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Abstract

In this paper, I compare two different approaches to model implicit contracting, the infinite-horizon approach typically used in the literature and a finite-horizon approach building on an adverse-selection model. I demonstrate that even the most convincing result of the infinite-horizon approach, namely that implicit contracting is improved, if the discount rate is lowered, does not carry over to the alternative modeling approach. Predictions of the first approach should therefore be handled with care and subject to a thorough reinvestigation.

Key words: Trust, finite horizon, infinite horizon, discounting, implicit contracting

JEL classification: D82, D83, J33, M52

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1 Introduction

Economists agree in that implicit contracts play an important role in real-world economic life. As explicit contracts are mostly incomplete, implicit agreements are oftentimes used to fill the resulting gaps. Examples are ubiquitous: Baker et al. (1994) report on firms tying their employees' remunerations to subjective evaluations that are not verifiable by third parties (e.g. a court).[1] Similarly, Holmström & Roberts (1998) describe the pattern of relations between Japanese manufacturing firms and their suppliers. This pattern is characterized by long-term, close relations that substitute for ownership in protecting specific assets.

Driven by the wish to understand implicit contracts and their properties, economists have tried to incorporate these contracts into their models.[2] At the core of these models lies the notion of trust. As implicit contracts are not enforceable by a court, the parties have to trust each other that the contract will not be reneged on. Trust is usually modeled by considering an infinitely repeated game, in which unworthy behavior of a party is punished by the other parties in form of an ultimate switch to the stage-game Nash-equilibrium strategy. As I discuss in a companion paper (Gürtler (2005)), this modeling approach is - besides its technical limitations - problematic since

[1] See also Gibbons (2005).

[2] There are many examples: Bull (1987) considers implicit contracts in a labor-market context. Baker et al. (1994), Schmidt & Schnitzer (1995) and Pearce & Stacchetti (1998) analyze the interaction of explicit and implicit contracts. Finally, Garvey (1995), Baker et al. (2002) and Halonen (2002) introduce implicit contracts into the property rights theory of the firm.

it cannot account for many real-world behavioral patterns. For instance, it cannot capture any evolution of trust. On the equilibrium path, all parties either always trust each other or they never do so. More realistic is a setting, where trust evolves dependent on past behavior. A party should permanently use incoming information to update its belief concerning the trustworthiness of other parties.

In this paper, I therefore elaborate a different trust modeling approach, which was touched by Hart & Holmström (1987) and which is in spirit similar to Kreps et al. (1982). A two-period principal-agent relationship is considered, where a fraction of all principals in the economy is assumed to be reliable. Reliable means that these principals always stick to their promises, i.e. that they always honor implicit agreements.[3] Unreliable principals, on the other hand, simply act in a profit-maximizing way. Hence, they will honor implicit agreements, only if it pays off for them. Agents are assumed to be unable to distinguish between principal types, this means, there is some kind of adverse-selection problem.

The two modeling approaches differ in one important aspect: In the infinite-horizon approach, agents know that all principals are unreliable. Contract negotiations therefore take place between agents and unreliable principals. In the finite-horizon approach, even unreliable principals pretend to be reliable; otherwise they could never enter any implicit agreement. As a

[3] This might be due to psychic costs they would incur, if exploiting an implicit agreement. See, for similar argumentations, e.g. Frank (1987), Huang & Wu (1994), Huck (1998), James Jr. (2002) or Sliwka (2003).

result, each principal has to act as a reliable one so that contract negotiations are in fact between agents and reliable principals. This difference partly leads to very different results. Most importantly, in the infinite-horizon approach an increase in the discount rate always leads to a worsening in implicit contracting in the sense that the achievable surplus (weakly) decreases. The reason is that a higher discount rate makes it less worthwhile to honor implicit agreements so that these are less easily sustainable. In the finite-horizon approach, on the other hand, implicit contracting might be improved, as the discount rate increases. The intuition for this result is as follows: A pooling equilibrium, that is, an equilibrium, where both types of principal honor the implicit agreement in the first period, entails benefits and costs for the reliable principals. In the first period, agents know that the implicit contract is never reneged on. Thus, given a contract offer, they choose more favorable actions than they would choose, if only reliable principals were expected to honor the agreement. This benefits the reliable principals. On the other hand, in a pooling equilibrium reliable principals miss the chance to separate from the unreliable ones so that pooling in the first period is costly for second-period contracting. An increase in the discount rate yields a decrease in these costs, and, as a result, a pooling equilibrium may become more likely. Furthermore, as, in such an equilibrium, more implicit agreements are honored in the first period and, in the second period, more implicit agreements are entered, the achievable surplus may increase in both periods.

To summarize, this paper demonstrates that the two trust modeling approaches may lead to very different predictions. A consequence is that conclusions that have been derived from the infinite-horizon approach have to be handled with care. It should be confirmed whether or not these conclusions hold under the alternative modeling approach.

A second contribution of this paper is that it explains the relation between the fraction of reliable principals in an economy and the structure of implicit contracting, which should be useful in explaining differences in contractual agreements in different societies. Two countervailing effects can, in this context, be isolated. First, as the percentage of reliable principals gets lower, it becomes more important for the reliable principals to pool with the unreliable ones. Second, however, it also becomes more difficult to induce unreliable principals to honor the first-period agreement, i.e. to actually achieve a pooling equilibrium. Both effects may be dominant so that a decrease in the percentage of reliable people may make a pooling equilibrium more or less likely.

Besides its implications concerning the modeling of trust, this paper contributes to the career concerns literature. As the present paper, work on career concerns (see e.g. Dewatripont et al. (1999a, b), Holmström (1999) or Fingleton & Raith (2005)) emphasizes the fact that people often choose actions in order to influence market perceptions about certain of their characteristics.

The paper is organized as follows: Section 2 presents, as a benchmark case, the infinite-horizon trust-modeling approach. Section 3 introduces its finite-horizon counterpart and contains the main results of the paper. Section 4 contains a concluding discussion.

2 The infinite-horizon approach

2.1 The stage game

Consider a relationship between a principal (she) and an agent (he), both being risk-neutral and unlimitedly liable. The agent chooses effort e at cost $C(e) = 0.5ce^2$, with $c > 0$, in order to produce output $y = e$ that completely accrues to the principal. The surplus to be realized is therefore given by $S = e - 0.5ce^2$ so that the first-best effort, that is, the effort maximizing S, is $e_{fb} = \frac{1}{c}$ (and the corresponding surplus $S_{fb} = \frac{1}{2c}$). Effort and, accordingly, output are assumed to be observable by the principal, but unverifiable to third parties (e.g. a court).[4] This assumption should be fulfilled in many practical settings, where the contribution of agents to firm value is impossible to assess. Further, we assume that there is no objective measure that might allow inference of the agent's effort. Hence, the principal cannot use explicit incentives to motivate the agent so that implicit contracts are the only thinkable incentive device.

In the market, there are only a few principals, but many agents. As a result, in negotiations, principals are supposed to have the complete bargaining power. Thus, a principal only has to make sure that the agent receives an expected payment equal to or higher than his reservation utility, which is normalized to zero.

[4]Note that the results to be derived in this paper do not depend on the assumption that output and effort are the same. As long as effort is observable by the principal, they would continue to hold, if output was given by $y = f(e, \varepsilon)$, with ε denoting some random components.

The solution to the stage game is very simple. The agent never exerts positive effort and the principal never pays anything to the agent. This solution can be derived by backward induction. If the principal promised the agent a reward for choosing positive effort, she would ex post, i.e. after realization of output, claim that effort was zero and deny the reward. The agent would anticipate the principal's reaction and choose zero effort. If, conversely, the principal would pay the agent a wage before the latter chose effort, optimal effort would be zero. Again, this is anticipated by the principal who then chooses a zero wage.

In order to enable a more satisfying solution, we have to make implicit agreements feasible. The most prominent approach is to consider the infinitely repeated version of the analyzed stage game. This is what we do next.

2.2 The infinitely repeated version of the stage game

To derive the subgame perfect equilibrium of the infinitely repeated version of the stage game, some additional assumptions have to be made. First, we assume that payoffs are discounted at a rate r. That is, a payoff of 1 unit in the next period is worth $\delta := \frac{1}{1+r} \in (0,1)$ units in the present one. The discount rate may account for the interest rate, at which the parties can lend or borrow money or simply for the parties' impatience.

Due to Levin (2003), we can focus on stationary contracts, i.e. contracts, where the players' actions do not change from period to period. Without loss of generality, we then concentrate on per-period contracts of the form

$(s, b, \hat{e})$. In words, at the beginning of each period, the principal pays the agent a fixed wage s. Further, she promises to pay a bonus b, if effort is at least as high as $\hat{e}$. The agent then chooses his effort, and, thereafter, output is realized. Finally, the principal decides on whether or not to pay the promised bonus.[5]

The parties are assumed to employ a Grim-trigger strategy. Roughly speaking, they start by cooperating and continue cooperation, unless one party defects. In this case, they refuse cooperation forever after. Although these strategies have technical shortcomings (they are e.g. not renegotiation-proof)[6], they are - partly due to their simplicity - very prominent and mainly used in the literature on implicit contracting.

We are now able to solve the model. To do so, consider an arbitrary period and start by assuming that the agent trusts the principal, i.e. the agent believes that an effort choice of at least $\hat{e}$ is rewarded by a bonus

[5] Notice that the time structure cannot be changed such that the principal acts before the agent. This follows from the assumption that the agent has no bargaining power and, accordingly, no incentive to honor the implicit agreement.

[6] The problem that, under the Grim-Trigger strategies, there is scope for renegotiation after a deviation of one party can be solved as follows (see e.g. the textbook by Bolton & Dewatripont (2005), p. 467): Instead of playing the stage-game Nash equilibrium in the punishment phase, the parties could play jointly efficient punishments, but change the division of the surplus after a deviation. In particular, the division should be changed such that the deviating party receives exactly the same payoff as in the Nash-equilibrium of the stage game. Note that the results to be derived remain the same, if we assume the parties to follow this second type of strategy.

payment. He then chooses effort equal to

$$e = \begin{cases} \hat{e}, \text{ if } b \geq 0.5c\hat{e}^2 \\ 0, \text{ otherwise} \end{cases} \tag{1}$$

It directly follows that the principal will never set the bonus lower than the costs entailed by effort, as, in this case, no effort could be induced. Note further that it is also weakly dominated to set $b > 0.5c\hat{e}^2$. Compared to the choice of $b = 0.5c\hat{e}^2$, the reneging temptation of the principal increases, while no further effort is induced. Hence, we can restrict attention to the case, where $b = 0.5c\hat{e}^2$ so that the agent's expected utility becomes $EU = s$. As, in the optimum, the participation constraint of the agent is binding, the fixed wage equals zero yielding the following per-period profit for the principal:

$$\pi = \sqrt{\frac{2b}{c}} - b \tag{2}$$

The principal wants to choose b so as to maximize this profit. Thereby, however, she has to consider a non-reneging constraint. This constraint ensures that, after observation of effort, the principal does not refuse to pay the bonus. Otherwise, the agent would anticipate this refusal, and the implicit contract was completely worthless. The non-reneging constraint says that the principal's gain from not paying the bonus, namely b, must not exceed the corresponding loss. The latter is given by the present value of the loss in future profits: As the agent refuses to cooperate forever after, implicit contracts would no longer be feasible so that future profits were all zero. Hence, the non-reneging constraint can be written as

$$b \leq \frac{1}{r}\left(\sqrt{\frac{2b}{c}} - b\right) \tag{3}$$

The principal maximizes (2) subject to constraint (3). The Lagrangian to this maximization problem is $L = \sqrt{\frac{2b}{c}} - b + \lambda \left(br - \sqrt{\frac{2b}{c}} + b \right)$. The Kuhn-Tucker conditions are given by (3) and

$$(1-\lambda)\left(0.5\sqrt{\frac{2}{bc}} - 1\right) + \lambda r = 0 \tag{4}$$

The optimal solution depends on whether or not (3) is binding. If it is not ($\lambda = 0$), $b = \frac{1}{2c}$ so that the first-best solution will be achieved. If, on the other hand, $\lambda > 0$, a second-best solution might be possible, where the optimal bonus can be derived from the binding version of condition (3). It is the maximal value for b solving $br - \sqrt{\frac{2b}{c}} + b = 0$, which is $b = \frac{2}{c(1+r)^2}$. To summarize, the optimal implicit bonus equals

$$b = \left\{ \begin{array}{c} \frac{1}{2c}, \text{ if } \ r \leq 1 \\ \frac{2}{c(1+r)^2}, \text{ if } \ r > 1 \end{array} \right\} \tag{5}$$

Note that, under the assumptions made, the surplus is equal to the principal's profit and given by

$$S = \pi = \left\{ \begin{array}{c} \frac{1}{2c}, \text{ if } \ r \leq 1 \\ \frac{2r}{c(1+r)^2}, \text{ if } \ r > 1 \end{array} \right\} \tag{6}$$

From (6), it is straightforward to derive the subsequent proposition:

Proposition 1 *The optimal surplus S is (weakly) decreasing in r.*

Proof. If $r \leq 1$, S is independent of r. For $r = 1$, the two expressions for the surplus are the same, namely $\frac{1}{2c}$. To see what happens, if, then, r increases, differentiate $\frac{2r}{c(1+r)^2}$ with respect to r. This yields $\frac{2(1-r)}{c(1+r)^3}$, which is negative for $r > 1$. ■

Proposition 1 has a very simple intuition.[7] If r increases, the value of future transactions with the agent decreases for the principal. An increase in r yields a decrease in the value of future profits, as these are discounted more heavily. As a consequence, it becomes less worthwhile for the principal to honor the implicit agreement since the punishment for reneging on the implicit bonus decreases. The implicit bonus to be sustainable thus (weakly) decreases implying that the agent's effort decreases as well. As the effort is never chosen inefficiently high, this implies that the expected surplus gets lower, too.

Although this result seems very intuitive, we will show in the following section that it depends on the used modeling approach. The finite-horizon approach we consider next, might lead to quite different findings.

3 The finite-horizon approach

We now consider a setting, where the stage-game from Section 2.1 is repeated once, i.e. we have two periods $t = 1, 2$. Additionally, we assume that a fraction θ of all principals in the economy is reliable. Reliable means that these principals always stick to their promises, hence they never break any implicit agreement. As mentioned before, this might be due to psychic costs they would incur, if doing so. The remaining principals as well as all agents in the economy are supposed to not incur such costs. Note that an implicit agreement again requires the agents to act before the principals, as, otherwise, the

[7] Notice that the argumentation in this paragraph is very similar to the idea, which is behind the folk theorem.

agents would refuse to stick to their promise. A contract again consists of a triple $(s_t, b_t, \hat{e}_t)$, with s_t denoting a fixed payment from principal to agent and b_t an implicit bonus to be paid, if the agent has chosen effort of at least $\hat{e}_t$.

Deriving the model solution, we work backwards and start with period 2. As the game ends after period 2, unreliable principals will always refuse to pay the bonus b_2, while reliable ones stick to their promise, and so pay the agent the agreed upon bonus. At the beginning of the second period, the agent may still not know his principal's type. Therefore, let Q_2 denote the agent's probability assessment that he works together with a reliable principal. In other words, the agent believes that, with probability Q_2, a choice of effort weakly exceeding $\hat{e}_2$ is followed by the bonus payment b_2. Being offered a contract $(s_2, b_2, \hat{e}_2)$, the agent hence chooses effort equal to

$$e_2 = \begin{cases} \hat{e}_2, \text{ if } Q_2 b_2 \geq 0.5c\left(\hat{e}_2\right)^2 \\ 0, \text{ otherwise} \end{cases} \tag{7}$$

It directly follows that any bonus below $\frac{0.5c(\hat{e}_2)^2}{Q_2}$ cannot be optimal for the principal, as no incentives would be induced by such a bonus. Bearing this in mind, the agent's expected utility is given by $EU_2 = s_2 + Q_2 b_2 - 0.5c\left(\hat{e}_2\right)^2$. Again, principals are assumed to have complete bargaining power and make a take-it-or-leave-it offer so that $s_2 = -Q_2 b_2 + 0.5c\left(\hat{e}_2\right)^2$. Note that, at the beginning of the second period, each principal has an interest to claim to be of the reliable type. Otherwise, she could not enter any implicit agreement. This implies that, although the agent might not be sure that he works together with a reliable principal, contract negotiations are between an agent and a principal who acts like a reliable one. Stated differently, although an unreliable principal would like to offer a different contract than a reliable one, she cannot, as this would reveal her type. Formally, the optimal contract therefore solves

$$\begin{aligned} \underset{b_2,\hat{e}_2}{Max}\ \pi_{2r} &= \hat{e}_2 - 0.5c\left(\hat{e}_2\right)^2 - \left(1 - Q_2\right) b_2 \\ s.t.\ Q_2 b_2 &\geq 0.5c\left(\hat{e}_2\right)^2 \end{aligned} \tag{8}$$

, where π_{2r} denotes the second-period profit of a reliable principal. One can easily see that, for $Q_2 < 1$, the incentive compatibility constraint $Q_2 b_2 \geq 0.5c\left(\hat{e}_2\right)^2$ is always binding in the optimum. For $Q_2 = 1$, it may or may not be binding. In the latter case, the agent is sure that he works with a reliable principal. Then, we always obtain the first-best solution. The principal promises to compensate the agent for the costs, entailed by effort and she can either do this by choosing the minimum bonus $b_2 = 0.5c\left(\hat{e}_2\right)^2$ together with the maximum fixed wage $s_2 = 0$ or by increasing the bonus and decreasing the fixed wage by the same amount. When continuing with our maximization problem, we can therefore assume that $Q_2 b_2 = 0.5c\left(\hat{e}_2\right)^2$, as, for any $Q_2 \in [0, 1]$, there exists an optimum, where this condition holds. The principal's maximization-problem then simplifies to

$$\underset{\hat{e}_2}{Max}\ \pi_{2r} = \hat{e}_2 - \frac{0.5c\left(\hat{e}_2\right)^2}{Q_2} \tag{9}$$

which leads to the solution

$$\hat{e}_2 = \frac{Q_2}{c},\ b_2 = \frac{Q_2}{2c} \tag{10}$$

Note that, for $Q_2 < 1$, $b_2 < \frac{1}{2c}$ and $\hat{e}_2 < \frac{1}{c}$ so that both, the bonus and the agent's effort, are inefficiently low. The reason is the following: If the agent is in doubt about the type of the principal he works with, he will not choose effort as high as he would in case he knew for sure that the principal was reliable. Implicit incentives offered by a reliable principal are then not as effective as they were, could the principal reveal her type. Yet, a reliable principal always sticks to her promise and hence bears the full bonus costs. This situation has the same structure as a free-rider problem and leads to inefficiently low bonus choices. Clearly, b_2 is increasing in Q_2. The more the agent thinks he is dealing with a reliable principal, the higher will be the bonus and, accordingly, the agent's effort, as trustworthy principals get more for what they pay for.

Let us conclude the second period by calculating the principals' second-period profits.[8] These are as follows:[9]

$$\pi_{2r} = \frac{Q_2}{2c} \tag{11}$$

$$\pi_{2u} = \frac{Q_2}{c} \tag{12}$$

Note that both, π_{2r} as well as π_{2u}, are increasing in Q_2. As mentioned before, reliable principals are better off being identified as reliable since they can set incentives more efficiently. On the other hand, unreliable principals gain by fooling the agents and pretending to be reliable, for they can then install implicit incentives, which they are not going to pay for. In the first period, there may thus be some kind of conflict between both types of principals. Unreliable principals may want to signal to be reliable, while reliable princi-

[8]The second-period surplus may differ from second-period profit. We derive this surplus explicitly after having analyzed the first period.

[9]π_{2u} denotes the second-period profit of an unreliable principal.

pals may want to prevent this. To explore the implications of this interest conflict more clearly, we next turn to period 1.

In the first period, after effort observation, reliable principals stick to their promise and pay the bonus b_1 as determined in the implicit contract. Unreliable principals, however, will only stick to their promise, if the involved gain in reputation outweighs the bonus costs. We assume, in this context, that principals who pay their agent a bonus lower than the one agreed upon, are stigmatized as being unreliable.[10] Unreliable principals therefore decide between paying the bonus b_1 or refusing to pay anything at all. In the first case, the agents do not learn anything new about the principals so that $Q_2 = \theta$. This case is denoted as a pooling equilibrium. In the latter case, we have a separating equilibrium (as the agents perfectly learn the principal types), where $Q_2 = 1$ for reliable principals and $Q_2 = 0$ for unreliable ones. Unreliable principals will thus fulfill the implicit agreement, if and only if the following condition holds:[11]

$$b_1 \leq \frac{1}{1+r}\frac{\theta}{c} =: X \tag{13}$$

The agent's optimal effort choice is obtained from (7) by replacing the subscript 2 by the subscript 1. Here, Q_1 does not denote the probability that the agent faces a reliable principal, but the probability that he faces a principal

[10] Recall our assumption that reliable principals always stick to their promise. It is therefore natural to stigmatize principals as unreliable, if they deviate from the promised bonus. Moreover, note that the results are not sensitive to the assumption that principals paying more than the promised bonus are not stigmatized as being unreliable.

[11] We implicitly assume that an unreliable principal will honor the first-period agreement, if he is indifferent between doing and not doing so.

that will honor the first-period implicit agreement. As indicated in the preceding discussion, even unreliable principals may find it worthwhile to honor implicit agreements in the first period.

In contract negotiations, the same argumentation as in the second period applies. Again, all principals pretend to be reliable so that bargaining is factually between agents and reliable principals. Note that therefore reliable principals decide on whether or not to pool with their unreliable counterparts. Before presenting the solution explicitly, it is therefore convenient to say something about the benefits and costs of a pooling equilibrium for the reliable principals. A pooling equilibrium (compared to a separating equilibrium) improves contracting in the first period, but worsens the situation for reliable principals in the second. The reason is very simple. In a pooling equilibrium, agents know that all first-period agreements are honored, whereas, in a separating equilibrium, the fraction $1-\theta$ of all implicit agreements is reneged on. In the second period, we have a reverse argumentation. In a separating equilibrium, agents know the type of the principal they are dealing with. Reliable principals are thus able to implement the first-best solution. In a pooling equilibrium, on the other hand, reliable principals are worse off since agents know that only with probability θ the implicit agreement is honored.

Summarizing, a pooling equilibrium is, for reliable principals, accompanied by probability assessments $(Q_1 = 1, Q_2 = \theta)$, while the corresponding probabilities in a separating equilibrium are $(Q_1 = \theta, Q_2 = 1)$. It is further important to note that first-period contracting might be deteriorated, as condition (13) has to be taken into account. If reliable principals want to achieve a pooling equilibrium, they will have to make sure that it is in the interest of unreliable principals to honor the implicit agreement. This may affect the optimal contract choice.

Let us now solve the model explicitly. Consider first the case of a pooling equilibrium. Here, the maximization problem is given by[12]

$$\begin{aligned} \underset{b_1,\hat{e}_1}{Max}\ \pi_{1r,P} &= \hat{e}_1 - 0.5c\left(\hat{e}_1\right)^2 \\ s.t.\ b_1 &\geq 0.5c\left(\hat{e}_1\right)^2, \\ b_1 &\leq X \end{aligned} \tag{14}$$

It is not restrictive at all to replace the condition $b_1 \geq 0.5c\left(\hat{e}_1\right)^2$ by its binding version $b_1 = 0.5c\left(\hat{e}_1\right)^2$. A bonus b_1 higher than $0.5c\left(\hat{e}_1\right)^2$ would make the fulfillment of the pooling constraint (13) less likely and is therefore (weakly) dominated. It follows that the solution to the maximization problem only depends on whether or not (13) is binding. The achievable profit in either case is

$$\pi_{1r,P} = \begin{cases} \frac{1}{2c}, \text{ if } \frac{1}{2c} \leq X \\ \frac{\sqrt{2\theta}}{c\sqrt{1+r}} - \frac{\theta}{c(1+r)}, \text{ otherwise} \end{cases} \tag{15}$$

In words, in a pooling equilibrium, the first-best solution is feasible, if, under the first-best bonus $b_1 = \frac{1}{2c}$, the pooling constraint is fulfilled. Otherwise, the bonus is given by $b_1 = \frac{1}{1+r}\frac{\theta}{c}$ leading to profit $\pi_{1r,p} = \frac{\sqrt{2\theta}}{c\sqrt{1+r}} - \frac{\theta}{c(1+r)}$.

The analysis in the case of a separating equilibrium is similar. Here, the

[12] The second entry in the subscript accompanying π indicates, whether the case of a pooling (P) or separating (S) equilibrium is considered.

maximization problem is given by

$$\begin{aligned}
\underset{b_1,\hat{e}_1}{Max}\ \pi_{1r,S} &= \hat{e}_1 - 0.5c\left(\hat{e}_1\right)^2 - (1-\theta)\,b_1 \qquad (16)\\
s.t.\ \theta b_1 &\geq 0.5c\left(\hat{e}_1\right)^2,\\
b_1 &> X
\end{aligned}$$

In a separating equilibrium, the condition $\theta b_1 \geq 0.5c\left(\hat{e}_1\right)^2$ need not be binding. If the separating condition requires a bonus that, together with $\theta b_1 = 0.5c\left(\hat{e}_1\right)^2$, would lead to inefficiently high effort, then it would be better to make the condition $\theta b_1 \geq 0.5c\left(\hat{e}_1\right)^2$ slack.[13] Therefore, the achievable profit depending on the parameter values is[14]

$$\pi_{1r,S} = \begin{cases} \frac{\theta}{2c}, \text{ if } \frac{\theta}{2c} > X \\ \frac{\sqrt{2\theta}}{c\sqrt{1+r}} - \frac{\theta}{c(1+r)}, \text{ if } \frac{\theta}{2c} \leq X \leq \frac{1}{2c\theta} \\ \frac{1}{2c} - \frac{(1-\theta)\theta}{(1+r)c}, \text{ otherwise} \end{cases} \qquad (17)$$

It remains to be shown, under what circumstances the reliable principals select a pooling equilibrium and when they prefer to play a separating equilibrium. The following proposition gives an answer to this question:

Proposition 2 *There will be a pooling equilibrium, if either (i)* $\sqrt{2\theta\,(1+r)} - 0.5 - \theta\,(1+0.5r) > 0$ *and* $r > 1$*, (ii)* $\sqrt{2\,(1+r)}\left(\sqrt{\theta}-\theta\right) - 0.5\,(1-\theta) > 0$ *and* $2\theta - 1 < r \leq 1$ *or (iii)* $r \leq 2\theta - 1$.

[13] Note that, in the opposite case of a pooling equilibrium, effort is never chosen inefficiently high, as the pooling constraint imposes a downward pressure on the bonus to be installed.

[14] In case $\frac{\theta}{2c} \leq X$, the reliable principals have to determine the bonus slightly above X in order to separate from the reliable ones. To simplify calculations, however, we assume $b_1 = X$, when determining the profit. All results to be derived should qualitatively be the same, if this bonus would be replaced by $b_1 = X + \kappa$, with $\kappa > 0$ and $\kappa \to 0$.

Proof. Depending on the parameter constellations, there may arise four different cases: In case (i), $X < \frac{\theta}{2c}$, in the second case, $\frac{\theta}{2c} \leq X < \frac{1}{2c}$, in case (iii) $\frac{1}{2c} \leq X \leq \frac{1}{2c\theta}$ and in case (iv), $X > \frac{1}{2c\theta}$. In the first case, the present value of the two pooling profits for a reliable principal is given by[15]

$$PV_{P,i} = \frac{\sqrt{2\theta}}{c\sqrt{1+r}} - \frac{\theta}{c\,(1+r)} + \frac{1}{1+r}\frac{\theta}{2c}$$

, while the corresponding value in the case of separation is given by

$$PV_{S,i} = \frac{\theta}{2c} + \frac{1}{1+r}\frac{1}{2c}$$

Pooling will be optimal, if the former value exceeds the latter resulting in $\sqrt{2\theta\,(1+r)} - 0.5 - \theta\,(1+0.5r) > 0$. Note further that the condition $X < \frac{\theta}{2c}$ is equivalent to $r > 1$. In the second case, the pooling profits are the same as before, whereas the present value of the separating profits changes to

$$PV_{S,ii} = \frac{\sqrt{2\theta}}{c\sqrt{1+r}} - \frac{\theta}{c\,(1+r)} + \frac{1}{1+r}\frac{1}{2c}$$

Pooling will then be preferred, if $\frac{\sqrt{2\theta}}{c\sqrt{1+r}} - \frac{\theta}{c(1+r)} + \frac{1}{1+r}\frac{\theta}{2c} > \frac{\sqrt{2\theta}}{c\sqrt{1+r}} - \frac{\theta}{c(1+r)} + \frac{1}{1+r}\frac{1}{2c}$, which is equivalent to $\sqrt{2}\left(\sqrt{\theta} - \theta\right)\sqrt{1+r} - 0.5\,(1-\theta) > 0$. Furthermore, the restrictions on X in the second case are equivalent to $2\theta - 1 < r \leq 1$. Further, in the third case, the present value of the profits under separation is the same as in the second case. The present value of the pooling profits, on the other hand, changes to

$$PV_{P,iii} = \frac{1}{2c} + \frac{1}{1+r}\frac{\theta}{2c}$$

[15] The first entry in the subscript distinguishes between whether a pooling (P) or a separating (S) equilibrium is played. The second entry describes the respective case, which is considered.

The condition guaranteeing that pooling is preferred can be shown to be equivalent to $0.5r + 1.5\theta - \sqrt{2(1+r)\theta} > 0$. Moreover, $\frac{1}{2c\theta} \geq X \geq \frac{1}{2c} \Leftrightarrow 2\theta^2 - 1 \leq r \leq 2\theta - 1$. It is easy to show that, for $2\theta^2 - 1 \leq r \leq 2\theta - 1$, the first condition $0.5r + 1.5\theta - \sqrt{2(1+r)\theta} > 0$ is always fulfilled. To demonstrate this, define $F(r,\theta) := 0.5r + 1.5\theta - \sqrt{2(1+r)\theta}$. Then, $\frac{\partial F}{\partial r} = 0.5 - \frac{\theta}{\sqrt{2(1+r)}}$ so that F has a minimum at $\tilde{r}$, with $0.5 - \frac{\theta}{\sqrt{2(1+\tilde{r})}} = 0 \Leftrightarrow \tilde{r} = 2\theta^2 - 1$. It follows that $F(\tilde{r},\theta) = 1.5\theta - \theta^2 - 0.5$. The condition $r \leq 2\theta - 1$ implies that θ cannot be (weakly) lower than 0.5, i.e. $\theta > 0.5$. It remains to be shown that, $F(\tilde{r},\theta) > 0$, for $\theta > 0.5$. $\frac{\partial F(\tilde{r},\theta)}{\partial \theta} = 1.5 - 2\theta$, which is positive, iff $\theta < 0.75$. As $F(\tilde{r}, 0.5) = 0$ and $F(\tilde{r}, 1) = 0$, $F(\tilde{r},\theta)$ has to be strictly positive for all $\theta \in (0.5, 1)$.

In the fourth case, the pooling profits are the same as in case (iii). In contrast, the present value of the separating profits changes to

$$PV_{S,iv} = \frac{1}{2c} - \frac{(1-\theta)\theta}{(1+r)c} + \frac{1}{1+r}\frac{1}{2c}$$

Pooling is therefore preferred, iff $\frac{1}{2c} + \frac{1}{1+r}\frac{\theta}{2c} > \frac{1}{2c} - \frac{(1-\theta)\theta}{(1+r)c} + \frac{1}{1+r}\frac{1}{2c} \Leftrightarrow 1.5\theta - 0.5 - \theta^2 > 0$. Notice that $X > \frac{1}{2c\theta} \Leftrightarrow 2\theta^2 - 1 > r$. This implies that $\theta > \sqrt{0.5}$. As seen before, $1.5\theta - 0.5 - \theta^2$ is then strictly positive. This completes the proof of Proposition 2. ■

It is convenient to illustrate the parameter constellations, for which a pooling equilibrium is chosen, in a figure. To simplify the exposition, we replace the discount rate r by the discount factor δ, making use of the relation $r = \frac{1-\delta}{\delta}$. The following figure depicts the choice of equilibrium (the shaded area depicts the set of parameters (δ, θ), which leads to a pooling equilibrium):

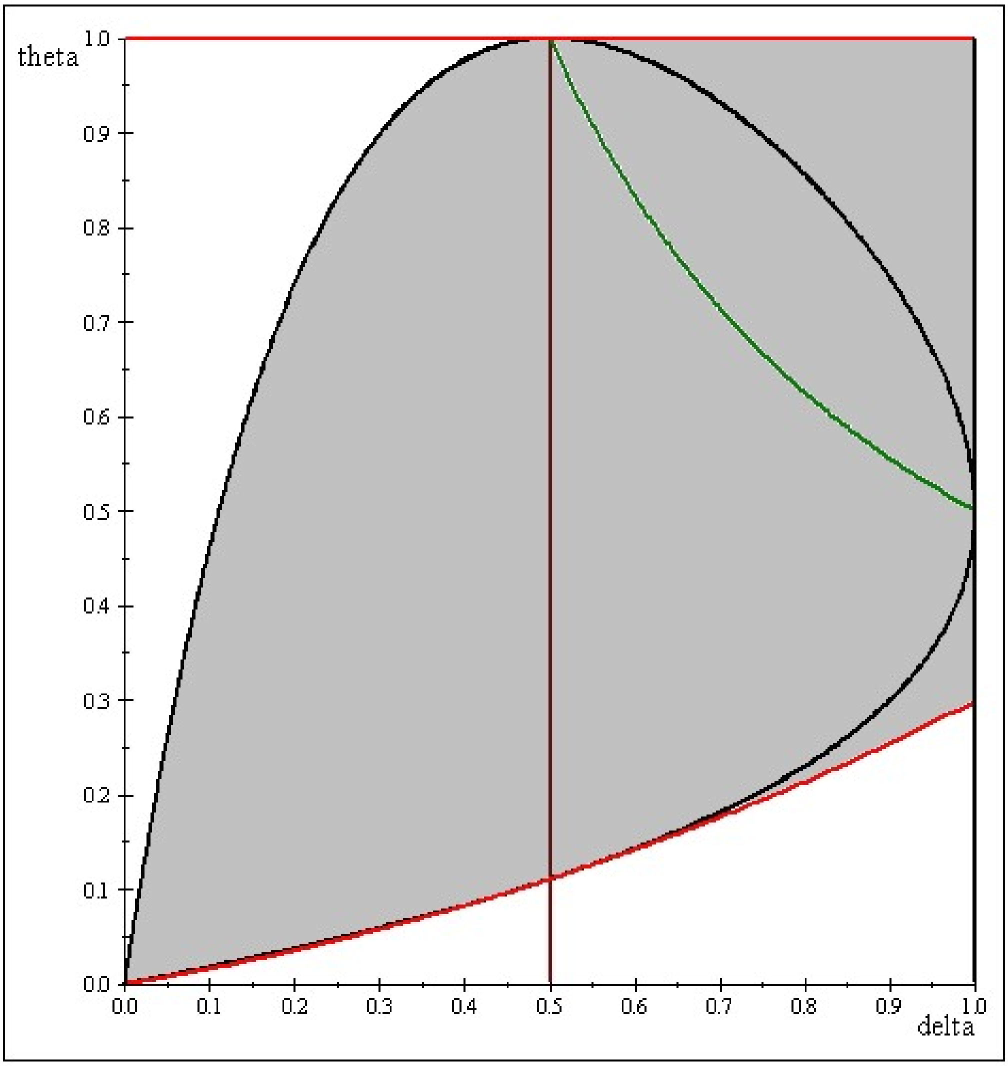

Figure 1: Type of equilibrium depending on parameter constellations

From Figure 1, we see that an increase in r (or a decrease in δ) may make a pooling equilibrium more or less likely. To explain this, notice that an increase in r has two effects on the likelihood of a pooling equilibrium. The first effect is very similar to the argumentation from Section 2. If r increases, it is more difficult to convince the unreliable principals to honor the implicit agreement. The gain of these principals from honoring the agreement is a higher profit in the second period. If r gets higher, this profit is discounted more heavily and a deviation from the agreement is more profitable. Therefore, the implicit bonus to be sustained in a pooling equilibrium must decrease, which makes the separating equilibrium (relatively) more attractive.

There is, however, a second effect that was absent in the model from Section 2 and that works into the opposite direction. As mentioned before, reliable principals decide on whether or not to play a pooling equilibrium. Their benefit from such an equilibrium is a better contract in the first period, while they suffer from a worse contract in the second. Hence, if r increases, the costs of a pooling equilibrium are discounted more heavily so that it becomes more profitable. As illustrated in Figure 1, this effect may well be dominant. A pooling equilibrium may therefore become more likely, the higher the discount rate.

To conclude the analysis, I am now going to present explicit formulas for the first-period profits and for the surplus in both, period 1 and period 2. Before doing so define the set of parameter constellations (r, θ), for which a pooling (separating) equilibrium is chosen, by A (B). The two profits are then given by[16]

$$\pi_{1r} = \left\{ \begin{array}{c} \frac{1}{2c}, \text{ if } \quad (r,\theta) \in A \text{ and } r \leq 2\theta - 1 \\ \frac{\sqrt{2\theta}}{c\sqrt{1+r}} - \frac{\theta}{c(1+r)}, \text{ if } \quad (r,\theta) \in A \text{ and } r > 2\theta - 1 \\ \frac{\theta}{2c}, \text{ if } \quad (r,\theta) \in B \text{ and } r > 1 \\ \frac{\sqrt{2\theta}}{c\sqrt{1+r}} - \frac{\theta}{c(1+r)}, \text{ if } \quad (r,\theta) \in B \text{ and } r \leq 1 \end{array} \right\} \tag{18}$$

$$\pi_{1u} = \left\{ \begin{array}{c} \pi_{1r}, \text{ if } \quad (r,\theta) \in A \\ \frac{\theta}{c}, \text{ if } \quad (r,\theta) \in B \text{ and } r > 1 \\ \frac{\sqrt{2\theta}}{c\sqrt{1+r}}, \text{ if } \quad (r,\theta) \in B \text{ and } r \leq 1 \end{array} \right\} \tag{19}$$

The surplus to be achieved in the first period is simply given by effort minus costs entailed by effort.[17] In the second period, however, one has to take into account that, in a separating equilibrium, only the fraction θ of all principal-agent relationships is continued. Second-period surplus therefore equals $e_2 - 0.5c\,(e_2)^2$ in a pooling and $\theta\left(e_2 - 0.5c\,(e_2)^2\right)$ in a separating equilibrium. The following expressions describe the two surpluses S_1 and S_2 depending on the parameter constellations:

$$S_1 = \left\{ \begin{array}{c} \pi_{1r} = \pi_{1u}, \text{ if } \quad (r,\theta) \in A \\ \frac{\theta}{2c}\left(2 - \theta\right), \text{ if } \quad (r,\theta) \in B \text{ and } r > 1 \\ \frac{\sqrt{2\theta}}{c\sqrt{1+r}} - \frac{\theta^2}{c(1+r)}, \text{ if } \quad (r,\theta) \in B \text{ and } r \leq 1 \end{array} \right\} \tag{20}$$

[16] Recall that, for $r \leq 2\theta - 1$, there will never arise a separating equilibrium. Therefore, when deriving the profits in a separating equilibrium, we only have to consider the two cases, where the constraint $\theta b_1 \geq 0.5c\,(\hat{e}_1)^2$ is binding.

[17] One argument for principals to be reliable was that these principals would feel psychic costs, if exploiting trust. As these psychic costs are never incurred, they do not have to be considered in the surplus formula.

$$S_2 = \left\{ \begin{array}{c} \frac{\theta}{2c}(2-\theta)\,, \text{ if } \quad (r,\theta) \in A \\ \\ \frac{\theta}{2c}, \text{ if } \quad (r,\theta) \in B \end{array} \right\} \tag{21}$$

From (20) and (21), one can derive the following proposition:

Proposition 3 *S_1, S_2 as well as both, S_1 and S_2, may be increasing in r.*

Proof. Consider an increase in r and suppose that this increase induces the reliable principals to switch from a separating equilibrium to a pooling one. From (21), it can directly be seen that S_2 then increases. To demonstrate that S_1 may also be increasing in r we consider a concrete example, where $\theta = 0.2$. Further, we assume that r increases from $r_1 = 0$ to $r_2 = 0.4$. We are therefore in the second case described in Proposition 2. At the initial discount rate, a separating equilibrium is selected resulting in surplus $S_{1s} = \frac{0.2\sqrt{2}}{c} - \frac{0.04}{c} = \frac{0.243}{c}$. For $r_2 = 0.4$, the reliable principals switch to a pooling equilibrium with the corresponding surplus $S_{1p} = \frac{\sqrt{0.4}}{c\sqrt{1.4}} - \frac{0.2}{1.4c} = \frac{0.392}{c}$, which is clearly higher than S_{1s}. ■

This proposition shows that the main result from Section 2 does not carry over to the alternative trust modeling approach. In particular, optimal surplus may be increasing in the discount rate. That is, discounting is not necessarily bad from the viewpoint of implicit contracting. The reason is that an increase in the discount rate may induce the reliable principals to switch from a separating to a pooling equilibrium. Such a pooling equilibrium is beneficial for two reasons: First, agents know that all first-period contracts are honored, which may improve the situation in the first period. Second, more agreements are entered in the second period. While in a separating equilibrium only the relationships between agents and reliable principals are continued, in a pooling equilibrium, unreliable principals enter second-period agreements as well. Although this is detrimental for second-period contracting of the reliable principals, the effect that more contracts are entered is dominant so that, in period 2, a pooling equilibrium leads to higher welfare than a separating one.

Furthermore, the fact that different trust modeling approaches lead to different results is also important, as most models make use of the infinite-horizon approach. As seen in this paper, implications derived from these models have to be handled with care. It is therefore useful to analyze, whether these implications are robust to an alternative modeling of trust.

Until now, we have analyzed the impact of the discount rate r on the implicit contracts chosen by the parties. To complete the analysis, one should take a look at the parameter θ and its influence on the contract choice. The effect of θ is twofold: First, we have already mentioned that, for reliable principals, a pooling (separating) equilibrium is accompanied by probability assessments $(Q_1 = 1, Q_2 = \theta)$ $((Q_1 = \theta, Q_2 = 1))$. Hence, the benefit and cost a pooling equilibrium entails are decreasing in θ. To be more concrete, this effect says that a pooling equilibrium is preferable, if θ is low and r very high. In this case, a pooling equilibrium yields a high benefit (a much better contract in the first period), while the corresponding cost (the lower profit in period 2) is heavily discounted. A second effect of θ is that it affects the pooling condition (13). The right-hand-side of (13) is strictly increasing in θ. This is intuitive. The higher θ, the more convinced are the agents that the implicit contract will be honored and the more do unreliable principals, in the second period, gain from cheating on the agents. Therefore, the higher (lower) θ, the more difficult it is for the reliable principals to separate from (pool with) the unreliable ones. Put differently, the lower θ, the less likely it is that, in the first period, the optimal pooling contract can be installed.

For $r \to 0$, the first effect is (almost) absent and pooling becomes optimal, if θ is sufficiently large. This can be seen from Figure 1.[18] If r increases, the first effect becomes relevant, too. As is clear from the preceding discussion, the two effects are then countervailing. While a lower θ in principle makes a pooling equilibrium more beneficial, it may prevent an efficient bonus choice so that a separating equilibrium may become preferable. Altogether, from Figure 1 we see that a pooling equilibrium is likely to be chosen unless both, θ and r, are relatively low or both are relatively high.

4 Concluding discussion

In this paper, we opposed two different approaches to model implicit contracting, an infinite-horizon approach that is frequently used in economic models and a finite-horizon approach based on an adverse-selection model. It was found that even the most convincing result of the infinite-horizon approach, namely that implicit contracting is improved in case of a lower discount rate, does not necessarily hold under the alternative approach. Therefore, predictions derived from the first modeling approach should be handled with care

[18] Notice that $r \to 0 \Leftrightarrow \delta \to 1$.

and subject to a reevaluation with the second approach.

A nice extension of the finite-horizon approach would be to apply it to the theory of the firm.[19] Agents' incentives would then stem from two sources: Asset ownership and implicit agreements. As reliable principals that have separated from unreliable ones, can perfectly motivate the agents by means of implicit contracting, asset ownership is likely to be assigned to these principals. Therefore, there could exist an equilibrium with the following intuitively appealing structure: In the first period, agents do not know the principal types and are thus protected by asset ownership. After the first period, principal types are revealed (i.e. the equilibrium is separating). Then, the assets of those agents who work with reliable principals are removed to the principals, while the remaining agents keep their assets. This would nicely explain changes in ownership structures over time. If this equilibrium may indeed exist, however, has to be shown analytically. I leave this for future research.

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[19] As far as I know, all models introducing implicit contracts into the theory of the firm make use of the infinite-horizon approach.

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