1-\delta_{\mathrm{L}})^2\, \frac{\lambda_{\mathrm{HH}}\,(1-\delta_{\mathrm{H}})^2}{\lambda_{\mathrm{HH}}\,(1-\delta_{\mathrm{H}})^2 + \lambda_{\mathrm{LL}}\,(1-\delta_{\mathrm{L}})^2}.$$

Especially, if consumers believe that only high quality is provided, profits are

$$\Pi_{\mathrm{HH}} = 2\, v + 2\, v - 2\, c,$$

$$\Pi_{\mathrm{HL}} = 2\,v + 2\,v\,(1-\delta_{\mathrm{L}}) + v\,\delta_{\mathrm{L}}\,\delta_{\mathrm{H}} - c,$$
$$\Pi_{\mathrm{LL}} = 2\,v + 2\,v\,(1-\delta_{\mathrm{L}})^2.$$

We have $\Pi_{\mathrm{HH}} \geq \Pi_{\mathrm{HL}}$ iff $c \leq v\,(2-\delta_{\mathrm{H}})\,\delta_{\mathrm{L}}$, and $\Pi_{\mathrm{HH}} \geq \Pi_{\mathrm{LL}}$ iff $c \leq v\,(2-\delta_{\mathrm{L}})\,\delta_{\mathrm{L}}$. Hence if $\delta_{\mathrm{H}} < \delta_{\mathrm{L}}$, providing only high quality is an equilibrium as long as $c \leq v\,(2-\delta_{\mathrm{L}})\,\delta_{\mathrm{L}}$; otherwise, it becomes profitable to produce both products in low quality instead. If alternatively $\delta_{\mathrm{H}} > \delta_{\mathrm{L}}$, providing only high quality is an equilibrium as long as $c \leq v\,(2-\delta_{\mathrm{H}})\,\delta_{\mathrm{L}}$; otherwise, it becomes profitable to produce one (not both) product in low quality instead.

Hence, we have generalized our earlier result with respect to umbrella branding and pure-strategy equilibria, namely that umbrella branding improves the provision of quality (see also Figure 5). We summarize these findings in the following proposition.

Proposition 5 (Extension of High-Quality Provision) *For any $\delta_{\mathrm{L}} > 0$ and $\delta_{\mathrm{H}} < 1$, umbrella branding increases the range of parameter values for which there is an equilibrium in which products are provided in high quality.*

In the other pure-strategy equilibrium, consumers believe that only low quality is provided. Profits then simplify to

$$\Pi_{\mathrm{HH}} = 2\,v\,\delta_{\mathrm{H}}\,(2-\delta_{\mathrm{H}}) - 2\,c,$$
$$\Pi_{\mathrm{HL}} = 2\,v\,\delta_{\mathrm{H}}\,(1-\delta_{\mathrm{L}}) + v\,\delta_{\mathrm{L}}\,\delta_{\mathrm{H}} - c,$$
$$\Pi_{\mathrm{LL}} = 0.$$

In this case, $\Pi_{\mathrm{LL}} \geq \Pi_{\mathrm{HL}}$ iff $c \geq v\,(2-\delta_{\mathrm{L}})\,\delta_{\mathrm{H}}$, and $\Pi_{\mathrm{LL}} \geq \Pi_{\mathrm{HH}}$ iff $c \geq v\,(2-\delta_{\mathrm{H}})\,\delta_{\mathrm{H}}$. Hence if $\delta_{\mathrm{H}} < \delta_{\mathrm{L}}$, providing only low quality is an equilibrium as long as $c \geq v\,(2-\delta_{\mathrm{H}})\,\delta_{\mathrm{H}}$; otherwise, it becomes profitable to produce both products in high quality instead. If alternatively $\delta_{\mathrm{H}} > \delta_{\mathrm{L}}$, providing only low quality is an equilibrium as long as $c \leq v\,(2-\delta_{\mathrm{L}})\,\delta_{\mathrm{H}}$; otherwise, it becomes profitable to produce one (not both) product in high quality instead. As a result, the regions with pure strategies equilibria overlap only for $\delta_{\mathrm{H}} < \delta_{\mathrm{L}}$. If $\delta_{\mathrm{H}} > \delta_{\mathrm{L}}$, there is a range in which only mixed strategy equilibria are possible, $\big(v\,(2-\delta_{\mathrm{H}})\,\delta_{\mathrm{L}};\, v\,(2-\delta_{\mathrm{L}})\,\delta_{\mathrm{H}}\big)$.

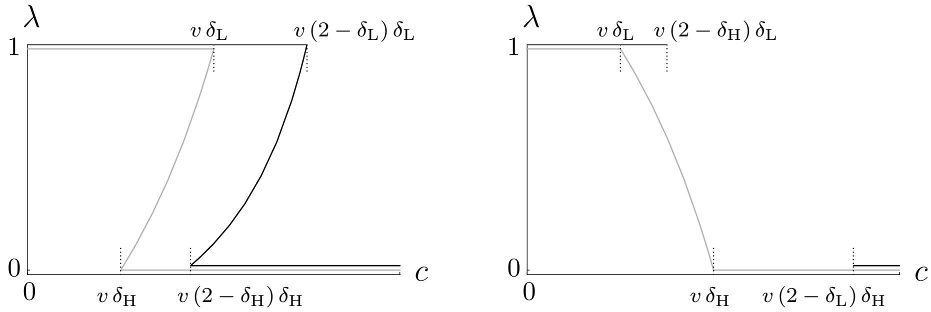
In the general mixed strategy case, we compute that $\Pi_{\mathrm{HH}} = \Pi_{\mathrm{LL}}$ iff

$$\lambda^*_{\mathrm{HH}} = \frac{(c-\delta_{\mathrm{H}}\,(2-\delta_{\mathrm{H}})\,v)\,(1-\delta_{\mathrm{L}})^2}{(v-c)\,(\delta_{\mathrm{L}}-\delta_{\mathrm{H}})\,(2-\delta_{\mathrm{L}}-\delta_{\mathrm{H}})}. \tag{5}$$

This critical λ^*_{HH} constitutes an equilibrium only if the detection of low quality exceeds that of high quality, $\delta_{\mathrm{L}} \geq \delta_{\mathrm{H}}$, as implied by the following remark (proof in the Appendix).

Remark 4 (Non-Existence of Correlated Mixed-Strategy Equilibria) *If $\delta_{\mathrm{H}} > \delta_{\mathrm{L}}$, there is no mixed-strategy equilibrium with correlated beliefs.*

Figure 5: Equilibrium Correspondence for λ, Multiple Products



Black graphs stand for positively correlated equilibria, gray graphs for uncorrelated equilibria (as in Figure 2). On the left, $0.25 = \delta_H < \delta_L = 0.5$. On the right, $0.25 = \delta_L < \delta_H = 0.5$.

As a consequence, we also have a robustness result for Section 3. Even if also high product quality can be detected with positive probability, qualitative findings remain unchanged. Therefore, the left of Figure 5 looks similar to Figure 2 (with $\delta_H = 0$), only that the lower kink of the Z-shaped graph becomes less pronounced for positive δ_H.

Not until the detection of high quality becomes more likely than that of low quality do equilibria change qualitatively. Remark 4 implies that for $\delta_H > \delta_L$ and for $c \in \big(v\,(2-\delta_H)\,\delta_L;\ v\,(2-\delta_L)\,\delta_H\big)$, there is no equilibrium with correlated beliefs. This is illustrated by the right of Figure 5; the black graph (depicting correlated equilibria) is unconnected. All equilibria with correlated beliefs are pure-strategy equilibria.

To sum up, for $\delta_L \geq \delta_H$, there are pure-strategy equilibria for all c, and the Pareto-dominant equilibrium is always in pure strategies. For an intermediate range $c \in \big(v\,\delta_L;\ v\,(2-\delta_L)\,\delta_L\big)$, the umbrella with high qualities is chosen in the Pareto-dominant equilibrium. In the opposite case $\delta_L < \delta_H$, there is only a mixed-strategy equilibrium with uncorrelated beliefs for $c \in \big(v\,(2-\delta_H)\,\delta_L;\ v\,(2-\delta_L)\,\delta_H\big)$. For smaller c, there are pure-strategy equilibria with $\lambda_{HH} = 1$, and for larger c, there are pure-strategy equilibria with $\lambda_{LL} = 1$. For an intermediate range $c \in \big(v\,\delta_L;\ v\,(2-\delta_H)\,\delta_L\big)$, the umbrella with high qualities is chosen in the Pareto-dominant equilibrium.

5 Conclusion

Marketing experts stress the potentials and dangers of umbrella branding. In particular, a well-meant brand extension can backfire if consumers feel deceived. As Aaker (1990, p. 52) illustrates, "producing other Tab flavors, such a ginger ale and root beer, seemed to make sense when Tab was Coca-Cola's diet drink entry and the firm wanted to compete for other flavor categories. The concept failed in part because substantial numbers of potential consumers felt that Tab had a disagreeable taste. It was perceived as a low-quality product by large parts of the target segment."

In this paper we have analyzed the interplay between the use of umbrella branding and the choice of product quality. To this end we presented a simple symmetric model in which two products are sold over two periods. In an initial period, the firm commits itself to product qualities of its products and decides whether to sell the products under an umbrella brand. In the first period, consumers make their purchasing decision after being informed whether umbrella branding is used. After the first period they observe low quality of a product with a certain probability and then again decide which products to buy. Here, umbrella branding allows consumers to pool their experiences across products. In particular, in an equilibrium with high quality and correlated beliefs, consumers conclude that a product that is sold under the same umbrella brand as another product that turns out to be low quality must also be low quality.

In this model we have characterized all perfect Bayesian equilibria with symmetric beliefs in which price is not used as signal. Then, selecting the Pareto-dominant equilibria, we have shown that umbrella branding mitigates the moral hazard problem, i. e. there is a range of parameter constellations in which the umbrella is chosen together with high quality for both products. Lacking the possibility to use umbrella branding, the firm would have chosen low quality. Here, umbrella branding is necessarily socially desirable. However, umbrella branding cannot fully solve the moral hazard problem, and for certain parameters there is social underprovision of quality.

We have extended this model in two directions. First, we have introduced asymmetric costs, valuations, and detection probabilities across products. In this generalized version, we have gained the following additional insights: The firm may use umbrella branding and high-quality for its products, while, lacking the possibility to use umbrella branding, the firm would have chosen one high and one low quality product. Hence, even if the moral hazard problem is solved for one of the products in any case, umbrella branding may provide incentives to increase the quality of the other product. We have also shown that, under the umbrella brand, a firm may want to sell one of the products below costs. It may want to subsidize this product because umbrella branding enables consumers to correlate their beliefs. Producing high quality instead of low quality leads to a positive spill-over to the other product, which overcompensates for the loss made from selling the product at a loss. This implies that in comparison to the first best, there may be a social overprovision of quality under umbrella branding. Second, we have introduced detection probabilities for high *and* low quality in our symmetric setting. Our earlier results in the setting in which only low quality could be detected ($\delta_{\mathrm{H}} = 0$) carry over to the case $0 < \delta_{\mathrm{H}} < \delta_{\mathrm{L}}$. If $\delta_{\mathrm{H}} > \delta_{\mathrm{L}}$, all mixed-strategy equilibria with umbrella branding disappear. Nevertheless, whenever $\delta_{\mathrm{L}} > 0$ and $\delta_{\mathrm{H}} < 1$, there is a range of parameter values where the Pareto-dominant equilibrium involves umbrella branding with high-quality products, and our main result that umbrella branding improves the provision of quality is confirmed. In the remainder we shortly discuss three other possible extensions.

Uninformed Choices. In this paper we have assumed that the firm sells the same number of units in both periods. As a consequence, we have shown that for $v < c < 2\,v$, it would be socially optimal to produce high quality but that this can never be implemented by an equilibrium. As the relative importance of period 1 becomes smaller, the range of costs in which the socially optimal outcome cannot be implemented shrinks in size.

Market Segmentation. In this paper we have assumed that both products are bought by the same consumers. However, the benefit of umbrella branding crucially depends on the correlating of beliefs across products. Clearly correlated beliefs are irrelevant if consumers fall into two dichotomous groups, one which only buys product 1 and the other which only buys product 2, and if there is no communication between those groups. Provided that umbrella branding is costly, one therefore obtains that umbrella branding can only be profit enhancing if a sufficiently large share of consumers who buy a particular product take into account the experience (by oneself or by others) with the other product.[8]

Heterogeneous Consumer Information. It seems worthwhile to investigate the role of umbrella branding in an environment in which consumers possess heterogeneous information. Our model can be extended to include an additional group of consumers who observe quality before purchase – call them experts. The other group of consumers receives information as has been modelled in this paper – call them amateurs. Suppose that if the firm were able to discriminate between experts and amateurs it would provide high quality to experts and low quality to amateurs. When it cannot discriminate, the firm can decide to extract the full surplus from experts and produce high quality products. If the share of experts is sufficiently large, this is indeed an equilibrium strategy, and also amateurs buy the products at the high price, correctly believing that the products are of high quality. However, for a share of experts below a critical level the provision of high quality can only be supported as an equilibrium outcome with umbrella branding, but not with independent selling. Again this shows that umbrella branding mitigates the moral hazard problem.

The analysis and the extensions suggest that the model lends itself as a workhorse for further research.

[8]For a formal analysis of the importance of consumer overlap, see Cabral (2001).

A Appendix

A.1 Proof of Proposition 2

Complete Mixing. Here, we determine all equilibria in which the firm randomizes between all four options. Let λ_{HH}, λ_{HL}, λ_{LH}, λ_{LL} be the probabilities with which the firm chooses each of the four options, then $\lambda_{\mathrm{HH}} + \lambda_{\mathrm{HL}} + \lambda_{\mathrm{LH}} + \lambda_{\mathrm{LL}} = 1$. Then the expected quality of product 1 in $t = 1$ is $\lambda_{\mathrm{HH}}+\lambda_{\mathrm{HL}}$; that of product 2 is $\lambda_{\mathrm{HH}}+\lambda_{\mathrm{LH}}$. In $t = 2$, consumers update their beliefs according to the performance of the products in $t = 1$. If no product has defaulted, the expected quality of product 1 is

$$\frac{\lambda_{\mathrm{HH}} + (1-\delta)\,\lambda_{\mathrm{HL}}}{\lambda_{\mathrm{HH}} + (1-\delta)\,\lambda_{\mathrm{HL}} + (1-\delta)\,\lambda_{\mathrm{LH}} + (1-\delta)^2\,\lambda_{\mathrm{LL}}}.$$

If product 2 has defaulted, the expected quality of product 1 is updated as

$$\frac{\delta\,\lambda_{\mathrm{HL}}}{\delta\,\lambda_{\mathrm{HL}} + \delta\,(1-\delta)\,\lambda_{\mathrm{LL}}} = \frac{\lambda_{\mathrm{HL}}}{\lambda_{\mathrm{HL}} + (1-\delta)\,\lambda_{\mathrm{LL}}}.$$

If product 1 has defaulted, the expected quality drops to zero, no matter how product 2 has performed. Quality expectations for product 2 are updated analogously. Then depending on the quality choice of both products, expected profits are

$$\begin{aligned}
\Pi_{\mathrm{HH}} &= 2\,v\,(\lambda_{\mathrm{HH}} + \lambda_{\mathrm{HL}}) + 2\,v\,\frac{\lambda_{\mathrm{HH}} + (1-\delta)\,\lambda_{\mathrm{HL}}}{\lambda_{\mathrm{HH}} + 2\,(1-\delta)\,\lambda_{\mathrm{HL}} + (1-\delta)^2\,\lambda_{\mathrm{LL}}} - 2\,c,\\
\Pi_{\mathrm{HL}} &= 2\,v\,(\lambda_{\mathrm{HH}} + \lambda_{\mathrm{HL}}) + (1-\delta)\,2\,v\,\frac{\lambda_{\mathrm{HH}} + (1-\delta)\,\lambda_{\mathrm{HL}}}{\lambda_{\mathrm{HH}} + 2\,(1-\delta)\,\lambda_{\mathrm{HL}} + (1-\delta)^2\,\lambda_{\mathrm{LL}}}\\
&\quad + \delta\,v\,\frac{\lambda_{\mathrm{HL}}}{\lambda_{\mathrm{HL}} + (1-\delta)\,\lambda_{\mathrm{LL}}} - c,\\
\Pi_{\mathrm{LL}} &= 2\,v\,(\lambda_{\mathrm{HH}} + \lambda_{\mathrm{HL}}) + (1-\delta)\,2\,v\,\frac{\lambda_{\mathrm{HH}} + (1-\delta)\,\lambda_{\mathrm{HL}}}{\lambda_{\mathrm{HH}} + 2\,(1-\delta)\,\lambda_{\mathrm{HL}} + (1-\delta)^2\,\lambda_{\mathrm{LL}}}\\
&\quad + 2\,\delta\,(1-\delta)\,v\,\frac{\lambda_{\mathrm{HL}}}{\lambda_{\mathrm{HL}} + (1-\delta)\,\lambda_{\mathrm{LL}}}.
\end{aligned}$$

Considering that $\lambda_{\mathrm{LL}} = 1 - \lambda_{\mathrm{HH}} - 2\,\lambda_{\mathrm{HL}}$, we have the two determining equations $\Pi_{\mathrm{HH}} = \Pi_{\mathrm{LL}}$ and $\Pi_{\mathrm{HH}} = \Pi_{\mathrm{HL}}$ for the two endogenous variables λ_{HH} and λ_{HL}. Multiplying each equation with the denominators, we get two new equations of degree 2 for each variable. Algebraic geometry tells us that generically there are exactly four (possibly real) solutions. All four have rather simple expressions,

$$\begin{aligned}
&\lambda^1_{\mathrm{HH}} = \Big(\frac{c}{v-c}\,\frac{1-\delta}{\delta}\Big)^2, \quad \lambda^1_{\mathrm{HL}} = \frac{c}{v-c}\,\frac{1-\delta}{\delta}\,\frac{\delta\,v-c}{\delta\,(v-c)},\\
&\lambda^2_{\mathrm{HH}} = 1, \quad \lambda^2_{\mathrm{HL}} = 0, \quad \text{and}\\
&\lambda^{3,4}_{\mathrm{HH}} = -\frac{1-\delta}{\delta}, \quad \lambda^{3,4}_{\mathrm{HL}} = \frac{(1-\delta)^2}{\delta},
\end{aligned}$$

where the third solution is a double root. All four solutions are real. We can exclude the second and third solution: In both cases, one of the expected profits Π_{HH}, Π_{HL} or Π_{LL} is undefined. As a consequence, we are left with the first (unique) solution.

Note that in comparison with the mixed strategy equilibrium of the one product case, $\lambda^*_{\mathrm{HH}} = (\lambda^*)^2$ and $\lambda^*_{\mathrm{HL}} = \lambda^*\,(1-\lambda^*)$. We have "rediscovered" the uncorrelated equilibrium: Consumers believe that products are independent, and the firm randomizes product quality independently.

Case $\lambda_{\mathrm{HL}} = 0$. In this case,

$$\Pi_{\mathrm{HH}} = 2\,v\,\lambda_{\mathrm{HH}} + 2\,v\,\frac{\lambda_{\mathrm{HH}}}{\lambda_{\mathrm{HH}} + (1-\delta)^2\,\lambda_{\mathrm{LL}}} - 2\,c,$$

$$\Pi_{\mathrm{HL}} = \Pi_{\mathrm{LH}} = 2\,v\,\lambda_{\mathrm{HH}} + 2\,v\,(1-\delta)\,\frac{\lambda_{\mathrm{HH}}}{\lambda_{\mathrm{HH}} + (1-\delta)^2\,\lambda_{\mathrm{LL}}} - c,$$

$$\Pi_{\mathrm{LL}} = 2\,v\,\lambda_{\mathrm{HH}} + 2\,v\,(1-\delta)^2\,\frac{\lambda_{\mathrm{HH}}}{\lambda_{\mathrm{HH}} + (1-\delta)^2\,\lambda_{\mathrm{LL}}}.$$

Considering that $\lambda_{\mathrm{LL}} = 1 - \lambda_{\mathrm{LL}}$, we get $\Pi_{\mathrm{HH}} = \Pi_{\mathrm{LL}}$ iff

$$\lambda^*_{\mathrm{HH}} = \frac{c}{v-c}\,\frac{(1-\delta)^2}{\delta\,(2-\delta)}.$$

For this value of λ^*_{HH}, we get $\Pi_{\mathrm{HH}} - \Pi_{\mathrm{HL}} = c\,\delta/(2-\delta) > 0$, hence we have indeed found an informative equilibrium where products are always made in the same quality.

Case $\lambda_{\mathrm{LL}} = 0$. Assuming $\lambda_{\mathrm{LL}} = 0$ and looking for a solution to $\Pi_{\mathrm{HH}} = \Pi_{\mathrm{HL}}$, we get

$$\lambda_{\mathrm{HL}} = \frac{\delta\,v - c}{2\,\delta\,(v-c)}.$$

For this solution, we can check that $\Pi_{\mathrm{LL}} \geq \Pi_{\mathrm{HL}}$ holds whenever $c \geq v\,\delta$. However, in this case λ_{HL} becomes negative. To sum up, there is no equilibrium with $\Pi_{\mathrm{LL}} < \Pi_{\mathrm{HL}} = \Pi_{\mathrm{HH}}$ because whenever $\Pi_{\mathrm{HL}} = \Pi_{\mathrm{HH}}$, the corresponding probabilities of options are between zero and one only if $\Pi_{\mathrm{LL}} > \Pi_{\mathrm{HH}}$. The argument for the case $\lambda_{\mathrm{HH}} = 0$ is analogous, but algebraically more involved.

There are three more cases: $\lambda_{\mathrm{HH}} = 1$, $\lambda_{\mathrm{LL}} = 1$ and $\lambda_{\mathrm{HL}} = 1/2$. The first two cases have already been discussed in the main text. The third case is $\lambda_{\mathrm{HL}} = 1/2$, and as a result $\lambda_{\mathrm{HH}} = \lambda_{\mathrm{LL}} = 0$. Expected profits are $\Pi_{\mathrm{HH}} = v + v - 2\,c$, $\Pi_{\mathrm{HL}} = v + v - c$, and $\Pi_{\mathrm{LL}} = v + (1-\delta)^2\,v$. Clearly, we have $\Pi_{\mathrm{HL}} > \Pi_{\mathrm{HH}}$. Furthermore, $\Pi_{\mathrm{HL}} \geq \Pi_{\mathrm{LL}}$ iff $c \leq \delta^2\,v$. Hence there is an equilibrium where consumers believe that either product 1 or product 2 is made in high quality; the strategy is indeed optimal. ■

A.2 Proof of Remark 4

Part 1: Non-existence of mixed-strategy equilibria with $\lambda_{\mathrm{HL}} = \lambda_{\mathrm{LH}} = 0$. Note that $\lambda^*_{\mathrm{HH}} \in (0;\ 1)$ iff c is between $v\,\delta_{\mathrm{H}}\,(2-\delta_{\mathrm{H}})$ and $v\,\delta_{\mathrm{L}}\,(2-\delta_{\mathrm{L}})$. As a result,

c is larger than $v\,\delta_{\mathrm{L}}\,\delta_{\mathrm{H}}$. To prove that (5) actually constitutes an equilibrium, we must still check whether $\Pi_{\mathrm{HL}} \leq \Pi_{\mathrm{HH}}$ holds for this quality choice. For $\lambda_{\mathrm{HH}} = \lambda^*_{\mathrm{HH}}$, we get

$$\Pi_{\mathrm{HH}} - \Pi_{\mathrm{HL}} = \frac{(\delta_{\mathrm{L}} - \delta_{\mathrm{H}})\,(c - v\,\delta_{\mathrm{L}}\,\delta_{\mathrm{H}})}{2 - \delta_{\mathrm{L}} - \delta_{\mathrm{H}}}.$$

Provided that $c > v\,\delta_{\mathrm{L}}\,\delta_{\mathrm{H}}$, this expression is positive if and only if $\delta_{\mathrm{L}} > \delta_{\mathrm{H}}$.

Part 2: Non-existence of other positively correlated mixed-strategy equilibria. We must ask whether there are other mixed-strategy equilibria in the region $c \in \big(v\,(2-\delta_{\mathrm{H}})\,\delta_{\mathrm{L}};\ v\,(2-\delta_{\mathrm{L}})\,\delta_{\mathrm{H}}\big)$. The answer is negative: Assume first that c is slightly larger than $v\,(2-\delta_{\mathrm{H}})\,\delta_{\mathrm{L}}$. Then, given the belief that only HH is played, HH is (slightly) dominated by HL, whereas LL is still suboptimal. For continuity reasons, the firm may mix between HH and HL, putting a high probability on HH. In this case, we have

$$\begin{aligned}
\Pi_{\mathrm{HH}} &\approx 2\,v + 2\,v - 2\,c,\\
\Pi_{\mathrm{HL}} &\approx 2\,v + 2\,v\,(1-\delta_{\mathrm{L}}) + v\,\delta_{\mathrm{L}} - c,\\
\Pi_{\mathrm{LL}} &\approx 2\,v + 2\,v\,(1-\delta_{\mathrm{L}})^2 + v\,(2-\delta_{\mathrm{L}})\,\delta_{\mathrm{L}}.
\end{aligned}$$

For $c \approx v\,(2-\delta_{\mathrm{H}})\,\delta_{\mathrm{L}}$, we find that $\Pi_{\mathrm{HL}} - \Pi_{\mathrm{LL}} \approx -v\,\delta_{\mathrm{L}}\,(1-\delta_{\mathrm{H}}+\delta_{\mathrm{L}}) < 0$. The firm optimally deviates to LL, hence a mix between HH and HL cannot be an equilibrium. Let us provide some intuition. For $c \leq v\,(2-\delta_{\mathrm{H}})\,\delta_{\mathrm{L}}$, consumers may consistently believe that all firms play HH, and, if they observe low quality for one product, infer that also the other product is of low quality. If consumers believe that firms mix between HH and HL (even if the probability of HL is low), they must infer *high* quality for the other product if they observe low quality for one product. Hence at $c \leq v\,(2-\delta_{\mathrm{H}})\,\delta_{\mathrm{L}}$, there is a discontinuity of beliefs if $\delta_{\mathrm{H}} > \delta_{\mathrm{L}}$.

The analogue applies if c is slightly smaller than $v\,(2-\delta_{\mathrm{L}})\,\delta_{\mathrm{H}}$. A continuity argument requires that firms mix between LL and HL, putting only a small probability on HL. Therefore,

$$\begin{aligned}
\Pi_{\mathrm{HH}} &\approx v\,\delta_{\mathrm{H}}\,(2-\delta_{\mathrm{H}}) + 2\,v\,\delta_{\mathrm{H}}^2 - 2\,c,\\
\Pi_{\mathrm{HL}} &\approx v\,\delta_{\mathrm{H}} - c,\\
\Pi_{\mathrm{LL}} &\approx 0.
\end{aligned}$$

Now if $c \approx v\,(2-\delta_{\mathrm{L}})\,\delta_{\mathrm{H}}$, we get $\Pi_{\mathrm{HL}} - \Pi_{\mathrm{LL}} \approx -v\,\delta_{\mathrm{H}}\,(1-\delta_{\mathrm{L}}) < 0$. Firms deviate to playing *only* LL. To give some intuition, as long as $c \geq v\,(2-\delta_{\mathrm{L}})\,\delta_{\mathrm{H}}$, consumers may consistently believe that firms produce LL only, and if they observe high quality infer that also the other product is good. For $c < v\,(2-\delta_{\mathrm{L}})\,\delta_{\mathrm{H}}$, this belief would induce firms to produce HL. Now consumers, observing high quality for one product, would have to infer low quality for the other. Under these circumstances, firms choose LL. The way out of this apparent contradiction is the belief that firms treat their products independently and produce low quality only (the gray line in Figure 5). ■

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