Inferior Products and Profitable Deception*

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Abstract

We analyze conditions facilitating profitable deception in a simple model of a competitive retail market. Firms selling homogenous products set up-front prices that consumers understand and additional prices that naive consumers ignore unless revealed to them by a firm, where we assume that there is a binding floor on the up-front prices. We show that our reduced-form framework captures more elaborate models of the credit-card and mutual-fund markets, and argue that it may apply to bank accounts and mortgages as well. Our main results establish that “bad” products (those with lower social surplus than an alternative) tend to be more reliably profitable than “good” products. Specifically, (1) in a market with a single socially valuable product and sufficiently many firms, a deceptive equilibrium—in which firms hide additional prices—does not exist and firms make zero profits. But perversely, (2) if the product is socially wasteful, then a profitable deceptive equilibrium always exists. Furthermore, (3) in a market with multiple products, since a superior product both diverts sophisticated consumers and renders an inferior product socially wasteful in comparison, it guarantees that firms can profitably sell the inferior product by deceiving consumers.

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1 Introduction

In this paper, we investigate circumstances under which firms sell products by deceiving some consumers about the products’ full price, focusing (in contrast to much of the literature) on deception that leads to positive equilibrium profits in seemingly competitive industries. We identify a novel, perverse aspect of profitable deception: products that generate lower social surplus than the best alternative facilitate deception precisely because they would not survive in the market if consumers understood their full price, and therefore firms often make profits on exactly such bad products but not on good products. We argue that—in seeking these profits—firms disproportionately push bad products and might enter markets for bad products in potentially massive numbers, creating further economic inefficiencies.

We develop our insights in the reduced-form model of Section 2.1, which builds on the seminal theory of Gabaix and Laibson (2006). Firms are engaged in simultaneous-move price competition to sell a single homogenous product. Each firm charges a transparent up-front price as well as an additional price, and unless at least one firm (costlessly) unshrouds the additional prices, naive consumers ignore these prices when making purchase decisions. To capture the notion that for some products firms cannot return all profits from later charges by lowering initial charges, we deviate from most existing work and posit that there is a floor on the up-front price. We argue that a profitable deceptive equilibrium—wherein all firms shroud additional prices—is the most plausible equilibrium whenever it exists: it is then the unique equilibrium in the variant of our model in which unshrouding carries a cost (no matter how small), and all firms prefer it over an

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1 Hidden fees have often enabled firms to reap substantial profits despite seemingly considerable competition, at least at the price-competition stage when entry and marketing costs have been paid and customer bases have been identified and reached. Investigating trade and portfolio data from a large German bank, for example, Hackethal, Inderst and Meyer (2010) document that “bank revenues from security transactions amount to €2,560 per customer per year” (2.4 percent of mean portfolio value), a figure likely well above the marginal cost of serving a customer. Similarly, based on a number of measures, including the 20-percent average premium in interbank purchases of outstanding credit-card balances, Ausubel (1991) argues that credit-card companies make large profits. Ellison and Ellison (2009) describe a variety of obfuscation strategies online computer-parts retailers use, and document that such strategies can generate surprisingly large profits given the near homogeneity of products. These observations, however, do not mean that the net economic surplus taking all operating costs into account are large or even positive in these markets: for example, fixed entry costs can dissipate any profits from the later stage of serving consumers.

2 Armstrong and Vickers (2012), Ko (2012), and Grubb (forthcoming) also analyze models with variants of our price-floor assumption.
unshrouded-prices and therefore zero-profit equilibrium. Hence, in our discussion we presume that a deceptive equilibrium is played whenever it exists.

In Section 2.2, we establish that our reduced-form framework captures more elaborate models of two important markets that have been invoked as conducive to hidden charges: credit cards and mutual funds. In our model of credit cards, issuers make offers consisting of an up-front price and an interest rate to naive time-inconsistent borrowers, who derive a convenience benefit from using a credit card and understate how much interest they will pay. In the case of indifference, each consumer chooses whether to get or charge a card by lexicographically applying an exogenously given preference over cards. Then, any consumer who prefers a firm’s card will get the card if the firm charges an up-front price of zero. Any additional consumers a negative up-front price attracts, therefore, will not use the firm’s card, and hence these consumers are unprofitable. As a result, firms act as if they were facing a price floor of zero.

In our model of mutual funds, firms choose front loads investors understand and management fees investors ignore unless explained to them by a firm. We argue that a key section of the Investment Company Act of 1940, which prohibits the dilution of existing investors’ shares in favor of new investors, prevents funds from offering discounts to new consumers, so that the front load must be non-negative. Finally, we briefly and informally lay out several other applications, including mortgages with changing terms, bank accounts, printers, and hotels, emphasizing that in some of these markets there may not be a binding floor on the up-front price.

Section 3 presents our basic results. As two benchmark cases, we show that if the price floor is not binding, only a zero-profit deceptive equilibrium exists, and note that if consumers are sophisticated in that they observe and take into account additional prices, firms again earn zero profits. If the price floor is binding and consumers are naive, however, profitable deception may occur. If other firms shroud and the up-front price is at the floor, a firm cannot compete on the up-front price and can compete on the total price only if it unshrouds—but because consumers who learn of the additional prices may not buy the product, the firm may find the latter form of competition unattractive. If this is the case for all firms, a profitable deceptive equilibrium exists. Otherwise, additional prices are unshrouded with probability one, and firms earn zero profits.
The above condition for a firm to find unshrouding unattractive has some potentially important implications for when profitable deception occurs. First, if the product is socially wasteful (its value relative to consumers’ outside option is lower than its production cost), a firm that unshrouds cannot go on to profitably sell its product, so no firm ever wants to unshroud. Perversely, therefore, in a socially wasteful industry a profitable deceptive equilibrium always exists. But if the product is socially valuable, a firm that would make sufficiently low profits from deception can earn higher profits from unshrouding and capturing the entire market, so if there is such a firm only a non-deceptive, zero-profit equilibrium exists. Hence, because in an industry with many firms some firm earns low profits, entry into socially valuable industries makes these industries more transparent; and whenever deceptive practices survive in an industry with many firms, our model says that the industry is socially wasteful. We also show that if consumers initially underappreciate the additional price because they are unaware of a valuable add-on they want to spend on, only an unshrouded-prices equilibrium exists. From the perspective of our model, therefore, the observation that unshrouding has not taken place in the credit-card market implies that excessive credit-card borrowing is not only unanticipated, but unwanted by consumers.

In Section 4, we extend our model by assuming that there are both sophisticated and naive consumers in the market. Consistent with the predictions of Gabaix and Laibson (2006) and Armstrong and Vickers (2012) that sophisticated consumers facilitate transparency and efficiency, we find that if the product is socially valuable and there are sufficiently many sophisticated consumers, only a non-deceptive equilibrium exists. In contrast to received wisdom, however, we also show that in a multi-product market with a superior and an inferior product, often sophisticated and naive consumers self-separate into buying the former and the latter product, respectively, and sophisticated consumers exert no pressure to unshroud the inferior product’s additional prices. Worse, because the superior product renders the inferior product socially wasteful in relative terms, it guarantees that profitable deception in the market for the inferior product can be maintained. This observation has a striking implication: all it takes for profitable deception to occur in a competitive industry is the existence of an inferior product with a shroudable price component and a binding floor on the up-front price, and firms’ profits derive entirely from selling this inferior product.
In Section 5, we turn to extensions and modifications of our framework. We show that if a firm has market power in the superior-product market, it may have an incentive to unshroud to attract naive consumers to itself, especially if—similarly to for instance Vanguard in the mutual-fund market—it sells mostly the superior product. But if the firm’s market power is limited and unshrouding is costly, the extent to which the firm educates consumers is also limited.

In Section 6, we discuss some policy implications of and evidence for our model. Since deception is likely to be more common and have more adverse welfare consequences when the price floor is binding, policymakers should concentrate their regulatory efforts on industries with a binding price floor—of which supranormal profits is a telltale sign. We also note that a policymaker can improve outcomes for consumers if she can lower additional prices. And although the overarching prediction of our paper that inferior products facilitate profitable deception and hence should go together with it is difficult to test directly, some auxiliary predictions of our model are consistent with existing evidence. In particular, our framework predicts that firms disproportionately push—for instance through commission-driven intermediaries or persuasive advertising—inferior products and may enter the market for inferior products in mass numbers, and that the regulation of hidden fees may not result in increases in other fees, and hence benefits consumers.

In Section 7, we discuss the behavioral-economics and classical literatures most closely related to our paper. While a growing theoretical literature investigates how firms exploit naive consumers by charging hidden or unexpected fees, our paper goes beyond all of the existing work in making the central prediction that socially wasteful and inferior products tend to be sold more profitably than better products, and identifying a number of additional implications of this insight. We conclude in Section 8 by pointing out important further questions raised by our model.

2 Basic Model and Applications

2.1 Setup

$N \geq 2$ firms compete for naive consumers who value each firm’s product at $v > 0$ and are looking to buy at most one item. Consumers have an outside option with utility $u$. Firms play a simultaneous-
move game in which they set up-front prices $f_n$ and additional prices $a_n \in [0, \bar{a}]$, and decide whether to costlessly unshroud the additional prices. A consumer who buys a product must pay both prices associated with that product—she cannot avoid the additional price. If all firms shroud, consumers make purchase decisions as if the total price of product $n$ was $f_n$. If at least one firm unshrouds, all firms’ additional prices become known to all consumers, and consumers make purchase decisions based on the true total prices $f_n + a_n$.\footnote{Our assumption on unshrouding—that a single firm can educate all consumers about all additional prices at no cost—is in the context of most real settings unrealistically extreme, especially if (as in our credit-card application) incurring the additional prices depends partly on the consumer’s own behavior. This extreme case is theoretically useful for demonstrating that education often does not happen even if it is very easy. We show in our working paper (Heidhues, K"oszegi and Murooka 2012b) that if unshrouding is somewhat costly or reaches only a fraction of consumers, a deceptive equilibrium is more likely to occur, but our qualitative results do not change.} If consumers weakly prefer buying and are indifferent between a number of firms, each of these firms gets a positive market share, with this share being $s_n \in (0,1)$ if consumers are indifferent between all firms.

Although we focus most of our analysis and discussion on price misperceptions, we also allow for consumers to misperceive the product’s value, as when investors overestimate the gross return of a managed mutual fund. Paralleling our setup for additional prices, we assume that if all firms shroud, consumers perceive the value of the product to be $\tilde{v}$, and if at least one firm unshrouds, consumers correctly understand the value to be $v$.

Firm $n$’s cost of providing the product is $c_n > 0$. We let $c_{\min} = \min_n \{c_n\}$, and—to ensure that our industry is competitive in the corresponding classical Bertrand model—assume that $c_n = c_{\min}$ for at least two firms $n$. In addition, we assume that $\max\{v, \tilde{v} + \bar{a}\} > c_n$ for all $n$; a firm with $\max\{v, \tilde{v} + \bar{a}\} < c_n$ cannot profitably sell its product, so we think of it as not participating in the market. And to avoid uninteresting qualifications, we suppose that $v - u \neq c_{\min}$.

Deviating from much of the literature, we impose a floor on the up-front price: $f_n \geq \underline{f}$. We assume that $\underline{f} \leq \min\{\tilde{v}, v\} - u$, which means that whether or not unshrouding occurs, consumers are willing to buy the product if they perceive the total price to be $\underline{f}$. We also assume that $\underline{f} \leq c_{\min}$, so that firms are not prevented from setting a zero-profit total price.

Our main interest is in studying the Nash-equilibrium outcomes of the above game played between firms. While we fully characterize equilibrium outcomes in our main propositions, in discussing our results we focus on conditions for and properties of deceptive equilibria—equilibria
in which all firms shroud additional prices with probability one. Because no firm has an incentive
to shroud if at least one firm unshrouds, there is always an unshrouded-prices equilibrium. When a
deceptive equilibrium exists, however, it is more plausible than the unshrouded-prices equilibrium
for a number of reasons. Most importantly, we show in Appendix A that whenever a deceptive
equilibrium exists in our model with no unshrouding cost, it is the unique equilibrium in the
variant of our model in which unshrouding carries a positive cost, no matter how small the cost is.
In addition, a positive-profit deceptive equilibrium is preferred by all firms to an unshrouded-prices
equilibrium. Finally, for the lowest-priced firms, the strategy they play in an unshrouded-prices
equilibrium is weakly dominated by the strategy they play in a positive-profit deceptive equilibrium.

To simplify our statements, we define the Bertrand outcome as the outcome of a Bertrand price-
competition game with rational consumers and transparent prices: consumers buy if and only if
\[ v - u > c_{\text{min}} \]
and pay a total price equal to \( c_{\text{min}} \) whenever they buy, and firms earn zero profits.

2.2 Applications

2.2.1 Credit Cards

Our first economic application is credit cards. We build on a variant of Heidhues and Köszegi’s
(2010) model of a credit market with naive time-inconsistent borrowers.\(^4\) Firms and consumers
interact over four periods, \( t = 0, 1, 2, 3 \). In period \( t = 0, 1, 2 \), a consumer aims to maximize total
utility \( u_t/\beta + \sum_{\tau=t+1}^{3} u_\tau \), where \( u_\tau \) is instantaneous utility in period \( \tau \). The short-term discount
factor \( \beta \) is distributed in the population according to the cumulative distribution function \( Q \) on
\([0, 1] \), where \( \overline{\alpha} \equiv \max_{\beta \in (0,1]} Q(\beta)(1/\beta - 1) \) exists.\(^5\) We posit that the consumer’s period-0 total
utility is relevant for welfare evaluations.

\(^4\) A substantial body of research in behavioral economics documents that individuals often have a taste for
immediate gratification (Laibson, Repetto and Tobacman 2007, Augenblick, Niederle and Sprenger 2013, for instance),
that such taste is associated with credit-card borrowing (Meier and Sprenger 2010), and that individuals may be
naive about their taste (Laibson et al. 2007, Skiba and Tobacman 2008). And consistent with the specific hypothesis
that consumers underestimate costly borrowing on their credit cards, Ausubel (1991) finds that consumers are less
responsive to the post-introductory interest rate in credit-card solicitations than to the teaser rate, even though the
former is more important in determining the amount of interest they will pay.

\(^5\) For presentational simplicity, we use a specification in which \( \beta \) divides current utility instead of the (behaviorally equivalent)
more conventional specification in which \( \beta \) multiplies future utility. A sufficient condition for
\( \max_{\beta \in (0,1]} Q(\beta)(1/\beta - 1) \) to exist is that \( Q(\cdot) \) is continuous and \( \lim_{\beta \to 0} Q(\beta)/\beta = 0 \).
Issuer \( n \)'s cost of managing a consumer's account is \( c_n < \bar{c} \), and all issuers acquire funds at an interest rate of zero. In period 0, each issuer chooses salient charges (such as annual fees) \( f_n \in \mathbb{R} \) to be paid in period 3, and a gross interest rate \( R_n \geq 1 \). Simultaneously with its pricing decision, each firm decides whether to unshroud the costs associated with credit-card use. If no firm unshrouds, consumers assume that they have \( \beta = 1 \). If a firm unshrouds, consumers learn that their \( \beta \)'s are drawn from \( Q(\cdot) \), but they do not learn their individual \( \beta \)'s.

Observing firms' offers, in period 0 a consumer decides whether to get a credit card, valuing its future convenience use at \( v \), and the outside option (such as getting a debit card) at \( u \). A consumer can get multiple cards, but needs only one for convenience use, and gets other card(s) only if she sees a strict further benefit. If a consumer gets at least one credit card, she runs up charges of 1 in period 1.\(^6\) If a consumer is indifferent between multiple credit-card deals, she lexicographically applies an exogenously given preference over cards to decide which one to get and charge.\(^7\) A unit of credit-card spending generates a unit of instantaneous utility, and repaying a unit of outstanding debt costs a unit of instantaneous utility.

We now show that the above credit-card pricing game simplifies to our reduced-form model. To facilitate the statement, we say that outcomes in two Nash equilibria are the same if the two equilibria differ at most in prices at which consumers do not buy.

**Lemma 1.** The sets of Nash-equilibrium outcomes when \( f_n \) is unrestricted \((f_n \in \mathbb{R})\) and when

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\(^6\) We assume that the consumer makes no charging decision. This makes little difference to the logic of our results. Since a naive consumer does not anticipate that she will delay repayment, she is happy to charge her card for its convenience benefit, and does not care about the interest rate on her card. Furthermore, an endogenous charging decision makes unshrouding less desirable for a firm, since a consumer who is aware that she might pay interest can avoid this by not using her card. Finally, for the same reason as below, a firm would not want to offer a negative up-front price when shrouding occurs.

\(^7\) This assumption has the stark implication that if a competitor gets a consumer to sign up for a second, less preferred card, it gets no interest payments from the consumer. This implication plays a central role in our proof that there is a price floor. A price floor can arise whenever consumers with multiple cards split their expenditures across cards. All that is necessary is that the cost of serving additional consumers below the price floor is greater than the revenue the firm generates from the charges it attracts from holders of multiple cards.

\(^8\) For simplicity, we assume that a consumer does not transfer balances to another credit card in period 2, which—under the assumption that there is a market for transfers—she would prefer to do in our three-period model unless the effort cost of doing so was high. As predicted by O'Donoghue and Rabin (2001) and is consistent with evidence in Shui and Ausubel (2004), however, in a more realistic, long-horizon, setting naive borrowers may procrastinate for a long time before finding or taking advantage of favorable balance-transfer opportunities.
$f_n$ is restricted to be non-negative ($f_n \geq 0$) are the same. Furthermore, if $f_n$ is restricted to be non-negative, then defining $a_n \equiv Q(1/R_n)(R_n - 1)$, the game is strategically equivalent to (i.e., for any action profile generates the same payoffs as) our reduced-form model with $f = 0$.

The key observation is that to avoid unprofitable consumers, firms act as if they were facing a floor of zero on the up-front price. Intuitively, although setting $f_n < 0$ rather than $f_n = 0$ induces more consumers to get firm $n$’s card, it does not prevent them from getting and using an alternative card they prefer. Since firm $n$’s profits derive from consumers using rather than just getting its card, the consumers it attracts with the negative up-front price are unprofitable.

Given that $f_n \geq 0$ for all $n$, each consumer gets at most one card. The resulting interest payment on the card, then, acts as an additional price in that it adds to the firm’s profits and lowers the consumer’s utility by the same amount. Furthermore, if shrouding occurs, a consumer expects to repay early, so she expects to pay no additional price, whereas if unshrouding occurs, she fully understands how much she will pay in expectation. This logic also makes clear that beyond interest payments, a late fee or any other unanticipated fee that transfers utility from consumers to firms can also serve as an additional price.\footnote{Suppose, for example, that each firm $n$ can impose a fee $\hat{a}_n \geq 0$ for minor infractions such as paying the bill a few days late. A consumer will pay the fee with probability $Q(\hat{a}_n)$, but unless unshrouding occurs, at the time of selecting a card she believes this probability will be zero. We suppose that $Q(\hat{a})$ is continuous and $\max_{\hat{a}} Q(\hat{a})\hat{a}$ exists. Then, $a_n = Q(\hat{a}_n)\hat{a}_n$ acts like an additional price, with the maximum additional price being $\bar{a} = \max_\hat{a} Q(\hat{a})\hat{a}$.}

Notice that in our framework, an issuer prefers to undercut competitors’ non-negative up-front prices if it can require consumers to use only the issuer’s card in exchange, thereby generating interest payments down the line. This pricing strategy is consistent with the observation that many credit cards offer usage-contingent perks such as airline miles and rental-car insurance. But while such a strategy avoids unprofitable naive consumers, it may disproportionately attract unprofitable sophisticated consumers, so that to avoid these consumers firms may again act as if they were facing a price floor. Formally, suppose that a fraction $\lambda$ of consumers has convenience benefit $v' < v$ (e.g., because they like paying in cash more than others), and has $\beta = 1$ and hence does not pay interest. If firm $n$ lowers $f_n$ below $v'$, it attracts all these sophisticated consumers. If $\lambda(v' - c_{min}) + (1 - \lambda)(v' + \bar{a} - c_{min}) \leq 0$, therefore, the price cut is unprofitable.
2.2.2 Mutual Funds

As a second important application, we discuss mutual funds. The Investment Company Act of 1940 in the US regulates the institutional structure, including the trading prices and accounting, of mutual funds. A fund must calculate its net asset value (NAV) based on market valuations, and including investment advisory fees and other expenses to date, at least once daily. The price of a share is simply the NAV divided by the number of shares, and investors can buy or redeem shares only at the per-share NAV. In addition, a fund can impose purchase fees (front loads). Management fees are subject to a fiduciary duty, which we interpret as putting a legal bound on the management fee. While disclosure regulations are in place to make sure investors understand the costs of fund ownership, different types of empirical evidence suggest that investors do not fully understand the management fees, and that they appreciate front loads better.

Formally, we assume that funds and investors interact over three periods, $t = 0, 1, 2$. In period 0, funds choose up-front prices (front loads) $f_n$ and management fees $\hat{a}_n$ satisfying $0 \leq \hat{a}_n \leq \hat{A}$, where the upper bound comes from fiduciary duty. Simultaneously with this choice, firms make unshrouding decisions. Fund $n$’s cost is $c_n$ per consumer initially investing in the fund.

Consumers make investment decisions in period 0, starting off with wealth $v$. Consumers’

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10 See rules 2a-4 and 22c-1 adopted under the Investment Company Act, or the summaries by the Securities and Exchange Commission available at http://www.sec.gov/answers/nav.htm and http://www.sec.gov/divisions/investment/icvaluation.htm (accessed October 1, 2014). A fund can also impose redemption fees (rear loads), but since these are not important for our arguments, we do not incorporate them in our model.

11 See Section 36 of the Investment Company Act. The Act does not impose a specific maximum fee. Nevertheless, fiduciary duty means that a manager must act in a way that benefits investors, and a management fee that leads an investor into (say) an almost certain loss clearly violates this restriction. Indeed, some lawsuits have successfully challenged excessive management fees on the basis of violating fiduciary duty: http://www.pionline.com/article/20111212/PRINT/312129971/wal-mart-merrill-lynch-settle-401K-fee-suit (accessed October 1, 2014).

12 Using a natural experiment in India, Anagol and Kim (2012) show that mutual-fund investors are less sensitive to amortized “initial issue expenses” than to otherwise identical “entry loads” paid up front. In a laboratory experiment on portfolio choice between S&P 500 index funds with different fees, Choi, Laiobson and Madrian (2010) find that subjects, including Wharton MBA students, Harvard undergraduates, and Harvard staff members, overwhelmingly fail to minimize fees. Using administrative data from the privatized Mexican Social Security system, Duarte and Hastings (2012) show that investors were not sensitive to fees in choosing between funds, leading to high management fees despite apparently strong competition. Barber, Odean and Zheng (2005) find a strong negative relationship between mutual-fund flows and front loads, but no relationship between flows and operating expenses, and Wilcox (2003) also presents evidence that the overwhelming majority of consumers overemphasize loads relative to expense ratios. Based on a survey of 2,000 mutual-fund investors, Alexander, Jones and Nigro (1998) report that only 18.9% could provide any estimate of their largest mutual fund’s expenses, although 43% claim that they knew this information at the time of purchase.
outside option generates utility $u$ in period 2. We normalize the gross return a fund can achieve to 1, so that fund $n$’s net return is $1 - \hat{a}_n$, but consumers perceive the gross return to be $\tilde{R} \geq 1$. If a consumer selects fund $n$, then with probability $Q(\hat{a}_n)$, she becomes dissatisfied with managed funds in period 1 and switches to an alternative investment opportunity with a return of 1.\footnote{This assumption is one simple way to capture the notion that higher management fees generate lower performance, and lower performance—indeed, of whether a consumer realizes the importance of fees or understands gross returns—can trigger dissatisfaction, redemption, and finding better alternative investment opportunities. We can think of the alternative investment consumers switch to as a low-cost index fund. In Section 4, we endogenize consumers’ outside option, which they may initially choose in period 0, as a low-cost index fund as well, although this outside option may in principle also be different.} We suppose that $Q(\cdot)$ is continuous and increasing, and that if investing in the mutual fund is optimal (i.e., $v(1 - \hat{a}_n)^2 \geq u$), then $Q(\hat{a}_n) = 0$. When choosing a mutual fund, consumers are aware of the up-front prices, but unless at least one firm unshrouds, they ignore management fees. Unshrouding reveals the expected lifetime management fees consumers will pay, but—for simplicity—we suppose that conditional on selecting a mutual fund in period 0, consumers do not change their period-1 switching behavior in response.\footnote{If $v(1 - \hat{a}_n)^2 < u$ and unshrouding occurs, consumers select the superior outside option, so our assumption about period-1 switching behavior is inconsequential. Otherwise, consumers find the mutual fund optimal independently of whether unshrouding occurs, so they refrain from switching.}

We now show that this game is strategically equivalent to one in which firms choose appropriately defined additional prices:

**Lemma 2.** Defining $\tilde{v} = \tilde{R}^2 v$, $a_n \equiv v[\hat{a}_n + (1 - Q(\hat{a}_n))(1 - \hat{a}_n)\hat{a}_n]$ and $\overline{a} \equiv \max_{\hat{a} \in [0, \hat{A}]} v[\hat{a} + (1 - Q(\hat{a}))(1 - \hat{a})\hat{a}]$, the mutual-fund model is strategically equivalent to our reduced-form model.

The reason is essentially the same as in our credit-card model: the management fees generate profits to firms and lower consumers’ utility by the same amount, and consumers are aware of it if and only if unshrouding occurs.

Finally, we discuss the floor on the up-front price. The language of the rules interpreting the Investment Company Act, and in particular the phrase and definition of purchase fees, strongly suggests that these fees are intended to be non-negative, so that $f_n \geq 0$. Furthermore, the constraint $f_n \geq 0$ follows directly from the provision of the Investment Company Act aimed at protecting existing consumers from using their shares to attract new consumers. Section 22(a) requires funds to “eliminate or reduce so far as reasonably practicable any dilution of the value of other outstanding
securities” in favor of new consumers. One simple interpretation of this “no-dilution” rule is that new investors cannot be treated better than existing investors. Since both pay the management fees, an existing investor would strictly prefer to be treated as a new investor whenever \( f_n < 0 \), so \( f_n < 0 \) violates no dilution. What gives us extra confidence that this interpretation is correct is that the regulations on pricing were adopted as a way to interpret the no-dilution section of the Act. Indeed, consistent with a hard price floor of zero, we are unaware of any mutual fund that offers cash back, discounts, or other financial incentives to attract new investors.

2.2.3 Other Potential Applications

Mortgages. For mortgages with changing repayment terms, such as pay-option mortgages, interest-only mortgages, and mortgages with teaser rates, many consumers underestimate the payments they will have to make once an initial teaser period ends.\(^{15}\) In this environment, we think of the consumer’s repayment costs assuming terms do not change as the up-front price and of the increase in payments as the additional price. The additional price is limited by consumers’ ability to walk away from the mortgage when payments increase. As we show in an earlier version of our paper (Heidhues, Kőszegi and Murooka 2014), a floor on the up-front price can result because consumers would find an overly low up-front price suspicious, and—concluding that “there must be a catch”—would not buy. But because we need to know consumers’ beliefs to identify this price floor, it is difficult to tell from primitives where the floor is.

Bank accounts and hotels. In these applications, consumers buy a base product—account maintenance in the case of bank accounts and a room in the case of hotels—and can then buy an add-on—such as costly overdrafting in the case of bank accounts or gambling entertainment in the

\(^{15}\) For instance, the Option Adjustable-Rate Mortgage (ARM) allows borrowers to pay less than the interest for a period, leading to an increase in the amount owed and sharp increases in monthly payments when the mortgage resets to an amortizing payment schedule. See “Interest-Only Mortgage Payments and Payment-Option ARMs—Are They for You?,” information booklet prepared for consumers by the Board of Governors of the Federal Reserve System, available at http://www.federalreserve.gov/pubs/mortgage_interestonly/mortgage_interestonly.pdf. Based on existing evidence (Cruickshank 2000, Woodward and Hall 2012, Gerardi, Goette and Meier 2009, Gurun, Matvos and Seru 2013, Bucks and Pence 2008) it is plausible to assume that consumers have a limited understanding of mortgages in general, and of these mortgages in particular. And some features of these mortgages, such as the Option ARM’s one- or three-month introductory interest rate, seem to serve only the purpose of deceiving borrowers about the product’s cost.
case of Las Vegas hotels—whose cost they do not appreciate at the moment of purchase.\textsuperscript{16} We can think of the base product’s price as the up-front price and of the utility loss from unexpectedly high add-on payments as the additional price. The additional price is limited by consumers’ ex-post demand response to add-on prices. Banks likely face a floor on the up-front price close to zero for the same reason as do credit cards, but it is unclear whether hotels face a binding price floor.

Printers. In the printer market, consumers do not anticipate high cartridge prices.\textsuperscript{17} To model this situation, we think of the printer price as the up-front price, and suppose that consumers value a printer explicitly or implicitly assuming that cartridges are free. Then, the additional price is consumers’ loss (and firms’ gain) from positive cartridge prices. A price floor may arise from suspicion or other sources, but we do not know whether it is binding.

3 Profitable Deception

This section analyzes our basic model, and discusses key economic implications.

3.1 Benchmarks: Non-Binding Price Floor or Sophisticated Consumers

First, we characterize equilibrium outcomes when the floor on the up-front price is not binding. In our setting, this means that (even adding the maximum additional price) a firm cannot make profits if it chooses an up-front price equal to the floor.

Proposition 1 (Equilibrium with Non-Binding Price Floor). Suppose $f \leq c_{\text{min}} - \bar{a}$. For any $\psi \in [0, 1]$, there exist equilibria in which unshrouding occurs with probability $\psi$. If shrouding occurs in an equilibrium, firms earn zero profits, and consumers buy the product from a most-efficient firm, pay $f = c_{\text{min}} - \bar{a}, a = \bar{a}$, and get utility $v - c_{\text{min}}$. If unshrouding occurs in an equilibrium, the Bertrand outcome obtains.

\textsuperscript{16} For more detailed arguments that these products can be interpreted as deceptive, see Armstrong and Vickers (2012) and Gabaix and Laibson (2006), respectively.

\textsuperscript{17} For instance, Hall (1997) and the UK’s Office of Fair Trading report that a large majority of consumers buying a printer do not know the price of a cartridge. Gabaix and Laibson (2006) invoke cartridge prices as one of their prime examples of hidden fees.
Proposition 1 implies that if the price floor is not binding, there is always a deceptive equilibrium. Since in a deceptive equilibrium consumers do not take into account additional prices when choosing a product, firms set the highest possible additional price, making existing consumers valuable. Similarly to the logic of Lal and Matutes’s (1994) loss-leader model as well as that of many switching-cost (Farrell and Klemperer 2007) and behavioral-economics theories, firms compete aggressively for these valuable consumers ex ante, and bid down the up-front price until they eliminate net profits. In addition, since with these prices a firm cannot profitably undercut competitors, no firm has an incentive to unshroud.

By Proposition 1, any equilibrium outcome is identical either to that in the above deceptive equilibrium, or to that in an unshrouded-prices equilibrium, in which unshrouding occurs with probability one. If unshrouding occurs, consumers make purchase decisions based on the total price, so firms effectively play a Bertrand price-competition game, leading to a Bertrand outcome.

As a second benchmark, we note what happens when all consumers are sophisticated in that they observe the total prices $f_n + a_n$ and make purchase decisions based on these prices, and also understand that the total value of a product is $v$. Then, we have standard Bertrand competition in the total price, so standard arguments imply that the Bertrand outcome obtains. For this to be the case, our assumption that $f \leq c_{\min}$ is crucial.

3.2 Naive Consumers with a Binding Price Floor

Taken together, the benchmarks above imply that for profitable deception to occur, both naive consumers must be present and the price floor must be binding. We turn to analyzing our model when this is the case, assuming for the rest of the section that all consumers are naive and $f > c_n - \bar{a}$ for all $n$. This condition means that setting an up-front price equal to the floor does not preclude a firm from making profits on its consumers.

We first identify sufficient conditions for a deceptive equilibrium to exist. As above, in such an equilibrium all firms set the maximum additional price $\bar{a}$. Then, since firms are making positive profits and hence have an incentive to attract consumers, they bid down the up-front price to $f$. With consumers being indifferent between firms, firm $n$ gets market share $s_n$ and therefore earns a
profit of \( s_n(f + \bar{a} - c_n) \). For this to be an equilibrium, no firm should want to unshroud additional prices. If unshrouding occurs, consumers are willing to pay exactly \( v - u \) for the product, so firm \( n \) can make profits of at most \( v - u - c_n \) by unshrouding. Hence, unshrouding is unprofitable for firm \( n \) if the following “Shrouding Condition” holds:

\[
s_n(f + \bar{a} - c_n) \geq v - u - c_n. \tag{SC}
\]

A deceptive equilibrium exists if (SC) holds for all \( n \). Furthermore, since \( s_n < 1 \), (SC) implies that \( f + \bar{a} > v - u \), so that in a deceptive equilibrium consumers are worse off than with their outside option. Proposition 2 summarizes this result, and characterizes equilibria fully:¹⁸

**Proposition 2** (Equilibrium with Binding Price Floor). Suppose \( f > c_n - \bar{a} \) for all \( n \).

1. If (SC) holds with a strict inequality for all \( n \), there is a deceptive equilibrium, in which all firms offer the contract \( (f_n, a_n) = (f, \bar{a}) \) with probability one, consumers receive utility less than their outside option, and firms earn positive profits. In any other equilibrium, unshrouding occurs with probability one and the Bertrand outcome obtains.

2. If (SC) is violated for some \( n \), unshrouding occurs with probability one and the Bertrand outcome obtains.

The intuition for why firms might earn positive profits despite facing Bertrand-type price competition is in two parts. First, as in previous models and as in our model with a non-binding price floor, firms make positive profits from the additional price, and to obtain these ex-post profits each firm wants to compete for consumers by offering better up-front terms. But once firms hit the price floor, they exhaust this form of competition without dissipating all ex-post profits.

Second, since a firm cannot compete further on the up-front price, there is pressure for it to compete on the additional price—but this requires unshrouding and is therefore an imperfect substitute for competition in the up-front price. If a firm unshrouds and cuts its additional price by a little bit, a consumer learns not only that the firm’s product is the cheapest, but also that all products are more expensive than she thought. If the consumer receives negative surplus from

¹⁸ We ignore the knife-edge case in which no firm violates (SC) but (SC) holds with equality for some firm \( n' \). The equilibrium outcomes characterized in Case I of Proposition 2 remain equilibrium outcomes in this case, but there can be additional equilibrium outcomes.
buying at the current total price (i.e., if \( \overline{f} + \overline{a} > v - \underline{u} \)), this surprise leads her not to buy, so the firm can attract her by unshrouding only if it cuts the additional price by a discrete margin. Since this may be unprofitable, the firm may prefer not to unshroud.

As the flip side of the above logic, if purchasing at the total market price is optimal (i.e., if \( \overline{f} + \overline{a} \leq v - \underline{u} \)), then despite the negative surprise a consumer is willing to buy from a firm that unshrouts and undercuts competitors’ additional prices by a little bit, so in this case a deceptive equilibrium does not exist. Our model therefore says that deception—and positive profits for firms—must be associated with suboptimal consumer purchase decisions.\(^{19}\)

Part II of Proposition 2 establishes that (SC) is not only sufficient, but also necessary for profitable deception to occur. To understand the rough logic, assume toward a contradiction that all firms shroud with positive probability. Then, a firm can ensure positive profits by shrouding and choosing prices \((\overline{f}, \overline{a})\), so that in equilibrium firms earn positive expected profits. Since a firm that sets the highest total price when unshrouding has zero market share if some other firm unshrouts, to earn positive profits it must be that with positive probability all rivals set higher total prices when shrouding. Whenever a firm sets one of these high total prices, the only event in which it can make profits is when shrouding occurs, so we can think of its incentives by conditioning on this event. Doing so, arguments akin to those above imply that in this high range firms set prices \((\overline{f}, \overline{a})\), and hence firm \(n\) earns \(s_n(\overline{f} + \overline{a} - c_n)\). But firm \(n\) can earn \(v - \underline{u} - c_n\) by unshrouding, so a firm that violates (SC) prefers to unshroud.

Note that if a consumer learns about the additional price before paying any of it and can at that point switch without incurring any cost, then \(\overline{a} \leq v - \underline{u}\) must hold. This implies that if in addition \(\overline{f} = 0\), (SC) is violated, so a deceptive equilibrium cannot occur. In our main applications, however, these prerequisites for (SC) to fail are violated, and in particular there is no consideration

\(^{19}\) Part I of Proposition 2 also states that whenever a deceptive equilibrium exists, there is also an equilibrium in which unshrouting occurs with probability one and—because prices are then transparent and the standard Bertrand logic holds—the Bertrand outcome obtains. As in the case of a non-binding price floor (Proposition 1), since unshrouting is costless, if a firm unshrouts it is (weakly) optimal for other firms to unshroud as well. Unlike in the case of a non-binding price floor, however, there is no equilibrium in which unshrouting occurs with an interior probability. Our proof of this claim elaborates on the following contradiction argument. Suppose that shrouding occurs with an interior probability, and consider a firm that unshrouts with positive probability and sets the highest price of any firm conditional on unshrousing. When setting this highest price, the firm earns profits only if all other firms shroud, and even then it earns at most \(v - \underline{u} - c_n\). If it instead shrouds and sets \((\overline{f}, \overline{a})\), then it earns at least \(s_n(\overline{f} + \overline{a} - c_n)\) if all other firms also shroud. By (SC), therefore, the firm prefers to shroud with probability 1, a contradiction.
that limits the potential size of $\bar{p}$ relative to $v - u$. For credit cards, the consumer accumulates debt before she realizes her true $\beta$, so she cannot walk away without consequences. For mutual funds, the consumer pays management fees before realizing it. More generally, in many or most applications, going back to the pre-contracting situation is quite costly. In the next subsection, we analyze in more detail circumstances under which profitable deception does versus does not occur.

### 3.3 Key Economic Implications

We first distinguish two cases according to whether the product is socially valuable or wasteful.

**Non-vanishingly socially valuable product (there is an $\epsilon > 0$ such that $v - u > c_n + \epsilon$ for all $n$).** In this case, the right-hand side of (SC) is positive and bounded away from zero. Hence, a firm with a sufficiently low $s_n$ violates (SC)—since it earns low profits from deception, it prefers to attract consumers through unshrouding—and thereby guarantees that only a zero-profit unshrouded-prices equilibrium exists. This implies that a deceptive equilibrium can exist if the number of firms is sufficiently small, but not if the number of firms is large—as some firm will then violate (SC). Furthermore, an increase in the number of firms can induce a regime shift from a high-price, deceptive equilibrium to a low-price, transparent equilibrium.\(^{20}\)

**Socially wasteful product ($v - u < c_n$ for all $n$).** In this case, the right-hand side of (SC) is negative while the left-hand side is positive. Hence, a deceptive equilibrium exists regardless of the industry’s concentration and other parameter values:

**Corollary 1 (Wasteful Products).** Suppose $f > c_{\text{min}} - \bar{a}$ and $v - u < c_n$ for all $n$. Then, a profitable deceptive equilibrium exists.

This perverse result has a simple logic: since a socially wasteful product cannot be profitably sold once consumers understand its total price, a firm can never profit from coming clean.

Assuming that the outside option is an appropriately chosen alternative product, our main applications seem to fit this case of our model. A low-cost debit card tied to the consumer’s bank

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\(^{20}\) The reason for stating our result for non-vanishingly socially valuable products is that if the social value of a firm’s product $(v - u - c_n)$ could be arbitrarily small, then even a firm with low profits from deception might not be willing to unshroud. A precise condition for when unshrouding must occur is $N > (f + \bar{a})/\epsilon$. Then, $s_n < \epsilon/(f + \bar{a})$ for some $n$, and for this $n$ we have $s_n(f + \bar{a} - c_n) < \epsilon < v - u - c_n$, in violation of (SC).
account serves the same convenience benefit as a credit card, so it is a superior outside option for many consumers. Consistent with this perspective, Laibson et al. (2007) estimate that owning a credit card makes the average household whose head has a high-school degree but not a college degree worse off by the equivalent of paying $2,000 at age 20. Similarly, although not everyone agrees with this view, many researchers believe that because few mutual-fund managers can persistently and significantly outperform the market, most managed funds are inferior to low-cost index funds.²¹

And while a sharply increasing mortgage-payment schedule makes sense for the few consumers who can look forward to drastic increases in income (as argued by Cocco 2013), the vast majority in this market would have been better served by traditional mortgages. Our model says that such products are sold and remain profitable in a seemingly competitive market not despite, but exactly because they are socially inferior to alternatives.

Indeed, the pricing structure of credit cards and mutual funds is consistent with our deceptive equilibrium. DellaVigna and Malmendier (2004) document that most of the popular credit cards in the US offer a $0 annual fee and a high regular interest rate.²² And many mutual funds have zero salient front loads, but substantial management fees.²³

The different logic of socially valuable and socially wasteful industries in our model yields two potentially important further points. First, our theory implies that if an industry experiences a lot of entry and does not come clean in its practices, it is likely to be a socially wasteful industry. Second, our theory suggests a general competition-impairing feature in valuable industries that is not present in wasteful industries: to reduce the motive to deviate from their preferred positive-

²¹ See, for example, Gruber (1996), Carhart (1997), Kosowski, Timmermann, Wermers and White (2006), and French (2008). Berk and Green (2004) and Berk and van Binsbergen (2012), however, argue that the evidence is consistent with the hypothesis that most managers can outperform the market, and that managed funds are not inferior to index funds.

²² There are two main categories of exceptions to the $0 annual fee. First, charge cards—which require repayment within 30 days—often have an annual fee. This is consistent with the prediction of our multi-product model below, where sophisticated consumers who anticipate paying costly interest prefer to use a charge card with a higher up-front price to a credit card with a lower up-front price. Second, some cards offered to consumers with bad credit have an annual fee. This presumably compensates issuers for credit risk, a consideration that is outside our model.

²³ In the US market, our model best fits level-load and no-load mutual fund shares ($6,748 billion in total net assets in 2012 according to the ICI fact book 2013, Figure 5.11), which generally have zero front loads and hence face a binding floor on the up-front price. Front-end load shares ($1881 billion in total net assets in 2012) have positive front loads, and are therefore a less obvious fit for our model with a binding price floor. But the typical such fund uses the front load largely to pay intermediaries’ commissions, so the up-front price collected by the fund is often at zero and hence again at the binding floor.
profit deceptive equilibrium in a valuable industry, each firm wants to make sure competitors earn sufficient profits from shrouding. This feature is likely to have many implications beyond the current paper (as, for instance, for innovation incentives in Heidhues et al. 2012), and implies that wasteful industries may sometimes be more fiercely competitive than valuable ones.

*Misprediction of value.* We now turn to some implications of Proposition 2 for consumer mis-prediction of value. An economically relevant possibility is one where consumers underestimate the value by a significant amount.

**Corollary 2** (Unanticipated Valuable Add-On). If \( v - \tilde{v} \geq \bar{\pi} \), then in any equilibrium unshrouding occurs with probability one.

The condition \( v - \tilde{v} \geq \bar{\pi} \) holds if consumers are completely unaware of a valuable add-on that they can purchase after getting the product, and the outside option does not have such add-ons. In this case, a deceptive equilibrium does not exist. Intuitively, if consumers are unaware of something they would be happy to pay for, then unshrouding cannot make them less willing to buy the product, so unshrouding and undercutting competitors is a profitable deviation from a candidate deceptive equilibrium. For instance, if consumers are unaware of some valuable features of smartphones, producers have all the incentive to explain them.

Corollary 2 has a potentially important implication regarding the interpretation of unanticipated borrowing in the credit-card market. Suppose that—unlike in our model with naive time-inconsistent borrowers—unanticipated borrowing generates unanticipated value for a consumer, for instance by facilitating an unexpected important expense.\(^{24}\) Then, Corollary 2 implies that unshrouding should take place. From the perspective of our model, therefore, the observation that unshrouding has not taken place implies that expensive credit-card borrowing is not only unanticipated, but—consistent with a time-inconsistent setup—also initially unwanted by consumers.

Beyond time inconsistency, our applications identify two further reasons for an unanticipated payment not to go along with unanticipated product value. First, consumers may simply fail to

\(^{24}\) Formally, we can modify our model in Section 2.2 by assuming that the consumer is time consistent, values the convenience benefit of the card at \( \tilde{v} \), and can in period 2 spend 1 on something she values at \( 1 + v - \tilde{v} \). She does not anticipate this need ex ante, and if she decides to spend, she cannot repay her credit-card balance. This implies that she is willing to pay at most \( \bar{\pi} = v - \tilde{v} \) to delay repayment.
anticipate a charge—such as a late fee for credit cards or management fee for mutual funds—that is imposed without creating any additional value. Second, consumers may understand that there is a valuable add-on—e.g., that a printer accommodates cartridges for printing—but explicitly or implicitly underestimate the add-on price or not think about add-on expenses when buying the base product.

Now suppose that consumers overestimate the product’s value ($\bar{v} > v$). Holding other parameters constant, this does not affect (SC), and hence does not affect whether a deceptive equilibrium exists in our model. Going slightly beyond our assumptions, however, consumers’ overestimation of value can facilitate the creation of a socially wasteful industry by making consumers more willing to buy the product. In particular, if $v - f < u < \bar{v} - f$, then a profitable deceptive equilibrium exists when consumers perceive the value to be $\bar{v}$, but not when they perceive the value to be $v$. For instance, the popularity of managed mutual funds is probably due not only to consumers’ underestimation of their expenses, but also to consumers’ overestimation of their gross returns.

4 Sophisticated Consumers and Multi-Product Markets

In this section, we incorporate sophisticated consumers—who observe but cannot avoid additional prices—into our model, considering both single- and multi-product markets. We assume that the proportion of sophisticated consumers is $\kappa \in (0, 1)$, that the price floor is binding ($f > c_n - a$ for all $n$), and that consumers perceive the product’s value correctly ($\bar{v} = v$).

4.1 Sophisticated Consumers in Our Basic Model

Notice that in any deceptive equilibrium, sophisticated consumers do not buy the product: if they did, they would buy from a firm with the lowest total price, and such a firm would prefer to either undercut equal-priced competitors and attract sophisticated consumers, or (if there are no equal-priced competitors) to unshroud and attract all consumers. Hence, the total price of any firm exceeds $v - u$. This implies that if firm $n$ unshrouds, it maximizes profits by setting a total price equal to $v - u$, attracting all naive and sophisticated consumers. Combining these considerations,
unshrouding is unprofitable if

\[(1 - \kappa)s_n(f + \sigma - c_n) \geq v - \mu - c_n,\]  

and a deceptive equilibrium exists if and only if Condition (1) holds for all \(n\).

If the product is socially valuable, the condition for a deceptive equilibrium to exist is stricter if sophisticated consumers are present than if they are not. Intuitively, while sophisticated consumers do not buy the product when the additional price is high, they can be attracted by a price cut, creating pressure to cut the additional price—and by implication also to unshroud. This result is consistent with the insights of Gabaix and Laibson (2006) and Armstrong and Vickers (2012) that if the proportion of sophisticated consumers is sufficiently high, transparency and efficiency obtains. But if the product is socially wasteful, then as before a profitable deceptive equilibrium always exists. The reason is simple: sophisticated consumers never buy a socially wasteful product in equilibrium, so their presence is irrelevant—firms just exploit naive consumers.

**4.2 Sophisticated Consumers with an Alternative Transparent Product**

In many markets for deceptive products, alternatives that are more transparent than and arguably superior to the deceptive products exist. Motivated by this observation, we modify our model above by assuming that each firm has a transparent product in addition to the potentially deceptive product we have described. Because we think of the alternative product as an endogenous outside option, we set consumers’ utility from not buying, \(u\), to zero. Consumers’ value for the transparent product is \(w > 0\), and firm \(n\)’s cost of producing it is \(c_{wn}^w \geq 0\), with at least two firms having the lowest cost \(c_{\text{min}}^w = \min_n \{c_{n}^w\}\). Crucially, we posit that product \(w\) is socially valuable \((w - c_{\text{min}}^w > 0)\), and is not inferior to product \(v\): \(w - c_{\text{min}}^w \geq v - c_{\text{min}}\). Consumers are interested in buying at most one product. Firms simultaneously set the up-front and additional prices for product \(v\), the single transparent price for product \(w\), and decide whether to unshroud the additional prices of product \(v\). If consumers weakly prefer buying and are indifferent between a number of firms in the market for product \(v\) or \(w\), firms split the respective market in proportion to \(s_n\) or \(s_{wn}^w\), respectively; and if consumers are indifferent between products \(v\) and \(w\), a given positive fraction of them chooses
product \( w \). We define a Bertrand outcome as a situation in which consumers buy a social-surplus-maximizing product at a total price equal to the product’s lowest marginal cost, and firms earn zero profits; this is the outcome that would obtain in classical Bertrand price competition.

In the case of credit cards, we think of a credit card as the inferior product, and of a debit card as the superior product, with the convenience benefit of the two being the same \((v = w)\). In the case of mutual funds, we think of a high-cost managed fund as the inferior product, and of a low-cost index fund as the superior product. We let \( v \) and \( w \) be consumers’ respective values for the funds before expenses, with \( v > w \). There are two reasons that consumers’ valuation for a managed fund might be higher. First, similarly to the notion of “money doctors” in Gennaioli, Shleifer and Vishny (2012), investors might get an emotional benefit from leaving their money with a specific person. Second, it might be that managed funds produce higher gross returns, either real or (in a variant we argue below is almost equivalent) perceived.

We now turn to the analysis of our model.

**Proposition 3** (Profitability of Inferior Products). Suppose \( f_n > c_n - \bar{a} \) for all \( n \) and \( v - f_n > w - c^w_{min} \), and consider any proportion \( \kappa \) of sophisticated consumers and any shares \( s_n, s^w_n \). Then, a deceptive equilibrium exists. In any deceptive equilibrium, firms sell the superior product \( w \) to sophisticated consumers and earn zero profits on it, and they sell the inferior product \( v \) to naive consumers and earn positive profits on it. In any other equilibrium, unshrouding occurs with probability one and the Bertrand outcome obtains.

Proposition 3 hinges on the condition \( v - f_n > w - c^w_{min} \). For payment cards, \( v = w \), so the condition holds under the reasonable assumption that \( f_n < c^w_{min} \). For mutual funds, \( v > w \) and \( f = 0 \), so the condition holds under the reasonable assumption that \( c^w_{min} \) is very low. Proposition 3 says that a positive-profit equilibrium in which naive consumers are deceived then exists, and all profits firms earn (in any equilibrium) must derive from selling the inferior product.

The intuition is in two parts. First, \( v - f_n > w - c^w_{min} \) implies that because naive consumers ignore the additional price, they mistakenly find the inferior product \( v \) more attractive than the superior product \( w \). As a result, the two types of consumers separate, so—quite in contrast to the message of Section 4.1—sophisticated consumers do not create an incentive to unshroud the inferior
product’s additional prices. Second, if a firm unshrouded, naive consumers would simply switch to the superior product, so the firm would not attract anyone to its inferior product. Using the logic of our single-product model, the superior product guarantees a deceptive equilibrium with positive profits from the inferior product by rendering the inferior product socially wasteful in comparison.\(^{25}\)

The above insights imply that for socially valuable products, the existence of a superior product can expand the scope of profitable deception and make naive consumers worse off: while unshrouding and marginal-cost pricing can obtain when there is no other product in the market, the opposite happens with the superior product around. Similarly—but perhaps even more perversely—the addition of an inferior product can also expand the scope of profitable deception, creating firms’ profit base in (or shifting it to) the new, inferior product.

Note that Proposition 3 holds for any market shares \(s_n, s^w_n\) for the two products. In particular, this means that in our model not even a “specialist” in product \(w\)—a firm that sells mostly the superior product—has an incentive to unshroud. Intuitively, competition reduces the margin on the superior product to zero, so whether or not it unshrouds a specialist makes no money from the superior product.\(^{26}\)

The results of Proposition 3 are only strengthened if—as when investors overestimate the gross return of a managed mutual fund—consumers’ perceived value for the inferior product when shrouding occurs is \(\tilde{v} > v\). Then, the sorting condition becomes weaker—\(\tilde{v} - f > w - c^w_{\min}\)—as naive consumers are more prone to choose the inferior product. If the weaker sorting condition holds, a deceptive equilibrium exists for the same reason as in Proposition 3: given that a superior product

\(^{25}\) The full separation between naive and sophisticated consumers relies on the assumption that sophisticated consumers cannot avoid the additional price. In some situations, there are likely to be sophisticated consumers who can avoid the additional price. In our credit-card application, for instance, sophisticated consumers may not be tempted by credit-card usage, or may repay all charges within the grace period. These sophisticated consumers prefer the inferior product, and—as in previous work—are cross-subsidized by naive consumers. If there are sufficiently few of these consumers, however, the inferior product still remains profitable.

\(^{26}\) Although we have exogenously imposed that product \(w\) is transparent, this will often arise endogenously even if firms make an unshrouding decision regarding both products. Clearly, under the condition of Proposition 3, an equilibrium in which product \(v\) is shrouded and product \(w\) is unshrouded exists in that case as well. If in addition \(w > v\) and there are sufficiently many firms in the market, the only profitable equilibrium is the one in which the superior product is unshrouded and the inferior product is shrouded. Consider, for example, a candidate equilibrium in which the superior product \(w\) is shrouded. Then, naive consumers must be buying product \(w\); otherwise, a firm could attract all these naive consumers by setting prices \((f, \tilde{u})\) on product \(w\), and for a low-profit firm this would be a profitable deviation. But if naive consumers are buying product \(w\), a low-profit firm has an incentive to unshroud product \(w\) in order to capture this socially valuable market.
is available at marginal cost, no firm can make money once consumers perfectly understand the nature of the products, so no firm has an incentive to unshroud.

Going beyond the setting of our model, the insights above have an immediate implication for the marketing of superior and inferior products: because the inferior product is more profitable, firms have an incentive to push it on consumers who may not otherwise buy it, further decreasing social welfare by expending resources to sell an inferior good. First, as formally analyzed in Murooka (2013), firms may pay intermediaries to convince consumers to buy the inferior product. Second, firms may engage in persuasive advertising to induce demand for the inferior product, with the—to the best of our knowledge—novel implication that persuasive advertising is directed exclusively toward an inferior good. Third, firms may inform consumers unaware of the inferior product of the product’s existence, yet not do the same for the superior product. Fourth, firms may make costly (real or perceived) improvements to the inferior product to make it more attractive to consumers. In some markets, firms may burn all of their gross profits on pushing inferior products.

5 Extensions and Modifications

In this section, we discuss various extensions and modifications of our theory. Unless otherwise stated, we continue to assume that the price floor is binding.

5.1 Market Power

We consider the effect of market power of a simple kind—we assume that firm $n$ is strictly more efficient than competitors ($c_n < c_{n'}$ for all $n' \neq n$). In this case, the condition for when a deceptive equilibrium exists in our single-product model remains unchanged. Intuitively, competitors’ costs do not affect their prices when shrouding occurs, so these costs do not play a role in determining whether firm $n$ wants to unshroud.

Market power does have an interesting effect in our multi-product model. In particular, suppose that firm $n$ has market power in the market for the superior product: $c^w_{\min} - c^w_n \equiv M > 0$, where $c^w_{\min} \equiv \min_{n' \neq n} c^w_{n'}$. For this market, we make the common assumption that no firm charges a price below cost, so that in equilibrium firm $n$ charges $c^w_{\min}$ and attracts all consumers. Then:
Proposition 4 (Market Power in the Superior Product). Suppose \( f > c_n' - \alpha \) for all \( n' \) and \( v - f > w - c_w' \). Then, there is a deceptive equilibrium in which all firms sell product \( v \) to naive consumers at prices \( f, \alpha \) and firm \( n \) sells product \( w \) at price \( c_w'_{\min} \) to sophisticated consumers, if and only if \( M \leq s_n(f + \alpha - c_n) \).

Proposition 4 implies that if firm \( n \)'s market power is larger than its share of deceptive profits, then the deceptive equilibrium from our competitive model does not exist: firm \( n \) would rather unshroud and induce everyone to buy the superior product. Such a situation could arise, for instance, if firm \( n \) sells almost only the superior product \( (s_n \approx 0) \). This observation explains why some specialists in superior products—e.g., Vanguard in the market for mutual funds—have tried to educate consumers about the inferiority of alternative products. But if firm \( n \)'s market power is lower than its share of deceptive profits, then it prefers not to educate consumers. Similarly to Section 4.2, the intuition is that deception creates a large profit margin on the inferior product despite competition, and unshrouding the additional prices of this product only leads consumers to buy the less profitable superior one.

Going further, the extent of unshrouding can be even more limited if unshrouding is costly. Suppose, as often seems to be the case in reality, that competition in the superior product is relatively fierce \( (M \) is small). Then, firm \( n \) is unwilling to educate naive consumers unless the unshrouding cost is small. And if it is relatively cheap to educate a low fraction of consumers, but much more expensive to educate a significant fraction, firm \( n \) would choose the former, limited education. These considerations help explain why consumer education is often limited in practice.

Market power also has an interesting effect in our single-product model with a non-binding price floor when firms are asymmetric in their ability to exploit consumers, and the product is socially valuable. Suppose \( c_n < \min_{n' \neq n} c_{n'} \), and denote by \( \pi_n \) the maximum additional price firm \( n \) can charge. Then, in a deceptive equilibrium the lowest \( f_{n'} \) firm \( n' \) is willing to charge equals \( c_{n'} - \alpha_{n'} \), so firm \( n \) cannot make profits greater than \( \min_{n' \neq n}(c_{n'} - \alpha_{n'}) - (c_n - \pi_n) \). If it unshrouds, however, firm \( n \) can make \( \min_{n' \neq n} c_{n'} - c_n \), so if the latter is greater a deceptive equilibrium does not exist. Intuitively, if one firm has lower marginal cost but another can impose a higher additional price, then the former firm has an incentive to unshroud to gain a competitive edge. This suggests that
even when the price floor is not binding, a deceptive equilibrium is more likely to occur with socially wasteful products.

5.2 Inefficiencies in Collecting Revenue from Additional Prices

In many settings, there is a distortion associated with collecting additional prices. In the printer market, for example, the additional price results from high cartridge prices consumers initially do not understand; but once they do, they reduce their demand in response, generating an inefficiency in cartridge usage.

To capture such inefficiencies, we assume that when a firm chooses the additional price \( a \in [0, a] \), it imposes a utility loss of \( a \) on a consumer and earns profits of \( \pi(a) \), where \( \pi(0) = 0 \), \( \pi(a) \) is differentiable, and \( \pi'(a) < 1 \) for all \( a \). We denote the maximum profits a firm can collect from the additional price by \( \pi^* \), and let \( a^* \) be an additional price that achieves these profits.\(^{27}\)

This modification has little effect on our qualitative results. Suppose first that the price floor is not binding. If \( c_n = c_{min} \) for at least four firms, then for any \( \psi \in [0, 1] \) there exist equilibria in which unshrouding occurs with probability \( \psi \). In particular, suppose two firms choose prices \( f = c_{min} - \pi^*, a = a^* \), two choose \( f' = c_{min}, a' = 0 \), all others set \( f_n = c_n, a_n = 0 \), all but one of the firms shroud, and one firm unshrouds with probability \( \psi \). Then, consumers choose \( f, a \) if shrouding occurs and \( f', a' \) if unshrouding occurs, so in either case no firm can make positive profits, and hence no firm has an incentive to deviate.

If the price floor is binding, then, by the same argument we used to derive (SC), a deceptive

\(^{27}\) For completeness, we formally establish that this specification captures in a reduced form a setting where the additional price arises from an unexpectedly high add-on price. Suppose that a consumer’s utility from a printer is \( v(q) \), where \( q \) is the amount of cartridges she uses, \( v \) is twice differentiable and strictly concave, \( v'(0) \) is finite, and the consumer has a satiation point. When purchasing her printer, the consumer expects cartridges to be free (for simplicity), but when choosing the number of cartridges, she correctly perceives their price. Finally, the marginal cost of a cartridge is 0.

A consumer’s ex-ante value for the printer is \( v = v(v^{-1}(0)) \). If the cartridge price is \( \hat{a} \in [0, v'(0)] \), then the consumer chooses \( q(\hat{a}) \) satisfying \( v'(q(\hat{a})) = \hat{a} \), so that her true utility is \( v(v^{-1}(\hat{a})) - \hat{a}v^{-1}(\hat{a}) \). We define the additional price as the difference between her ex-ante value and the true value: \( a = \hat{a}v^{-1}(\hat{a}) + v(v^{-1}(0)) - v'(0) \). Since \( a \) is strictly increasing in \( \hat{a} \) and has the range \( [0, v(v^{-1}(0)) - v(0)] \), for any \( a \) in this range there is a unique \( \hat{a}(a) \) generating it. Hence, the firm’s profit from the additional price is \( \pi(a) = \hat{a}(a)v^{-1}(\hat{a}(a)) \). Differentiating \( a - \pi(a) = v(v^{-1}(0)) - v'(0) \) totally with respect to \( a \) shows that \( \pi'(a) < 1 \).
equilibrium exists if and only if

\[ s_n(f + \pi^* - c_n) \geq v - u - c_n \text{ for all } n. \]

Both with a non-binding and with a binding price floor, the inefficiency in imposing additional prices increases the welfare cost of deception compared to a situation where \( \pi(a^*) = a^* \). In the former case, consumers (who get utility \( v - c_{\text{min}} - (a^* - \pi^*) \)) bear the welfare cost, whereas in the latter case firms (who get revenue \( f + \pi^* \)) do.

## 5.3 Downward-Sloping Product Demand

Suppose there is heterogeneity in \( v, u = 0 \), and let \( D(f) \) be the demand curve induced by the distribution of naive consumers’ valuations. We suppose that \( D(f) > 0 \) and that there is a choking price (i.e., an \( f' \) such that \( D(f) = 0 \) for \( f \geq f' \)). Then, as an analogue of Proposition 2, a deceptive equilibrium with prices \((f, \pi)\) exists if and only if

\[ s_n D(f)(f + \pi - c_n) \geq \max_{f \in [\underline{f}, \overline{f} + \pi]} D(f)(f - c_n) \text{ for all } n. \tag{2} \]

The left-hand side of Inequality (2) is the profit firm \( n \) earns in the candidate deceptive equilibrium, while the right-hand side is the profit the firm earns when unshrouding and optimally undercutting competitors. To understand why unshrouding can be unattractive, note that unshrouding and slightly undercutting competitors leads consumers with values between \( \underline{f} \) and \( \underline{f} + \pi \) not to buy, discretely reducing industry demand. This means that the optimal unshrouding strategy discretely reduces the markup (from \( \underline{f} + \pi - c_n \) to \( f - c_n \)) or discretely reduces demand (from \( D(\underline{f}) \) to \( D(f) \)), or both, undermining the standard Bertrand argument. Furthermore, if the product is socially wasteful to produce (\( D(c_{\text{min}}) = 0 \)), the right-hand side of Inequality (2) is zero, so a deceptive equilibrium always exists. But if the product is socially valuable and there are sufficiently many firms, at least one firm would choose to unshroud, eliminating the deceptive equilibrium.\(^{28}\)

\(^{28}\)Consider also what happens when there are sophisticated consumers in the population who are not separated by a superior transparent product, and who are heterogeneous in \( v \). So long as a positive fraction of sophisticated consumers buys the product despite their knowing about the high additional price, a cut in the additional price attracts all these sophisticated consumers, so that an arbitrarily small fraction of these consumers induces some competition in the additional price. Whenever shrouding can be maintained, however, firms’ profits are not driven to
6 Discussion

6.1 Some Policy Implications

The suboptimal nature of equilibrium in our models raises the question of whether a social planner can improve outcomes. We discuss here a few issues related to potential policy interventions, though a fuller exploration of this difficult question is outside the scope of this paper.  

As a simple observation, the possibility in our single-product model that the industry shifts from deceptive to transparent pricing as the number of firms grows identifies a potential consumer-protection benefit of competition policy. Nevertheless, because in our multi-product model a firm specializing in the superior product has more incentive to educate consumers if it has market power than if it does not, competition is not uniformly beneficial in our model.

An alternative, and in the context of deception more direct, way of intervening in the market is to regulate hidden charges. To see the effects of such regulations in our setting, consider a policy that lowers the maximum additional price firms can charge from $\bar{a}$ to $\bar{a}' < \bar{a}$ in the range where the price floor is binding. If (SC) still holds with $\bar{a}$ replaced by $\bar{a}'$, firms charge $(f, \bar{a}')$ in the new situation, so that the decrease in the additional price benefits consumers one to one. And if the decrease in the additional price leads to some firm violating (SC), the market becomes transparent and prices drop further. This provides a counterexample to a central argument brought up against many consumer-protection regulations: that its costs to firms will be passed on to consumers.

Zero because—similarly to the “captive” consumers in Shilony (1977) and Varian (1980)—naive consumers provide a profit base that puts a lower bound on firms’ total profit. Furthermore, it is clear that these profits can be sufficient to deter unshrouding.

See Congdon, Kling and Mullainathan (2011) and especially Agarwal, Driscoll, Gabaix and Laibson (2009) for further discussions of the advantages and disadvantages of different types of policies aimed at mitigating the effects of consumer mistakes.

For example, the Credit Card Accountability, Responsibility, and Disclosure (Credit CARD) Act of 2009 limits late-payment, over-the-limit, and other fees to be “reasonable and proportional to” the consumer’s omission or violation, thereby preventing credit-card companies from using these fees as sources of extraordinary ex-post profits. Similarly, in July 2008 the Federal Reserve Board amended Regulation Z (implementation of the Truth in Lending Act) to severely restrict the use of prepayment penalties for high-interest-rate mortgages. Regulations that require firms to include all non-optional price components in the up-front price—akin to recent regulations of European low-cost airlines—can also serve to decrease $\bar{a}$.

It is important to note, however, that the direct regulation of additional prices is unlikely to be fully effective as a general approach to combating deception. As Agarwal et al. (2009) discuss in their setting, formulating ex-ante guidelines for which charges are acceptable seems extremely difficult, and individually researching and approving each new financial product is very costly. One potential response to this challenge is for researchers to develop portable...
Beyond the main theme we have developed above—that in equilibrium socially wasteful and inferior products might be sold profitably to consumers—a further important general message emerges from our results: that deception is likely to be more widespread and economically harmful in the case of a binding than in the case of a non-binding price floor. With a binding price floor, firms strictly prefer deception to no deception, so they may take steps, such as engaging in “exploitative innovation” (Heidhues, Kőszegi and Murooka 2012a), to ensure that a deceptive equilibrium exists. In addition, less efficient producers may enter the market and sell positive amounts, and firms may spend on pushing and hence expanding the market share of inferior products. Finally, if there is some market power and firms are asymmetric, for socially valuable products a deceptive equilibrium is more likely with a binding price floor. These observations have two policy implications. First, if policymakers need to decide where to concentrate their regulatory efforts, they should—for both distributional and efficiency reasons—prioritize industries where the price floor is binding. One sign of such an industry is the presence of surprisingly large profits given the level of competition. Second, in as much as the price floor arises for regulatory reasons, policymakers can attempt to relax the floor with improved regulation. In the context of mutual funds, for example, Ayres and Curtis (2014) propose a regulatory framework to facilitate competition through “relative performance guarantees”—whereby low-cost funds can offer to cover any shortfall in net performance relative to high-cost competitors. While there may be practical difficulties in implementation, this would allow funds to offer (what we call) negative up-front prices. If consumers’ values for the product are heterogeneous, however, a disadvantage of this kind of policy is that it can induce more consumers to purchase an inferior product.

empirical methods for detecting additional prices and consumer mistakes from commonly available data. As a simple example, the empirical approach of Chetty, Looney and Kroft (2009) takes advantage of the general observation that if a consumer reacts to changes in financially identical price components differently, then she is not paying sufficient (relative) attention to some components of the price. Another response to the same challenge is to develop policies that do not rely on the social planner knowing which price components consumers underappreciate. For example, Murooka (2013) shows that when financial intermediaries motivated by commissions are used by firms to sell deceptive products, capping commissions, or requiring commissions to be uniform across products, can improve welfare even if the regulator cannot identify additional prices.
6.2 Testable Predictions and Existing Evidence

The overarching main prediction of our paper is that wasteful and inferior products facilitate profitable deception and hence should go hand in hand with it. This central prediction is difficult to test directly. Nevertheless, when the price floor is binding, our theory has a number of auxiliary empirical predictions that are testable and supported by some evidence.

First, exactly in line with our prediction, firms disproportionately push inferior products is confirmed by evidence in a number of domains. Mullainathan, Nöth and Schoar (2011) document that (through intermediaries) firms tend to push inferior products in the mutual-fund market. Consistent with the perspective that this hurts consumers, Bergstresser, Chalmers and Tufano (2009) find that broker-sold funds deliver lower risk-adjusted returns than do direct-sold funds. And Agarwal and Evanoff (2013) document that borrowers eligible for cheaper and superior loans were steered into subprime-like mortgages by brokers.

Second, consistent with the prediction that consumers may benefit from regulatory controls on one component of the price (arguably hidden charges) that appears fully fungible with another component, Bar-Gill and Bubb (2012) and Agarwal, Chomsisengphet, Mahoney and Stroebel (forthcoming) document that the Credit CARD Act—while succeeding in lowering regulated fees—did not lead to an increase in unregulated fees or a decrease in the availability of credit, so that it lowered the total cost of credit to consumers.

Third, our model predicts that all firms produce the inferior product, but only the most efficient firms produce the superior product, so (to the extent that there is cost heterogeneity across firms) active firms’ market shares tend to be smaller in the market for the inferior product than in the market for the superior product. Although some alternative models may make the same prediction, this prediction is consistent with casual evidence that low-cost index funds tend to be larger than managed funds. For instance, while index funds hold 10% of mutual-fund assets, seven of the ten largest mutual funds are index funds.\(^{32}\)

Relatedly, our model predicts that (assuming entry costs are sufficiently low) inferior products

\(^{32}\)For the top mutual funds, see http://www.marketwatch.com/tools/mutual-fund/top25largest. The ICI Fact Book 2013 (http://www.ici.org/pdf/2013_factbook.pdf) reports that index funds held $1.3 trillion of the industry’s $13 trillion in net assets in 2012.
may attract massive entry. This prediction is consistent with long-term trends in the mutual-fund industry: since 1975, just before low-cost index funds were introduced in the market, the number of actively managed funds increased from 426 to 7,223. In a classical model, inferior products should of course experience massive exit rather than entry.

7 Related Theoretical Literature

In this section, we discuss theories most closely related to our paper. Our paper goes beyond all of the existing work in making the central prediction that socially wasteful and inferior products tend to be sold more profitably than better products, and identifying a number of additional implications of this insight. Indeed, in previous theories deceptive products are socially valuable.

In Gabaix and Laibson’s (2006) model, firms sell a base good with a transparent price and an add-on with a shrouded price, and consumers buying the base good can avoid the add-on by undertaking costly steps in advance. Gabaix and Laibson’s main prediction is that unshrouding the add-on prices can be unattractive because it turns profitable naive consumers (who fail to avoid the expensive add-on) into unprofitable sophisticated consumers (who avoid the add-on). Although the precise trade-off determining a firm’s decision of whether to unshroud is different, we start from a similar insight, and use it to explore new implications.

Clarifying and adapting Gabaix and Laibson’s theory, Armstrong and Vickers (2012) investigate a model of contingent charges in financial services, and apply it the UK retail banking industry. Consistent with our perspective, they argue that the “free if in credit” model—whereby firms charge nothing for account maintenance, and rely on contingent charges, such as overdraft protection, for revenue—can be naturally explained by the presence of naive consumers.

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33 See ICI Fact Book 2013 (http://www.ici.org/pdf/2013_factbook.pdf), page 142. The number of funds in 2012 was 7,596, of which 373 were index funds. This number does not include mutual funds that own other mutual funds.

34 Relatedly, Piccione and Spiegler (2010) characterize how firms’ ability to change the comparability of prices through “frames” affects profits in Bertrand-type competition. If a firm can make products fully comparable no matter what the other firm does—which is akin to unshrouding in our model and that of Gabaix and Laibson (2006)—the usual zero-profit outcome obtains. Otherwise, profits are positive. Piccione and Spiegler highlight that increasing the comparability of products under any frame through policy intervention will often induce firms to change their frames, which can decrease comparability, increase profits, and decrease consumer welfare. Investigating different forms of government interventions, Ko (2012) and Kosfeld and Schüwer (2011) demonstrate that educating
In research complementary to ours, Grubb (forthcoming) considers services, such as cellphone calls or bank-account overdraft protection, for which consumers may not know the marginal price, and asks whether requiring firms to disclose this information at the point of sale increases welfare. If consumers correctly anticipate their probability of running into high fees, such price-posting regulation can actually hurt because it interferes with efficient screening by firms. If consumers underestimate their probability of running into fees, in contrast, fees allow firms to extract more rent from consumers, and price posting prevents such exploitation.\footnote{Our theory also builds on a growing literature in behavioral industrial organization that assumes consumers are not fully attentive, mispredict some aspects of products, or do not fully understand their own behavior. See for instance DellaVigna and Malmendier (2004), Eliaz and Spiegler (2006), Spiegler (2006a, 2006b), Laibson and Yariv (2007), Grubb (2009), and Heidhues and Kőszegi (2010).}

Our theory is also related to the classical industrial-organization literature on markets in which firms sell a primary as well as a complementary product, consumers are locked in after the primary purchase, and consumers initially do not observe a firm’s price for the complementary good. The main goal of this literature is to analyze the effect of market power in the complementary-good market on prices and welfare. Closely related to our benchmark Proposition 1, Lal and Matutes (1994) show that in competing for consumers, sellers offset the high price for the complementary good with a low price for the primary good. Nevertheless, Shapiro (1995), Hall (1997), and Borenstein, Mackie-Mason and Netz (2000) show that social welfare may not be maximized due to deadweight losses from consumers’ reactions to the overly low price of the primary good and the overly high price of the complementary good. Our paper has a different objective—analyzing the scope for and implications of profitable deception in a competitive market—and shows that deception can lead to additional welfare losses from the systematic sale and pushing of inferior products.

Relatedly, in assuming that consumers can be induced to pay high additional fees once they buy a product, our theory shares a basic premise with the large literature on switching costs. But even if firms cannot commit to ex-post prices and there is a floor on ex-ante prices—so that positive profits obtain in equilibrium—our model’s main insights do not carry over to rational switching-cost models. Most importantly, we are unaware of any rational switching-cost model that predicts the systematic sale of inferior products in competitive markets, and under the natural assumption naive consumers in the Gabaix and Laibson’s (2006) framework can decrease welfare because formerly naive consumers may engage in inefficient substitution of the add-on.
that consumers know or learn product attributes, it is clear that a firm is indeed better off selling a superior product.\footnote{This point is immediate if consumers know the products’ values at the time of original purchase. As an illustration of the same point when consumers learn product values only after initial purchase, suppose that there are two products with values $v_H$ and $v_L$ and costs $c_H$ and $c_L$, respectively, product $H$ is strictly superior ($v_H - c_H > v_L - c_L$), and the switching cost is $k$. Consider a firm who could sell either product to a consumer at an ex-ante price of $p$ followed by an ex-post price of its choice. If the firm sells the superior product, it can charge an ex-post price of $c_H + k$ in a competitive market, so it makes profits of $p + c_H + k - c_H$. If the firm sells the inferior product, then (as the consumer realizes that she can switch to the superior product) it can charge an ex-post price of $c_H + k - (v_H - v_L)$, so it makes profits of $p + c_H + k - (v_H - v_L) - c_L$. It is easy to check that the firm prefers to sell the superior product.}

8 Conclusion

In our model, we have imposed exogenously that firms can unshroud additional prices. An important agenda for future research is identifying the kinds of education that convince naive consumers. This is likely to require a more elaborate understanding of naivete, as well as to open up further questions regarding who is able to influence consumers in equilibrium.

We have also taken the opportunity to deceive consumers—that is, the shroudable additional price component—as exogenous. In most real-world markets, however, someone has to come up with ways to hide prices from consumers, so that the search for deception opportunities can be thought of as a form of innovation. In our companion paper (Heidhues et al. 2012a), we study the incentives for such “exploitative innovation,” and contrast them with the incentives for innovation that benefits consumers. Here too we find a perverse incentive: because learning ways to charge higher additional prices increases the profits from shrouding and thereby lowers the motive to unshroud, a firm may have a strong incentive to make exploitative innovations and have competitors copy them. In contrast, the incentive to make an innovation that increases the product’s value to consumers is zero or negative if competitors can copy the innovation, and even if they cannot the incentive is strong only when the product is socially wasteful. In addition, we show that if the deceptive feature can be copied by competitors, the incentive to come up with deceptive products is stronger when the price floor is binding than when it is not. Since deception often seems based on contract features that can be easily copied, it is therefore likely to be more widespread when the price floor is binding, adding another reason that deception is a greater problem in this case than...
when the price floor is not binding. The possibility of exploitative innovation also adds caution to policies that directly regulate the maximum additional price: such a policy can greatly increase firms’ incentive to make new exploitative innovations, and hence may have a small net effect.

References


Appendices

A Costly Unshrouding

Consider the same game as in Section 2, except that firm \(n\) has to pay \(\eta \geq 0\) to unshroud additional prices.

**Proposition 5** (Unique Equilibrium with Costly Unshrouding). Fix all parameters other than \(\eta\), and suppose that \(f > c_n - \bar{a}\) for all \(n\) and a deceptive equilibrium exists for \(\eta = 0\). Then, for any \(\eta > 0\) there exists a unique equilibrium, and in this equilibrium all firms shroud and offer \((f, \pi)\) with probability one.

Proposition 5 says that if a deceptive equilibrium exists in our basic model, then it is the unique equilibrium in the variant of our model in which unshrouding carries a positive cost, no matter how small the cost is. This provides one justification for our presumption that firms play a deceptive equilibrium whenever it exists.

To see the logic of Proposition 5, suppose that some firm unshrouds with positive probability. Notice that since \(\eta > 0\), in order to unshroud a firm must make positive gross profits afterwards. Hence, no firm unshrouds with probability one—as this would lead to Bertrand-type competition and zero gross profits. Now for each firm that unshrouds with positive probability, take the supremum of the firm’s total price conditional on the firm unshrouding, and consider the highest supremum. At this price, a firm cannot make positive profits if any other firm also unshrouds. Hence, conditional on all other firms shrouding at this price, the firm must make higher profits from unshrouding than from shrouding. But this is impossible: if the firm has an incentive to shroud in this situation with zero unshrouding cost—which is exactly the condition for a deceptive equilibrium to exist—then it strictly prefers to shroud with a positive unshrouding cost.
B Proofs

Proof of Lemma 1. We first show that the sets of Nash-equilibrium outcomes when \( f_n \) is unrestricted and when \( f_n \) is restricted to be non-negative are the same. Suppose first that unshrouding occurs with probability one and \( v - u < c_{\text{min}} \). Then, in both the restricted and unrestricted games consumers buy with probability zero, so the outcomes are the same.

To prove the first statement in the lemma otherwise, we begin by arguing that any equilibrium in the restricted game is an equilibrium in the unrestricted game by establishing that firm \( n \) does not strictly prefer to charge \( f_n < 0 \) when all other firms set non-negative up-front prices. We start with some preliminary observations regarding the consumers’ behavior. Note that a consumer’s charging decision depends only on the cards she accepts, the additional prices of these cards, and whether unshrouding occurs (and, conditional on the set of cards she accepts, not on up-front prices). Note also that if all firms shroud, consumers accept any card for which \( f_n < 0 \) and charge their favorite card \( n' \) among those they accept. When it comes to period 2, a consumer who charged her expenses to card \( n' \) delays repayment to period 3 if \( \beta R_{n'} < 1 \), or \( \beta < 1/R_{n'} \). As a result, firm \( n' \) earns an expected ex post profit of \( Q(1/R_{n'})(R_{n'} - 1) \) from the consumers who charged their expenses to card \( n' \). If unshrouding occurs, consumers accept all cards for which \( f_n < 0 \) if they prefer to accept some card, i.e., if \( \sum_n I_{\{f_n < 0\}} f_n + \min_{n'} \{I_{\{f_{n'} > 0\}} f_{n'} + Q(1/R_{n'})(R_{n'} - 1)\} \leq v - u \), where \( I_{\{\}} \) denotes the indicator function. And if unshrouding occurs and consumers get a card, they charge their expenses to their preferred card among those that minimize \( I_{\{f_{n'} > 0\}} f_{n'} + Q(1/R_{n'})(R_{n'} - 1) \).

Now to see why firm \( n \) does not strictly prefer to set \( f_n < 0 \), let firm \( n \) set \( f_n = 0 \) instead of \( f_n < 0 \) while leaving its total price \( f_n + Q(1/R_n)(R_n - 1) \) and its unshrouding decision the same. Any consumer who gets and charges firm \( n \)’s card with the former prices also does so with the latter prices, while with the latter prices no other consumer gets firm \( n \)’s card. Hence, the latter prices cannot be less profitable.

Next, we prove that any equilibrium in the unrestricted game is an equilibrium in the restricted game by proving that no firm charges \( f_n < 0 \) with positive probability in any Nash equilibrium of the unrestricted game. It follows from our observations about consumer behavior above that if two or more firms set \( f_n < 0 \), each consumer either accepts all of these cards or does not accept any
First, suppose shrouding occurs with positive probability in equilibrium. Then, every firm can ensure positive profits by shrouding and setting $f_n = 0$ and $R_n = \arg\max_R Q(1/R)(R - 1)$, which will lead all consumers whose favorite card is card $n$ to get it and charge it whenever shrouding occurs. Hence, all firms earn positive profits. Notice that if there exist some consumers who accept card $n'$ with $f_{n'} < 0$ but do not charge their expenses to card $n'$, then firm $n'$ can profitably deviate by raising $f_{n'}$ by a bit and lowering $R_{n'}$ while keeping $f_{n'} + Q(1/\bar{R}_{n'})\left(\bar{R}_{n'} - 1\right)$ constant—this ensures that both when shrouding occurs and when unshrouding occurs firm $n'$ loses no consumers who accept and charge card $n'$, and firm $n'$ increases profits from consumers who accept but do not charge card $n'$. Hence, if some consumers accept card $n'$ with $f_{n'} < 0$, all of the consumers charge their expenses to card $n'$.

We now show that at most one firm $n'$ can set a negative up-front price. To see why, suppose otherwise: firm $n'$ and $n''$ set negative up-front prices with positive probability. Since all consumers who accept card $n'$ with $f_{n'} < 0$ charge their expenses to card $n'$ and since each consumer either accepts all cards with negative up-front prices or does not accept any card, all consumers must accept no card with probability one whenever two firms set negative up-front prices. This requires that unshrouding occurs with probability one whenever both firms set negative up-front prices, and hence one firm, say $n'$, must unshroud with probability one whenever it sets a negative up-front price $f_{n'} < 0$. Since conditional on $f_{n'} < 0$ consumers do not accept card $n'$ whenever the other firm $n''$ also sets $f_{n''} < 0$, card $n'$ is not worth accepting for any consumer: $f_{n'} + Q(1/\bar{R}_{n'})(\bar{R}_{n'} - 1) > v - u$ whenever $f_{n'} < 0$. But then, firm $n'$ earns zero profits when setting $f_{n'} < 0$, a contradiction. Hence, at most one firm, say firm $n'$, sets a negative up-front price when shrouding occurs with positive probability.

Take any firm $n \neq n'$ that does not set a negative up-front price. Because all consumers accept only card $n'$ whenever $f_{n'} < 0$, firm $n$ must earn positive expected profits conditional on all firms setting a non-negative up-front price. To do so, firm $n$ cannot with positive probability set $f_{n} > v - u$ when it shrouds because with such an up-front price it will make zero sales both when shrouding occurs and when unshrouding occurs and all firms set non-negative up-front prices. Furthermore,
firm \( n \) cannot with positive probability set \( f_n + Q(1/R_n)(R_n - 1) > v - u \) when it unshrouds as otherwise firm \( n \) makes zero sales when all firms set non-negative up-front prices. Similarly, firm \( n' \) must with probability one set \( f_n' \leq v - u \) conditional on shrouding and \( f_n' + Q(1/R_n')(R_n' - 1) \leq v - u \) conditional on unshrouding. This implies that all consumers with probability one accept a card both when shrouding occurs and when unshrouding occurs, and therefore consumers always accept any card that charges a negative up-front price. Thus, firm \( n' \) that charges a negative up-front price \( f_n' < 0 \) with positive probability can gain by replacing each negative up-front price \( f_n' \) through a higher up-front price \( f_n'/2 \) and keeping its strategy otherwise unchanged. Hence, in any equilibrium all firms charge nonnegative prices with probability one if shrouding occurs with positive probability.

Second, suppose unshrouding occurs with probability one and \( v - u > c_{min} \). It is easy to see that all consumers must accept some card with probability one in this case. As above, all consumers who accept card \( n \) with \( f_n < 0 \) must charge their expenses to card \( n \), and hence with probability one firm \( n \) must set the lowest total price whenever setting \( f_n < 0 \). Using this fact, standard Bertrand-type arguments applied to the total price \( f_n + Q(1/R_n)(R_n - 1) \) establish that in equilibrium at least two most-efficient firms must charge a total price \( f_n + Q(1/R_n)(R_n - 1) = c_{min} \) with probability one, no firm charges a total price strictly below \( c_{min} \) with positive probability, and consumers buy with probability one. But then any firm \( n \) that charges \( f_n < 0 \) attracts some consumers that do not charge their expenditures with the firm; firm \( n \) would be strictly better off when charging the same total expected payment and setting \( f_n \geq 0 \). This completes the argument that no firm sets a negative up-front price in any Nash equilibrium of the unrestricted game, and thereby the proof that the sets of Nash-equilibrium outcomes in the restricted and unrestricted games are the same.

We now move on to the second part of the lemma. Given that \( f_n \geq 0 \) for all \( n \), each consumer gets at most one card, and if she does charges it. As a result, firm \( n \) earns an expected ex post profit of \( a_n \equiv Q(1/R_n)(R_n - 1) \) from all its consumers. The maximum ex-post profit per such consumer the firm can earn is \( \bar{a} = \max_R Q(1/R)(R - 1) \), which exists by assumption. Notice that (i) firm \( n \)'s profit from a consumer is \( f_n + a_n \); (ii) a consumer obtains utility \( v - f_n - a_n \) from card \( n \); (iii) in period 0, consumers expect to receive utility \( v - f_n \) from card \( n \) if shrouding occurs
and $v - f_n - a_n$ if unshrouding occurs. Hence, thinking of the firms as choosing $a_n \in [0, \bar{a}]$ rather than $R_n$ generates the same payoffs as in our reduced-form model, making the games strategically equivalent. Finally, note that consumers’ lexicographic preferences over cards induce exogenously given market shares when consumers are indifferent between cards.

\[ \text{Proof of Lemma 2.} \] If a consumer invests in fund $n$, she receives

\[
-f_n + Q(\hat{a}_n) v(1 - \hat{a}_n) + (1 - Q(\hat{a}_n)) v(1 - \hat{a}_n)^2 = v - f_n - v \left[ a_n + (1 - Q(\hat{a}_n))(1 - \hat{a}_n)\hat{a}_n \right] = a_n
\]

in period 2. It is now easy to see that the payoffs to both parties are the same as in our reduced-form model. First, in a shrouded market investors perceive that they will have $\hat{v} - f_n$ in period 2, whereas in reality they get $v - f_n - a_n$. Second, if a firm unshrouds, investors—who now know how much they will pay in management fees in total—correctly perceive their returns. Third, a firm’s revenue from the consumer is $f_n + a_n$. Fourth, a consumer prefers to take a fund over her outside option if and only if her perceived utility from it is at least $u$. Furthermore, $a_n \leq \max_{\hat{a} \in [0, \hat{A}]} v[\hat{a} + (1 - Q(\hat{a}))(1 - \hat{a})\hat{a}] = \bar{a}$.

\[ \text{Proof of Proposition 1.} \] First, we establish that for any $\psi \in [0, 1]$ there exists an equilibrium with the properties stated in the proposition. Let every firm offer the contract $f_n = c_n - \bar{a}, a_n = \bar{a}$ and unshroud with probability $\gamma$, where $(1 - \gamma)^N = 1 - \psi$. In this equilibrium, every firm makes zero profits independent of whether shrouding occurs or not, and—again independent of whether shrouding occurs or not—no firm can attract customers at a total price above its marginal cost. In what follows, we establish that any equilibrium has the properties specified in the proposition.

Suppose unshrouding occurs with probability $\psi = 1$. Then we have Bertrand competition in the total price, and standard arguments imply that the Bertrand outcome obtains.

We are thus left to consider the case in which unshrouding occurs with probability $\psi < 1$. Note first that no firm makes sales at a total price $f_n + a_n < c_{\min}$ with positive probability in equilibrium because such a firm could profitably deviate by moving all probability mass from total prices strictly below $c_{\min}$ to a total price of $c_{\min}$.

Next, we prove by contradiction that no firm makes sales at a total price $f_n + a_n > c_{\min}$ with positive probability. If this is not the case, then any most-efficient firm, i.e., any firm with
cost $c_n = c_{\text{min}}$, can copy this firm’s strategy and thereby earn positive profits. Hence, any most-efficient firm makes positive profits in equilibrium. Denote by $\pi^*_n > 0$ a most-efficient firm $n$’s equilibrium profits. Let $\bar{f}$ be the supremum of the up-front prices of most-efficient firms conditional on shrouding. Since $f + a \leq c_{\text{min}}$, in order for a most-efficient firm to make profits it must charge $f > f_*$ so that $\bar{f} > f_*$. 

Suppose first that some most-efficient firm $n$ sets the up-front price $\bar{f}$ with positive probability. Denote by $\hat{a}$ the supremum of firm $n$’s additional price conditional on shrouding and setting an up-front price of $\bar{f}$. Then, it cannot be the case that firm $n$ earns positive expected profits when it sets $\bar{f}$ and shrouding occurs: this would only be possible if another most-efficient firm also set $\bar{f}$ with positive probability when shrouding, but then firm $n$ would benefit from minimally undercutting the up-front price. Hence, conditional on charging $(\bar{f}, a_n)$ and shrouding, firm $n$ can earn positive expected profits only when unshrouding occurs and all rivals charge a total price equal to or above $\bar{f} + a_n$ with positive probability. Thus, with positive probability all rivals must charge total prices weakly above $\bar{f} + \hat{a}$. This implies that if firm $n$ charges $\bar{f} + \hat{a}$ with positive probability, then all rivals must charge total prices strictly above $\bar{f} + \hat{a}$ with positive probability; if rivals charged total prices equal to $\bar{f} + \hat{a}$ with positive probability, then firm $n$ would strictly increase profits by minimally lowering its up-front price.

Let $\bar{t}_n'$ be the supremum of the total prices of firm $n'$ conditional on unshrouding. Let $\bar{t} = \max\{\bar{t}_{n'} | c_{n'} = c_{\text{min}}\}$. With slight abuse of notation, say firm $n'$ is the most efficient firm that achieves $\bar{t}$ when unshrouding. First, suppose firm $n'$ sets $\bar{t}$ when unshrouding with positive probability. Because no firm other than $n'$ can set a total price $\bar{t}$ with positive probability (otherwise, firm $n'$ would prefer to minimally lower its total price), firm $n'$ makes zero profits if any other most-efficient firm unshrouds, so firm $n'$ must earn positive profits when all other most-efficient firms shroud. Hence, all most-efficient firms $n'' \neq n'$ must with positive probability set a total price $f_{n''} + a_{n''} > \bar{t}$ when shrouding. Consider the supremum of total prices $\bar{f} + \hat{a}$ of a most-efficient firm when shrouding. A firm $\tilde{n}$ setting this total price with positive probability makes zero profits when a most-efficient firm unshrouds, so it must earn positive profits conditional on all most-efficient firms shrouding, and can do so only when shrouding occurs. Hence, $\bar{f} < \bar{f}$ and $\bar{f}$ is with positive
probability the lowest up-front price. But then the most-efficient firm setting \( \bar{f} \) can profitably deviate by moving the probability mass from \( \bar{f} \) to \( \hat{f} \) in such a way that the total price distribution remains unchanged (which it can do because \( \hat{f} + \hat{a} > \bar{f} + \bar{a} \)), a contradiction. Hence no firm sets \( \hat{f} + \hat{a} \) with positive probability. Let firm \( \tilde{n} \) be a most-efficient firm that achieves the supremum \( \hat{f} + \hat{a} \) when shrouding. Consider a sequence of optimal total prices \( \tilde{f}_k + \tilde{a}_k \) set by firm \( \tilde{n} \) when shrouding that converges to \( \hat{f} + \hat{a} \). Then firm \( \tilde{n} \)'s expected profits conditional on unshrouding occurring go to zero, and hence it must earn positive expected profits conditional on shrouding occurring. This implies that there exists a subsequence \( \tilde{f}_k \) converging to some \( \tilde{f} < \bar{f} \) with the property that \( \tilde{f} \) is with positive probability the lowest up-front price. But then again the most-efficient firm setting \( \bar{f} \) can profitably deviate as above. We conclude that no firm sets \( \bar{f} \) with positive probability.

Next, suppose that no firm sets \( \bar{f} \) with positive probability. Consider a sequence \( t_{n'} \to \bar{f} \) of optimal total prices by firm \( n' \) when unshrouding; then conditional on a most efficient firm unshrouding, firm \( n' \)'s expected profits go to zero as \( t_{n'} \to \bar{f} \); hence, to earn its equilibrium profit level, firm \( n' \)'s expected profits conditional on all most-efficient firms shrouding must be strictly positive and bounded away from zero. Hence, all other most-efficient firms \( n'' \neq n' \) must with positive probability set a total price \( f_{n''} + a_{n''} \geq \bar{f} \) when shrouding. By the same argument as in the preceding paragraph, we arrive at a contradiction. Therefore, we conclude that no firm sets \( \bar{f} \) with positive probability when shrouding.

Suppose, thus, that no most-efficient firm charges \( \bar{f} \) with positive probability when shrouding. Let firm \( n \) be a firm that achieves this supremum. Note that firm \( n \)'s expected profits conditional on shrouding occurring go to zero as \( f_n \to \bar{f} \). Let \( \hat{a} = \limsup_{\epsilon \downarrow 0} \{a| (f, a) \text{ optimal and } f > \bar{f} - \epsilon \} \). Consider a sequence \( (f_n, a_n) \) of optimal prices that converges to \( (\bar{f}, \hat{a}) \). For sufficiently high \( l \), firm \( n \) must make expected profits conditional on unshrouding occurring and all rivals charging a total price at or above \( f_n + a_n \). From now on, we follow the same arguments as the two preceding paragraphs (for the case in which some firm sets \( \bar{f} \) with positive probability when shrouding) to establish (in the above notation) that \( \tilde{f} < \bar{f} \) and \( \tilde{f} \) is with positive probability the lowest up-front price. Then for sufficiently high \( l \), firm \( n \)'s profits when charging an optimal \( f_n, a_n \) and shrouding occurring go to zero; but firm \( n \) could lower the up-front price from \( f_n \) to \( \hat{f} \) and still charge the
same total price $f_n^l + a_n^l$, increasing its expected profits conditional on shrouding occurring and not affecting it otherwise, contradicting the optimality of $f_n^l, a_n^l$. This contradiction establishes that $\bar{f} > \bar{f}$ is impossible.

We have thus established that no firm sells a contract at a total price other than $c_{\text{min}}$. Next, observe that consumers must buy the product at a total price of $c_{\text{min}}$ with probability one if $v - u > c_{\text{min}}$; otherwise, all firms charge a price strictly greater than $c_{\text{min}}$ with positive probability, and then a most-efficient firm can make positive profits, which we showed above is impossible. Note also that if $v - u < c_{\text{min}}$ and unshrouding occurs, then consumers do not buy the product. This implies that if unshrouding occurs, the Bertrand outcome obtains.

Finally, we show that conditional on shrouding occurring, consumers must pay $f = c_{\text{min}} - \bar{a}, a = \bar{a}$ with probability one, and therefore get utility $v - u - c_{\text{min}}$. Since no firm makes sales at total prices below $c_{\text{min}}$ with positive probability, no firm sells at an up-front price below $c_{\text{min}} - \bar{a}$ with positive probability. Now suppose that consumers buy at an up-front price above $c_{\text{min}} - \bar{a}$ with positive probability conditional on shrouding occurring. Then, there exists $\epsilon > 0$ for which consumers buy at an up-front price greater than $c_{\text{min}} - \bar{a} + \epsilon$ conditional on shrouding occurring with positive probability, and a most-efficient firm could profitably deviate through shrouding and offering the contract $f = c_{\text{min}} - \bar{a} + (\epsilon/2), a = \bar{a}$, a contradiction.

Proof of Proposition 2. I. Note first that if some firm unshrouds with probability one, all other firms are indifferent between shrouding and unshrouding. Thus, an equilibrium in which unshrouding occurs with probability one always exists. In any such equilibrium consumers observe and take the additional price into account, so that our game reduces to a standard Betrand game in which the consumers’ willingness to pay is $v - u$. Hence, standard arguments imply that the Bertrand outcome obtains.

Now consider deceptive equilibria, i.e., equilibria in which shrouding occurs with probability one. In case firm $n$ has a positive probability of sales in equilibrium, it must set $a_n = \bar{a}$ as otherwise it could increase its profits conditional on a sale by increasing $a_n$ without affecting the probability of selling. The same argument as in the text establishes that if (SC) holds for all $n$, then there is a deceptive equilibrium in which all firms set $(f_n, \bar{a})$. We now provide a formal argument for why
firms set \( f \) with probability one in any deceptive equilibrium. The proof is akin to a standard Bertrand-competition argument. Take as given that all firms shroud with probability one, and that all firms set the additional price \( \pi \). Note that by setting \( f_n = \underline{f} \), firm \( n \) can guarantee itself a profit of \( s_n(\underline{f} + \pi - c_n) > 0 \). As a result, no firm will set \( f_n > v - \underline{u} \), because then no consumer would buy from it. Take the supremum \( \overline{f} \) of the union of the supports of firms’ up-front price distributions.

We consider two cases. First, suppose that some firm sets \( \overline{f} \) with positive probability. In this case, all firms have to set \( \overline{f} \) with positive probability; otherwise, a firm setting \( \overline{f} \) would have zero market share and hence zero profits with probability one. Then, we must have \( \overline{f} = \underline{f} \); otherwise, a firm could profitably deviate by moving the probability mass to a slightly lower price. Second, suppose that no firm sets \( \overline{f} \) with positive probability. Let firm \( n \)’s price distribution achieve the supremum \( \overline{f} \). Then, as \( f_n \) approaches \( \overline{f} \), firm \( n \)’s expected market share and hence expected profit approaches zero—a contradiction.

We now establish that if the strict inequality

\[
s_n(\underline{f} + \pi - c_n) > v - \underline{u} - c_n
\]

holds for all \( n \), in any equilibrium shrouding occurs either with probability one or zero. Suppose, toward a contradiction, that all firms shroud with probability strictly between 0 and 1. If all competitors shroud with positive probability, firm \( n \) can guarantee itself positive profits by shrouding itself and offering the contract \((\underline{f}, \pi)\), which attracts consumers since \( v - \underline{u} \geq \underline{f} \) and makes positive profits since \( \underline{f} + \pi > c_n \). To earn positive profits when unshrouding, firm \( n \) must set a total price \( t_n \leq v - \underline{u} \). Consider the supremum of the total price \( \hat{t}_n \) set by firm \( n \) when unshrouding, and let \( \hat{t} = \max_n \{\hat{t}_n\} \). Note that there exists at most one firm that sets this price with positive probability; if two or more firms did so, then some firm could increase profits by moving this probability mass to slightly below \( \hat{t} \). First, consider the case in which one firm puts positive probability mass on \( \hat{t} \), and let this firm be firm \( n \). Firm \( n \) earns zero profits conditional on some other firm unshrouding. Conditional on all others shrouding it earns less than

\[
\hat{t} - c_n \leq v - \underline{u} - c_n < s_n(\underline{f} + \pi - c_n),
\]

and hence would increase its profits by deviating from unshrouding and charging the total price \( \hat{t} \) to
shrouding and charging \((f, \bar{a})\). Second, consider the case in which no firm puts positive probability mass on \(\hat{t}\). Let \(n\) be a firm that achieves this supremum. Consider a sequence \(t_k \to \hat{t}\) of optimal total prices by firm \(n\) when unshrouding, and denote the corresponding expected profits conditional on some other firm unshrouding by \(\epsilon_k\). Then, \(\epsilon_k \to 0\). Notice that conditional on all other firms shrouding, firm \(n\) earns at most \(t_k - c_n \leq v - u - c_n\), so that the (unconditional) expected profit of firm \(n\) is less than \(v - u - c_n + \epsilon_k\), which by Condition (3) is strictly less than \(s_n(f + \bar{a} - c_n)\) for a sufficiently small \(\epsilon_k > 0\). Hence, firm \(n\) is strictly better off shrouding and charging \((f, \bar{a})\)—a contradiction. Thus, if Condition (3) holds, unshrouding occurs with probability one or zero in equilibrium.

II. We establish by contradiction that if (SC) is violated for some firm, then in any equilibrium additional prices are unshrouded with probability one. Note again that if unshrouding occurs with probability one, we have Bertrand competition in the total price, and hence the Bertrand outcome obtains. The proof that unshrouding occurs with probability one in this case proceeds in three steps.

Step (i): All firms earn positive profits. If shrouding occurs with positive probability, then firms must earn positive profits: if all competitors shroud the additional prices, firm \(n\) can guarantee itself positive profits by shrouding and offering the contract \((f, \bar{a})\), which attracts consumers since \(v - u \geq f\) and makes positive profits since \(f + \bar{a} > c_n\).

Step (ii): All firms choose the up-front price \(f\) whenever they shroud. Consider the supremum of the total price \(\hat{t}_n\) set by firm \(n\) when unshrouding, and let \(\hat{t} = \max_n \{\hat{t}_n\}\). Note that there exists at most one firm that sets this price with positive probability; if two did, then either could increase profits by moving this probability mass to slightly below \(\hat{t}\). Let \(n\) be the firm that puts positive probability mass on \(\hat{t}\) if such a firm exists; otherwise, let \(n\) be a firm that achieves this supremum. For firm \(n\) to be able to earn its equilibrium profits for prices at or close to \(\hat{t}\), all competitors of \(n\) must set a total price weakly higher than \(\hat{t}\) with positive probability. By the definition of \(\hat{t}\), this means that all competitors of \(n\) charge a total price weakly higher than \(\hat{t}\) with positive probability when shrouding.

First, suppose all firms other than \(n\) set a total price strictly higher than \(\hat{t}\) with positive
probability. Because each firm other than \( n \) makes zero profits when unshrouding occurs, it must make positive profits when shrouding occurs. In addition, since it only makes profits when shrouding occurs, it sets the additional price \( \overline{\alpha} \) with probability one. Take the supremum of firms’ up-front prices \( \bar{f}' \) conditional on the total price being strictly higher than \( \hat{i} \). Because consumers do not buy the product if the up-front price is greater than \( v - \underline{\alpha} \) and firms must earn positive profits by (i), \( \bar{f}' \leq v \). Note that \( \bar{f}' + \overline{\alpha} > \hat{i} \), so \( \overline{\alpha} > \hat{i} - \bar{f}' \).

We now show that \( \bar{f}' = f \) by contradiction. Suppose \( \bar{f}' > f \). If two or more firms set \( \bar{f}' \) with positive probability when shrouding, each of them wants to minimally undercut—a contradiction.

If only one firm \( n' \) sets \( \bar{f}' \) with positive probability, then firm \( n' \) has zero market share both when unshrouding occurs and when shrouding occurs and some firm other than \( n' \) sets a total price strictly greater than \( \hat{t} \). Because firm \( n' \) earns positive profits by (i) and is the only firm that sets \( \bar{f}' \) with positive probability conditional on the total price being strictly higher than \( \hat{t} \), every firm \( n'' \) other than \( n' \) sets its up-front price strictly higher than \( \bar{f}' \) and its total price weakly lower than \( \hat{t} \) when shrouding with positive probability. Suppose first \( n' = n \). Then, there exists a firm \( n'' \neq n \) that shrouds and sets an up-front fee \( f_{n''} > \bar{f}' \), \( a_{n''} \leq \hat{i} - f_{n''} \) with positive probability. Since \( \overline{\alpha} > \hat{i} - \bar{f}' \), firm \( n'' \) can increase its profits by decreasing all prices \( f_{n''} > \bar{f}' \) to \( \bar{f}' \) and by increasing its additional price holding the total price constant—a contradiction. Next, suppose \( n' \neq n \). Then, firm \( n \) shrouds, sets \( f_n > \bar{f}' \) with positive probability and charges an additional price \( a_n \leq \hat{i} - f_n \) with probability one when charging these up-front prices. For almost all of these up-front prices, firm \( n \) must earn strictly positive profits when shrouding occurs; otherwise firm \( n \) could unshroud with probability one and generate positive (and hence higher) profits when all rivals shroud and charge a total price above \( \hat{i} \). Thus, firm \( n' \) shrouds and sets \( f_{n'} \geq f_n > \bar{f}' \), \( a_{n'} \leq \hat{i} - f_{n'} \) with positive probability. Then, firm \( n' \) can increase its profits by decreasing all prices \( f_{n'} > \bar{f}' \) to \( \bar{f}' \) and increasing its additional price holding the total price constant—a contradiction.

If no firm sets \( \bar{f}' \) with positive probability, there exists firm \( n' \) that for any \( \epsilon > 0 \) sets up-front prices in the interval \( (\bar{f}' - \epsilon, \bar{f}') \) with positive probability. As \( \epsilon \rightarrow 0 \), the probability that firm \( n' \) charges the highest up-front price and the total price is strictly higher than \( \hat{i} \) goes to one. Also, by the definition of \( \hat{i} \), firm \( n' \) shrouds when setting these up-front prices. Therefore, the profits
go to zero when unshrouding occurs or when shrouding occurs and some other firm sets a total price strictly greater than \( \hat{t} \). Now follow the same steps as in the previous paragraph to derive a contradiction. Thus, we have established that \( \bar{f}' = \bar{f} \).

Because \( \bar{f}' = \bar{f} \), each firm \( n' \neq n \) sets an up-front price of \( \bar{f} \) with probability one conditional on its total price being strictly higher than \( \hat{t} \). Hence, \( \bar{f} + \bar{a} \geq \hat{t} \). We now show that whenever shrouding, any firm \( n' \neq n \) does not set up-front prices strictly above \( \bar{f} \) with positive probability. Suppose by contradiction that firm \( n' \) sets prices above \( \bar{f} \) with positive probability when shrouding. As \( n' \) sets \( \bar{f} \) with probability one when charging a total price strictly above \( \hat{t} \), the associated additional price must almost always satisfy \( a_{n'} \leq \hat{t} - f_{n'} \) when shrouding and setting the up-front price strictly above \( \bar{f} \). Since \( n' \) sets up-front prices strictly above \( \bar{f} \) with positive probability when shrouding, there exists an up-front price \( g' > \bar{f} \) such that firm \( n' \) sets prices above \( g' \) with positive probability.

There cannot be a competitor whose up-front price when shrouding falls on the interval \( [\bar{f}, g'] \) with positive probability; otherwise, firm \( n' \) could increase its profits by decreasing all prices above \( g' \) to \( \bar{f} \) and by increasing its additional price holding the total price constant. Note that by the first sentence of this paragraph, firm \( n' \) sets \( \bar{f} \) with positive probability when shrouding. But then, firm \( n' \) can raise its up-front price from \( \bar{f} \) to \( g' \) and increase profits—a contradiction. Thus, any firm \( n' \neq n \) sets the up-front price \( \bar{f} \) with probability one when shrouding.

Now suppose that firm \( n \) charges an up-front price strictly above \( \bar{f} \) when shrouding with positive probability. Then it can only earn profits when unshrouding occurs and hence must almost always charge a total price less than or equal to \( \hat{t} \) when shrouding and setting an up-front price strictly greater than \( \bar{f} \). But if it unshrouds and sets the same prices, it would also earn profits when all rivals shroud and set a price above \( \hat{t} \), thereby strictly increasing its profits—a contradiction. Hence firm \( n \) also must set \( \bar{f} \) with probability one when shrouding.

Second, suppose some firm \( n' \neq n \) sets its total price equal to \( \hat{t} \) with positive probability. Then, by the above argument no other firms set total price \( \hat{t} \) with positive probability. Take the supremum of firms’ up-front prices \( \bar{f}' \) conditional on the total price being greater than or equal to \( \hat{t} \). The remainder of the proof is the same as above.

*Step (iii): Additional prices are unshrouded with probability one.* Suppose not. Then, each firm
chooses to shroud with positive probability. Take the infimum of total prices $t$ set by any firm when shrouding. We consider two cases. First, suppose $t \leq v - u$. Take a firm that achieves the infimum. By (i), this firm earns positive profits. For any $\epsilon > 0$, take total prices below $t + \epsilon$ of the firm. By unshrouding and setting $t - \epsilon$ instead, the firm decreases its profits by at most $2\epsilon$ when one or more other firms unshroud, but (since by (ii) all firms set an up-front price of $f$ when shrouding) discretely increases its market share if all other firms shroud. Hence, for sufficiently small $\epsilon > 0$ this is a profitable deviation—a contradiction. Second, suppose $t > v - u$. Take firm $n$ that violates (SC). By (ii), firm $n$ charges the up-front price $f$ whenever it shrouds. Note that firm $n$’s profits are zero when a rival unshrouds, and its profits are at most $s_n(f + \bar{a} - c_n)$ when shrouding occurs. But then, deviating and setting a total price equal to $v$ is profitable because conditional on others shrouding firm $n$ would earn $v - u - c_n > s_n(f + \bar{a} - c_n)$.

Proof of Corollary 2. Since $f \leq \tilde{v} - u$ and $\bar{a} \leq v - \tilde{v}$, we have $f + \bar{a} \leq v - u$, which implies that (SC) is violated.

Proof of Proposition 3. First, if unshrouding occurs with probability one, all consumers are sophisticated and standard (Bertrand-competition) arguments imply that they buy a total-surplus-maximizing product at marginal cost.

Now suppose, toward a contradiction, that shrouding occurs with positive probability strictly less than one. Our proof that this is impossible has four steps. In Step (i) we show that if shrouding occurs with positive probability, firms earn positive profits. Step (ii), which is contained in Lemma 3, establishes that every firm sets an up-front price $f_n = f$ when shrouding. Since $v - f > w - c_{w_{\min}}$, Step (ii) implies that naive consumers buy the inferior product whenever shrouding occurs. Step (iii) uses Bertrand-competition-type arguments to show that sophisticated consumers always buy a total-surplus-maximizing product at marginal cost. This implies (Step (iv)) that firms can only earn profits if shrouding occurs, and hence all firms shroud with probability one, a contradiction.

We shall refer to the up-front price $f_n$ of the inferior product and the total price $t^w_n$ of the superior product as the perceived price (when shrouding) below. Also, if a firm does not offer a contract for the superior product we define $t^w_n = \infty$, and if it does not offer the inferior one we define $f_n = \infty$ and $a_n = 0$. 49
Step (i): Since shrouding occurs with positive probability, any firm can guarantee itself strictly positive profits through shrouding and offering the contract \((\bar{f}, \bar{a})\) for the inferior product and offering the superior product at or above marginal cost. In this case, the price floor ensures that no rival can offer a contract for the inferior good that naive consumers perceive as being strictly better, and naive consumers buy the superior good only if it is priced below the lowest marginal cost, and such purchase cannot occur with positive probability in equilibrium. Thus, all firms must earn positive expected profits in equilibrium. Let \(\pi^*_n > 0\) be firm \(n\)'s equilibrium expected profits.

Step (ii): Conditional on firm \(n\) shrouding, define the perceived deal offered to naive consumers as \(\tilde{d}_n = \max\{v - f_n, w - t^w_n\}\). If shrouding occurs, naive consumers choose a best such deal. Let \(\tilde{d}_n\) be the infimum of \(\tilde{d}_n\) conditional on shrouding, and let \(\tilde{d} = \min_n\{\tilde{d}_n\}\). A key step in our proof is the following:

Lemma 3. \(\tilde{d} = v - f\).

Proof of Lemma 3. Suppose, toward a contradiction, that \(\tilde{d} \neq v - f\). Since no firm can set an up-front price below the floor, \(\tilde{d} < v - f\). We begin by showing that there is a firm \(n'\) such that for any \(\epsilon > 0\), firm \(n'\) has an optimal action that involves shrouding and giving perceived deal \(\tilde{d}_{n'}(\epsilon)\) such that \(0 \leq \tilde{d}_{n'}(\epsilon) - \tilde{d} < \epsilon\) and the expected profits of firm \(n'\) when choosing this action and shrouding occurring is less than \(\epsilon\). Note that if a firm makes strictly greater expected profit on a sophisticated consumer than on a naive consumer conditional on shrouding, then it strictly prefers to unshroud, a contradiction. Hence, it is sufficient to prove our statement for naive consumers. This is trivial from the definition of \(\tilde{d}\) if no firm sets \(\tilde{d}\) with positive probability when shrouding, or exactly one firm sets \(\tilde{d}\) with positive probability when shrouding. Suppose, therefore, that two or more firms set \(\tilde{d}\) with positive probability when shrouding. Let \(n'\) be one of these firms. We will show that if firm \(n'\) earns positive expected profits from this action conditional on shrouding occurring, then it has a profitable deviation. If it earns positive expected profits on the inferior product only, lowering \(f_{n'}\) minimally while still shrouding and holding other prices constant is a profitable deviation; if it earns positive expected profits on both the inferior and the superior product, then lowering \(f_{n'}\) and \(t^w_{n'}\) minimally by the same amount is a profitable deviation; and if it earns positive expected profits on the superior product only, then unshrouding and lowering \(t^w_{n'}\),
minimally is a profitable deviation. This concludes our argument that a firm \( n' \) with the above properties exists. Denote a deal offered from a firm \( n \) to sophisticated consumers, or its “real deal,” by \( d_n = \max\{w - t_n, v - f_n - a_n\} \). For future reference, note two facts: (i) our undercutting argument implies that if firm \( n' \) chooses \( \tilde{d} \) and shrouds, conditional on shrouding occurring it earns zero profits; and (ii) we can choose sequences \( \epsilon' \to 0 \) and \( \tilde{d}_n(\epsilon') \) such that the corresponding real deals converge; denote the limit by \( d(\tilde{d}) \).

For a sufficiently small \( \epsilon > 0 \), the optimal action in which firm \( n' \) sets the perceived deal \( \tilde{d}_n(\epsilon) \) earns less than \( \epsilon \), so firm \( n' \) must earn \( \pi_n^* - \epsilon \) conditional on unshrouding occurring. Now for it to be the case that firm \( n' \) makes \( \pi_n^* - \epsilon \) when unshrouding occurs but at most \( \epsilon \) when shrouding occurs, there must exist a firm \( n'' \neq n' \) and an \( \epsilon' > 0 \) such that firm \( n'' \) offers a strictly better real deal than the real deal associated with \( \tilde{d}_n(\epsilon) \) with probability of at least \( 1 - \epsilon' \) when it shrouds but offers a worse real deal than that associated with \( \tilde{d}_n(\epsilon) \) with positive probability when it unshrouds. Furthermore, as \( \epsilon \to 0 \), we must have \( \epsilon' \to 0 \), and the probability with which firm \( n'' \) offers a worse real deal than that associated with \( \tilde{d}_n(\epsilon) \) when it unshrouds remains bounded away from zero. This implies that there is a firm \( n'' \) such that firm \( n'' \) offers a weakly better real deal than \( d(\tilde{d}) \) with probability \( 1 \) when it shrouds but offers a weakly worse real deal than \( d(\tilde{d}) \) with positive probability when it unshrouds.

For each firm \( n \) that unshrouds with positive probability, denote the infimum of real deals offered by firm \( n \) to consumers conditional on unshrouding by \( d_n \). Let \( d \) be the minimum of these real deals. Note that for a sufficiently small \( \epsilon > 0 \), the real deal associated with \( \tilde{d}_n(\epsilon) \) is strictly better than \( d \) as conditional on unshrouding occurring firm \( n' \) must make positive profits when offering the perceived deal \( \tilde{d}_n(\epsilon) \). This implies that \( d(\tilde{d}) \) is a weakly better deal than \( d \).

First, suppose that some firm offers \( d \) with positive probability when unshrouding. Observe that two firms cannot offer this deal with positive probability, as otherwise a firm would benefit from offering a minimally better deal since it must make positive expected profits when offering \( d \). We argue that if there is a single firm that offers \( d \) with positive probability, it must be firm \( n'' \). Suppose firm \( n''' \neq n'' \) offers \( d \) with positive probability when unshrouding. Then, conditional on shrouding firm \( n'' \) cannot offer a real deal equal to \( d \) with positive probability; otherwise, firm \( n'' \)
would prefer to undercut. Hence, using that firm \( n'' \) offers a weakly better real deal than \( d(\tilde{d}) \geq d \) with probability 1 when it shrouds, we conclude that firm \( n'' \) offers a strictly better real deal than \( d \) with probability 1 when it shrouds. This implies that firm \( n''' \) makes zero profits conditional on firm \( n'' \) shrouding. In addition, firm \( n''' \) makes zero profits conditional on firm \( n'' \) unshrouding as \( \tilde{d} \leq d_{n''} \), and both cannot be set with positive probability—a contradiction. This completes the argument that firm \( n'' \) must offer \( d \) with positive probability.

Again using that it cannot be the case that two firms set \( \tilde{d} \) with positive probability, all rivals of firm \( n'' \), including firm \( n' \), must with positive probability offer a strictly worse real deal than \( \tilde{d} \) conditional on shrouding. Choose an optimal such offer, and denote the real deal by \( d' < \tilde{d} \) and the perceived deal by \( \tilde{d}' \). Because \( d' < \tilde{d} \), when offering this deal firm \( n' \) earns zero expected profits conditional on unshrouding occurring, and hence it must earn more than \( \pi^*_{n'} \) conditional on shrouding occurring. Using the fact from above that firm \( n' \) earns zero profits if it sets \( \tilde{d}' \) and shrouding occurs, this implies that the perceived deal \( \tilde{d}' \) must be strictly better than \( \tilde{d} \). Also when shrouding occurs and firm \( n' \) makes this offer, firm \( n' \) cannot attract sophisticated consumers because firm \( n'' \) offers a better deal than \( d(\tilde{d}) \geq d > d' \) with probability 1 when shrouding. Thus, firm \( n' \) attracts only naive consumers when offering \( \tilde{d}' \). Notice that firm \( n' \) cannot attract only naive consumers to product \( w \) as naive consumers evaluate product \( w \) in the same way as sophisticated consumers do and evaluate the inferior one as a (weakly) better deal than sophisticated ones do. Hence, when firm \( n' \) makes the above offer, naive consumers buy product \( v \) from firm \( n' \) with positive probability. Now choose another optimal offer by firm \( n' \) in which it shrouds, offers a perceived deal \( \tilde{d} < \tilde{d}' \), makes lower profits conditional on shrouding occurring, and earns positive profits conditional on unshrouding occurring (this is possible because \( \tilde{d}' > \tilde{d} \)). Denote this offer by \( \hat{w}(\hat{f}, \hat{a}) \). The fact that firm \( n' \) earns positive expected profits conditional on unshrouding occurring means that the corresponding real deal \( \hat{d} \) must be weakly better than \( \tilde{d} \), so that \( \hat{d} > d' \). Recall that firm \( n' \) must with positive probability attract naive consumers to the inferior product when offering \( \tilde{d}' \). For the deal \( \tilde{d}' \) to be worse but at the same time to be perceived as better than
\( \hat{d} \), it must be that

\[
\begin{align*}
v - f' & > \max\{w - \hat{t}^w, v - \hat{f}\}, \\
v - f' - a' & < \max\{w - \hat{t}^w, v - \hat{f} - \hat{a}\},
\end{align*}
\]

where \((f', a')\) is a contract offer that corresponds to the perceived deal \( \tilde{d}' \). Since firm \( n' \) must attract naive consumers with positive probability when offering \( \tilde{d}' \), it is strictly better off changing the perceived deal \( \hat{d} \) to \( \tilde{d}' \) while holding the real deal constant. To complete the argument, we show that this deviation is feasible. In case \( f' + a' \geq \hat{f} + \hat{a} \), firm \( n' \) can do so by lowering \( \hat{f} \) to \( f' \) and raising the additional price so as to keep the total price of the inferior product fixed. If \( f' + a' < \hat{f} + \hat{a} \), then the second inequality above implies that \( \max\{w - \hat{t}^w, v - \hat{f} - \hat{a}\} = w - \hat{t}^w \) and hence firm \( n' \) can do so by changing \((\hat{f}, \hat{a})\) to \((f', a')\), thereby improving the perceived deal without affecting the real deal \( w - \hat{t}^w \) it offers to sophisticated consumers as well as naive consumers when unshrouding occurs. We thus have a contradiction.

Second, suppose that no firm offers \( d \) with positive probability. Note that in this case \( d(\tilde{d}) > \tilde{d} \); otherwise, for a sufficiently small \( \epsilon > 0 \) firm \( n' \) could not be making \( \pi^*_n \), when offering the perceived deal \( \hat{d}_{n'}(\epsilon) \), as it would make lower profits both when shrouding occurs and when unshrouding occurs. By the following argument, which mimics the one above, \( n'' \) must achieve the infimum \( \tilde{d} \).

To see it, note that if a firm \( n''' \neq n'' \) achieves this infimum, then since \( n'' \) offers a weakly better deal than \( d(\tilde{d}) > \tilde{d} \) when shrouding, firm \( n''' \) earns zero profits conditional on \( n'' \) shrouding; and as the real deals of \( n''' \) approach \( \tilde{d} \) the probability of \( n'' \) offering a better real deal when unshrouding goes to 1, so \( n''' \) expected profits conditional on \( n'' \) unshrouding go to zero. This contradicts that firm \( n''' \) must earn \( \pi^*_n > 0 \) for almost all offers. Hence, \( n'' \) achieves the infimum \( \tilde{d} \).

Take a sequence of real deals \( d_{n''}^l \rightarrow \tilde{d} \) that are optimal for firm \( n'' \) when unshrouding. Then, the expected profits firm \( n'' \) earns from unshrouding and choosing \( d_{n''}^l \) when firm \( n' \) unshrouds approach zero as \( l \rightarrow \infty \). Hence, it must be the case that conditional on shrouding, firm \( n' \) charges a weakly worse deal than \( \tilde{d} \) with positive probability. Choose an optimal such offer, and denote the real deal by \( d' \leq \tilde{d} \) and the perceived deal by \( \tilde{d}' \). From here, the logic is essentially the same as when a single firm charges \( \tilde{d} \) with positive probability, but we repeat a large part of it because there are minor changes. Because \( d' \leq \tilde{d} \) and no firm charges \( \tilde{d} \) with positive probability when
unshrouding, when offering this deal firm \( n' \) earns zero expected profits conditional on unshrouding occurring, and hence it must earn at least \( \pi^*_{n'} > 0 \) conditional on shrouding occurring. Using the fact from above that firm \( n' \) earns zero profits if it sets \( \tilde{d} \) and shrouding occurs, this implies that the perceived deal \( \tilde{d}' \) must be strictly better than \( \tilde{d} \). Also when shrouding occurs and firm \( n' \) makes this offer, firm \( n' \) cannot attract sophisticated consumers because firm \( n'' \) offers a weakly better deal than \( d(\tilde{d}) \) with probability 1 when shrouding, and \( d(\tilde{d}) > d > d' \). Thus, firm \( n' \) attracts only naive consumers when offering \( \tilde{d}' \). Notice that firm \( n' \) cannot attract only naive consumers to product \( w \) as naive consumers evaluate product \( w \) in the same way as sophisticated consumers do and evaluate the inferior one as a (weakly) better deal than sophisticated ones do. Hence, when firm \( n' \) makes the above offer, naive consumers buy product \( v \) from firm \( n' \) with positive probability. Now choose another optimal offer by firm \( n' \) in which it shrouds, offers a perceived deal \( \hat{\tilde{d}} < \tilde{d}' \), makes lower profits conditional on shrouding occurring, and earns positive profits conditional on unshrouding occurring (this is possible because \( \tilde{d}' > \tilde{d} \)). Denote this offer by \( \hat{\tilde{d}}' \), \((\hat{\tilde{f}}, \hat{\tilde{a}})\). The fact that firm \( n' \) earns positive expected profits conditional on unshrouding occurring means that the corresponding real deal \( \hat{d} \) must be strictly better than \( d \), so that \( \hat{d} > d' \). From here, the argument is exactly the same as when a single firm charges \( d \) with positive probability. This completes the proof of the lemma.

Lemma Q.E.D.

Hence, we conclude that \( \tilde{d} = v - \tilde{f} \), and therefore all firms set an up-front price \( \tilde{f} \) for the inferior product with probability one conditional on shrouding and all naive consumers buy the inferior product when shrouding occurs (since \( v - \tilde{f} > w - c_{\text{min}}^w \) and no firm in equilibrium sells the superior product at a price \( t_{\text{min}}^w < c_{\text{min}}^w \) with positive probability).

Step (iii): We next establish that there exists a firm \( n \) that offers a deal in which \( d_n = w - c_{\text{min}}^w \) with probability one. Suppose otherwise. Then there exists a real deal \( d^0 \) at which a most-efficient firm, i.e., a firm with cost \( c_{\text{min}}^w \) for the superior product, earns positive expected profits \( \pi^0 \) from selling to sophisticated consumers. Let \( \underline{d}_n \) be the infimum of the real deals set by firm \( n \) when either shrouding or unshrouding. Let \( d = \min_n \{ d_n | c_n^w = c_{\text{min}}^w \} \). First, if multiple firms offer \( \underline{d} \) with positive probability, a most-efficient firm that does so with positive probability when unshrouding must sell at \( \underline{d} \) with positive probability, and hence prefers to minimally raise \( \underline{d} \). If a most-efficient
firm sets \( d \) with positive probability when shrouding, it can minimally raise \( d \) keeping the price for the inferior product \((f, a_n)\) as well as its shrouding decision fixed; this increases its profits from sophisticated consumers without affecting the behavior of naive consumers when shrouding occurs, and at most minimally decreases its profits from selling to naive consumers when unshrouding occurs. Second, if a single most-efficient firm charges \( d \), it must be shrouding to earn positive profits. But then keeping the price for the inferior product \((f, a_n)\) as well as its shrouding decision fixed and at the same time offering the deal \( d^0 \) is a profitable deviation, because it does not affect the profits from selling to naive consumers when shrouding occurs, strictly increases the profits from selling to sophisticated consumers, and weakly increases the profits from selling to naive consumers if unshrouding occurs. If no most-efficient firm offers the deal \( d \) with positive probability, choose a most-efficient firm that achieves this infimum. Consider a sequence \( d^l \) of optimal real deals offered by this firm that converges to \( d \). For sufficiently large \( l \), the firm earns less than \( \pi^0 > 0 \) from sophisticated consumers. Hence, the firm must shroud when making these offers. But then again keeping the price for the inferior product \((f, a_n)\) as well as its shrouding decision fixed, and at the same time offering the deal \( d^0 \) is a profitable deviation. Hence, we conclude that there exists a firm \( n \) that offers a deal in which \( d_n = w - c_{\min}^w \) with probability one.

**Step (iv):** Since no firm can offer more than the maximal total surplus \( w - c_{\min}^w \) without making a loss, and only a most-efficient firm can make an offer in which \( d_n = w - c_{\min}^w \) without making a loss, with probability one all sophisticated consumers buy from a most-efficient firm. Furthermore, because no firm can earn positive profits if unshrouding occurs, every firm shrouds with probability one, a contradiction.

We have therefore established that shrouding cannot occur with an interior probability. To complete the proof of the proposition, we establish properties of a deceptive equilibrium. Step (i) still applies, so all firms earn positive expected profits. If shrouding occurs with probability one, then the same steps as in the first paragraph of the proof of Lemma 3 imply that if \( d < v - f \), there is a firm and an optimal action that does not earn the firm’s equilibrium profits, a contradiction. Hence, again all firms set an up-front price \( f \) for the inferior product with probability one and all naive consumers buy the inferior product. Now the exact same proof as in Step (iii) above implies
that there is a firm that offers a real deal $w - c_{\text{min}}^w$ with probability one. Furthermore, to earn positive profits, every firm must set a real deal for the inferior product that is strictly worse than $w - c_{\text{min}}^w$. Hence, only naive consumers buy the inferior product. Because every firm must attract some naive consumers to earn positive profits, every firm sets $f_n = \underline{f}$, and, since firm $n$’s market share is independent of $a_n$, every firm sets $a_n = \bar{a}$. Hence, any deceptive equilibrium must have the properties in the proposition. Finally, if at least two most-efficient firms charge $c_{\text{min}}^w$ for product $w$, then obviously no firm wants to deviate, so that a deceptive equilibrium indeed exists. This completes the proof of the proposition.

Proof of Proposition 4. We confirm that if $M \leq s_n(f + \bar{\pi} - c_n)$, then the equilibrium described in the proposition exists. By the sorting condition naive consumers buy product $v$. Furthermore, because $w$ is superior, we have $w - c_{n}^w \geq v - c_n$, and the condition on $M$ implies $c_{\text{min}}^w - c_n^w < f + \bar{\pi} - c_n$. Subtracting these inequalities gives $w - c_{\text{min}}^w > v - (f + \bar{\pi})$, so sophisticated consumers prefer product $w$, and buy since $w - c_{\text{min}}^w > 0$ by assumption. Independently of whether shrouding occurs or not, clearly no firm can benefit from changing its prices. If any firm other than $n$ unshrouds, it makes zero profits, so it does not unshroud. Finally, firm $n$ makes a profit of $M$ on every sophisticated consumer whether or not it unshrouds, so its unshrouding decision hinges on profits from naive consumers. If firm $n$ shrouds, it makes $s_n(f + \bar{\pi} - c_n)$ on the pool of naive consumers, and if it unshrouds, it can make exactly $M$. Hence, it does not prefer to unshroud.

By the converse of the above argument, if $M > s_n(f + \bar{\pi} - c_n)$, then firm $n$ strictly prefers to deviate from the candidate deceptive equilibrium in the proposition, so such an equilibrium does not exist.

Proof of Proposition 5. Recall that a deceptive equilibrium exists for $\eta = 0$ if and only if for all firms $n$,

$$s_n(f + \bar{\pi} - c_n) \geq v - \underline{u} - c_n.$$

This proof has five steps.

Step (i): No firm unshrouds the additional price with probability one. If a firm unshrouds with probability one, all consumers become sophisticated and hence buy from the firm with the
lowest total price $f + a$. Hence by standard Bertrand arguments, all consumers buy at a total price $f + a = c_{\text{min}}$ and no firm makes positive profits from selling to the consumers excluding the unshrouded cost. Then, the firm that chooses to unshroud makes negative profits—a contradiction.

**Step (ii): All firms earn positive profits.** According to (i), in any equilibrium there is positive probability that no firm unshrouds. Then, each firm $n$ can earn positive profits by shrouding the additional prices and offering $(f, \bar{a})$.

**Step (iii): The distributions of total prices are bounded from above.** Suppose firm $n$ sets the total price $f_n + a_n > v - u + \bar{a}$ with positive probability in equilibrium. When the additional prices are shrouded, consumers never buy the product from firm $n$ because this inequality implies $f_n > v - u$. When the additional prices are unshrouded, consumers never buy from firm $n$ because $f_n + a_n > v - u$. Firm $n$’s profits in this case is at most zero, a contradiction with (ii).

**Step (iv): No firm unshrouds the additional price with positive probability.** Let $\hat{t}_n$ be the supremum of the equilibrium total-price distribution of firm $n$ when unshrouding; set $\hat{t}_n = 0$ in case firm $n$ does not unshroud. Let $\hat{t} = \max_n \{\hat{t}_n\}$; by (iii), $\hat{t}$ is bounded from above and hence well-defined. Consider firm $n$ that unshrouds and for whom $\hat{t}_n = \hat{t}$. Note that in any equilibrium in which some firm unshrouds with positive probability, $\hat{t} > c_n$ by (ii).

First, suppose that firm $n$ charges the total price $\hat{t}$ with positive probability. If some other firm $n' \neq n$ also sets the total price $\hat{t}$ with positive probability, then firm $n$ has an incentive to slightly decrease its total price—a contradiction. Thus, only firm $n$ charges the total price $\hat{t}$ with positive probability. Because $\hat{t}$ is the supremum of the total-price distribution conditional on unshrouding, firm $n$ can earn positive profits only if all firms other than $n$ choose to shroud. Conditional on all other firms shrouding, $n$’s expected profits are no larger than $v - u - c_n - \eta$, because the additional price is unshrouded by firm $n$ and hence consumers never buy the product from firm $n$ if $\hat{t} > v - u$.

When firm $n$ shrouds and offers $(f, \bar{a})$, however, its profits conditional on all other firms shrouding are at least $s_n(f + \bar{a} - c_n)$. Thus, the equilibrium condition $s_n(f + \bar{a} - c_n) \geq v - u - c_n$ implies that deviating by shrouding and offering $(f, \bar{a})$ is not profitable—a contradiction.

Second, suppose that firm $n$ does not charge the total price $\hat{t}$ with positive probability. Then, for any $\varepsilon > 0$, firm $n$ charges a total price in the interval $(\hat{t} - \varepsilon, \hat{t})$ with positive probability. As $\varepsilon \to 0$, the
probability that firm $n$ conditional on some other firm unshrouding can attract consumers goes to zero, because $\hat{t}$ is the supremum of the total-price distribution conditional on unshrouding. Hence, firm $n$ cannot earn the unshrouding cost $\eta > 0$ conditional on some other firm unshrouding—i.e., it loses money in expectation relative to shrouding and offering $(f, \pi)$. In addition, conditional on all other firms shrouding firm $n$ earns less than $v - u - c_n$: the deviation profits in the no-unshrouding-cost case. Because shrouding is an equilibrium in the no-unshrouding-cost case, there is a profitable deviation for firm $n$—a contradiction.

Step (v): All firms offer the contract $(f, \pi)$ with probability one. By (iv), all firms choose to shroud with probability one. By the exact same argument as in Proposition 2 for the case in which shrouding occurs with probability one, all firms offer the contract $(f, \pi)$ with probability one. □